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# Molecular Design of Chemical Compounds with Prescribed Properties from QSAR Models Containing the Hosoya Index 

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# Molecular Design of Chemical Compounds with Prescribed Properties from QSAR Models Containing the Hosoya Index ${ }^{\#}$ 

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#### Abstract

A problem of computer generation of chemical structures with given property values is considered. It is supposed that there exists a structure-property correlation equation containing the Hosoya index and the numbers of different atom types. Such an equation allows us to reduce the aforementioned problem to the task of generation of chemical graphs containing definite structural fragments. An algorithm for solving this problem and examples of its application are considered.


Keywords. Quantitative structure-property relationships; chemical structure reconstruction; inverse problem in QSPR/QSAR; Hosoya index; topological indices.

## 1 INTRODUCTION

The studies of quantitative structure-property relationships (QSPR) can be considered as one of the important goals of modern theoretical/mathematical chemistry. The structure of molecules in QSPR studies is usually described in terms of topological, electronic, steric and other characteristics, and the property is correlated with these parameters for some training set of compounds. The main goal of constructing the correlating equations is to estimate the properties of the compounds not belonging to the training set in order to select structures for further synthesis of compounds with required properties.

The so-called inverse problem (IP) consists in the exhaustive generation of chemical structures of some class with a prescribed value of some property. The generation process is based on the preliminary constructed QSPR equation, which usually can be written as $y=f\left(x_{1}, \ldots, x_{N}\right)$, where $y$ is the property value, and $x_{1}, \ldots, x_{N}$ are some topological indices. The IP in this case can be reduced to

[^0]the procedure of generation of structures with definite values of parameters $x_{1}, \ldots, x_{N}$ which are determined from the correlation equation at the condition $y_{1} \leq y \leq y_{2}$.

Earlier the IP for correlation equations containing only one particular topological index (Randić, Wiener, centric Balaban index, etc.) or several indices of a special type (kappa-indices of molecular shape, information topological indices) were considered [1-11]. However, other topological descriptors are also widely used in QSPR [12]. Thus, a development of algorithms for solving the IP for other indices and for multiple linear regression is of great importance in practical QSPR studies [13].

One of the well known molecular descriptors used in these studies is the Hosoya index $Z$ defined as:

$$
\begin{equation*}
Z=\sum_{k=0}^{[n / 2]} p_{k} \tag{1}
\end{equation*}
$$

where $p_{k}$ is the number of subgraphs consisting of $k$ non-adjacent edges in a molecular graph $G$ representing a chemical structure, $p_{0}=1, n$ is the number of vertices in $G[14,15]$. The index $Z$ can be also defined by the formula:

$$
Z=i^{-n} M(i),
$$

where $M(x)=\sum_{k=0}^{[n / 2]}(-1)^{k} p_{k} x^{n-2 k}$ is the matching (or acyclic) polynomial of a graph $G$, and $i$ is the imaginary unit [16]. There are some extensions of index $Z$ to weighted graphs [17-21]. The Hosoya index has been used in correlations of thermochemical properties of alkanes, bond orders and $\pi-$ electron energies of conjugated hydrocarbons [22,23]. Also some linear correlations between boiling points and parameters $Z$ and $P=\prod_{i} v_{i}\left(v_{i}\right.$ is the degree of the $i$-th vertex in graph $\left.G\right)$ were found [12]. Note that for acyclic graphs the Hosoya index can be expressed via the coefficients of a characteristic polynomial of $G$ [14].

In the present paper an algorithm for solving the IP for the case of correlation equation

$$
\begin{equation*}
y=f\left(Z, n, n_{1}, n_{2}, n_{3}, n_{4}\right) \tag{2}
\end{equation*}
$$

where $n_{i}$ is the number of vertices of degree $i(i=1,2,3,4)$ in a graph is considered. It is assumed that a molecular graph $G$ is a simple one (without multiple edges and labeled vertices) and has no vertices of degree exceeding four. It is supposed that the Hosoya index can be expressed from Eq. (2), that is:

$$
\begin{equation*}
Z=f_{1}\left(y, n, n_{1}, n_{2}, n_{3}, n_{4}\right) \tag{3}
\end{equation*}
$$

for some function $f_{1}$. Evidently, the index $Z$ does not take into account heteroatoms and multiple bonds in molecules. However, the suggested algorithm may be used for such compounds if there
exists a sufficiently good correlation (2) for a particular case.
Some examples of solving the IP both for hydrocarbons and for molecules containing heteroatoms are given.

## 2 THEORETICAL RESULTS

Here we will present some graph-theoretical results concerning the numbers $p_{k}$ of $k$-matching in a graph $G(k \geq 1)$, obtained by us. Suppose that $G$ is a simple connected graph with vertex degrees not exceeding four. It is known [24] that the number of edges $p_{1}$ can be found by the following formula:

$$
\begin{equation*}
p_{1}=\frac{1}{2} \sum_{i=1}^{4} i \cdot n_{i} . \tag{4}
\end{equation*}
$$

## Theorem 1.

$$
\begin{equation*}
p_{2}=\frac{1}{2} p_{1}+\frac{1}{2} p_{1}^{2}-\frac{1}{2} \sum_{i=1}^{4} i^{2} \cdot n_{i} . \tag{5}
\end{equation*}
$$

Proof. Let us select an arbitrary edge $\gamma$ in graph $G$ and consider a graph $G_{1}$ obtained from $G$ by deleting the selected edge $\gamma$ and all edges adjacent to it. Denote degrees of vertices incident to $\gamma$ by $v_{1}(\gamma)$ and $v_{2}(\gamma)$. Then the number of edges in graph $G_{1}$ is equal to $p_{1}-\left(v_{1}(\gamma)+v_{2}(\gamma)-1\right)$. Denote by $p_{2 \mid \gamma}$ the number of pairs of non-adjacent edges in $G$, one of which is $\gamma$. Then, obviously, $p_{2 \mid \gamma}$ is equal to the number of edges in $G_{1}$, that is $p_{2 \mid \gamma}=p_{1}-\left(v_{1}(\gamma)+v_{2}(\gamma)-1\right)$. Summing these equalities over all possible edges $\gamma$, we get

$$
\begin{aligned}
\sum_{\gamma} p_{2 \mid \gamma} & =\sum_{\gamma}\left(p_{1}-\left(v_{1}(\gamma)+v_{2}(\gamma)-1\right)\right)= \\
& =\sum_{\gamma} p_{1}-\sum_{\gamma}\left(v_{1}(\gamma)+v_{2}(\gamma)\right)+\sum_{\gamma} 1=p_{1}^{2}-\sum_{\gamma}\left(v_{1}(\gamma)+v_{2}(\gamma)\right)+p_{1} .
\end{aligned}
$$

Each pair of non-adjacent edges is present in the sum $\sum_{\gamma} p_{2 \mid \gamma}$ twice, since each edge of $G$ may be considered both as $\gamma$ and as the edge of $G_{1}$. Hence $\sum_{\gamma} p_{2 \mid \gamma}=2 p_{2}$ and

$$
p_{2}=\frac{1}{2} p_{1}^{2}-\frac{1}{2} \sum_{\gamma}\left(v_{1}(\gamma)+v_{2}(\gamma)\right)+\frac{1}{2} p_{1} .
$$

Transform the sum $\sum_{\gamma}\left(v_{1}(\gamma)+v_{2}(\gamma)\right)$. Each vertex degree $v_{k}$ is present in it as many times as many edges are incident to the $k$-th vertex, that is:

$$
\sum_{\gamma}\left(v_{1}(\gamma)+v_{2}(\gamma)\right)=\sum_{k=1}^{n} v_{k} \cdot v_{k}=\sum_{i=1}^{4} i^{2} \cdot n_{i} .
$$

Using the last formula, one can easily get the formula (5).
Theorem 2. For $2 \leq k<[n / 2]$ the following inequalities are true:

$$
\begin{equation*}
\frac{1}{k+1} p_{k}\left(p_{1}-\sum_{i=1}^{4} \alpha_{i}(k) \cdot i+k\right) \leq p_{k+1} \leq \frac{1}{k+1} p_{k}\left(p_{1}-2 k\right) \tag{6}
\end{equation*}
$$

where

$$
\begin{aligned}
& \alpha_{4}(k)=\min \left\{n_{4}, 2 k\right\}, \\
& \alpha_{3}(k)=\min \left\{n_{3}, 2 k-\alpha_{4}\right\}, \\
& \alpha_{2}(k)=\min \left\{n_{2}, 2 k-\alpha_{4}-\alpha_{3}\right\}, \\
& \alpha_{1}(k)=\min \left\{n_{1}, 2 k-\alpha_{4}-\alpha_{3}-\alpha_{2}\right\} .
\end{aligned}
$$

Proof. Let us prove the left inequality in Eq. (6). Select and fix an arbitrary set $\gamma_{1}, \ldots, \gamma_{k}$ of $k$ nonadjacent edges of graph $G$. Denote by $G_{k}$ a subgraph of $G$ obtained by deleting from $G$ the selected edges, all edges adjacent to it and vertices that became isolated. Denote by $\bar{G}_{k}$ a subgraph of $G$ consisting of deleted edges and vertices incident to it. Denote by $q_{k}$ and $\bar{q}_{k}$ the numbers of edges in $G_{k}$ and $\bar{G}_{k}$, respectively. Then $q_{k}+\bar{q}_{k}=p_{1}$. Let us estimate $\bar{q}_{k}$. One can show that $\bar{q}_{k} \leq \sum_{i=1}^{k}\left(v_{1}\left(\gamma_{i}\right)+v_{2}\left(\gamma_{i}\right)-1\right)=\sum_{i=1}^{k}\left(v_{1}\left(\gamma_{i}\right)+v_{2}\left(\gamma_{i}\right)\right)-k$, where $v_{1}\left(\gamma_{i}\right)$ and $v_{2}\left(\gamma_{i}\right)$ are degrees of vertices incident to the edge $\gamma_{i}$. A maximum value of the sum $\sum_{i=1}^{k}\left(v_{1}\left(\gamma_{i}\right)+v_{2}\left(\gamma_{i}\right)\right)$ is reached when a maximum number of $2 k$ vertices in it are of degree 4 , a maximum number of remained vertices are of degree 3 , etc. Note that the number of vertices of degree 4 in $\gamma_{1}, \ldots, \gamma_{k}$ cannot be greater than $n_{4}$ nor $2 k$, similarly, the number of vertices of degree 3 cannot be greater than $n_{3}$ nor the number of the remained vertices, etc. Introduce the numbers $\alpha_{4}(k)=\min \left\{n_{4}, 2 k\right\}, \alpha_{3}(k)=\min \left\{n_{3}, 2 k-\alpha_{4}\right\}$, $\alpha_{2}(k)=\min \left\{n_{2}, 2 k-\alpha_{4}-\alpha_{3}\right\}, \alpha_{1}(k)=\min \left\{n_{1}, 2 k-\alpha_{4}-\alpha_{3}-\alpha_{2}\right\}$. Then:

$$
\sum_{i=1}^{k}\left(v_{1}\left(\gamma_{i}\right)+v_{2}\left(\gamma_{i}\right)\right) \leq \sum_{i=1}^{4} i \cdot \alpha_{i}(k) .
$$

Hence, $\bar{q}_{k} \leq \sum_{i=1}^{4} i \cdot \alpha_{i}(k)-k$ and:

$$
q_{k} \geq p_{1}-\sum_{i=1}^{4} i \cdot \alpha_{i}(k)+k
$$

The number of sets of $k+1$ non-adjacent edges at fixed edges $\gamma_{1}, \ldots, \gamma_{k}$ is equal to the number of edges in $G_{k}$, that is, $q_{k}$. Summing these inequalities for $q_{k}$ over all sets $\gamma=\left\{\gamma_{1}, \ldots, \gamma_{k}\right\}$, which number is equal to $p_{k}$, one can find:

$$
\sum_{\gamma} q_{k} \geq p_{k}\left(p_{1}-\sum_{i=1}^{4} i \cdot \alpha_{i}(k)+k\right)
$$

Each set of $k+1$ non-adjacent edges is present $k+1$ times in the sum $\sum_{\gamma} q_{k}$ : in each case $k$ edges of these $k+1$ ones are fixed as $\gamma_{1}, \ldots, \gamma_{k}$ and the $(k+1)$-th belongs to $G_{k}$. Hence $\sum_{\gamma} q_{k}=(k+1) p_{k+1}$. Using this relationship and the last inequality, we obtain the left inequality in Eq. (6).

Consider now the right inequality in Eq. (6). It is easy to show that $\bar{q}_{k} \geq 2 k$. Indeed, $\bar{q}_{k}$ is the number of edges in graph $\bar{G}_{k}$ containing $k$ edges $\gamma_{1}, \ldots, \gamma_{k}$ and at least $k$ edges adjacent to it. Hence, $q_{k} \leq p_{1}-2 k$. Summing these inequalities over all sets $\gamma=\left\{\gamma_{1}, \ldots, \gamma_{k}\right\}$ in the same manner as in the first part of this proof, we obtain the right inequality in Eq. (6).

## 3 AN ALGORITHM FOR SOLVING THE INVERSE PROBLEM

In this section we will give the general methodology of the IP solution for the case of equation (2) for simple molecular graphs with vertex degrees not exceeding four. The solution process can be subdivided into the following steps.

1. Let $\left[y_{1}, y_{2}\right]$ be an interval of possible values of property under consideration, $n$ is a number of graph vertices. Find all partitions of $n$ into 4 integers (including zero): $n=n_{1}+n_{2}+n_{3}+n_{4}$, where $n_{i}$ is the number of vertices with degree $i(i=1,2,3,4)$.
2. For the sets $\left\{n_{i}\right\}$ check necessary and sufficient conditions for the existence of molecular graphs with given distribution of vertex degrees and eliminate unsuitable variants. Let $v_{1} \geq v_{2} \geq \ldots \geq v_{n}>0$ be an ordered set of numbers. A molecular graph with vertex degrees $\left\{v_{i}\right\}$ exists if and only if [24]:
a. $\sum_{i=1}^{n} v_{i}=\sum_{i=1}^{4} i \cdot n_{i}$ is even;
b. for any integer $r, 1 \leq r \leq n-1$,

$$
\sum_{i=1}^{r} v_{i} \leq r(r-1)+\sum_{i=r+1}^{n} \min \left(r, v_{i}\right) ;
$$

c. $\sum_{i=1}^{n} v_{i} \geq 2(n-1), n \geq 3$.
3. Calculate $p_{1}$ and $p_{2}$ using formulas (4) and (5). On this step some additional criteria related to the considered class of compounds may be also used for elimination of unsuitable sets $\left\{n_{i}\right\}$ (for example, $p_{1}=n-1$ for acyclic structures, $p_{1}>n-1$ for cycle-containing ones; $p_{2}=n_{2}+3 n_{3}$ for simple graphs corresponding to cata-condensed benzenoid hydrocarbons, etc.).
4. For the remaining sets $\left\{n_{i}\right\}$ using Eq. (3) and given interval of property values $\left[y_{1}, y_{2}\right]$ define an interval of possible values of index $Z$. Further the following elimination criteria are used:

$$
\begin{equation*}
1+p_{1}+p_{2} \leq Z \leq 1+p_{1}+p_{2}+\sum_{k=3}^{[n / 2]} p_{k, \max } \tag{7}
\end{equation*}
$$

where $p_{2, \text { max }}=p_{2}, \quad p_{k+1, \text { max }}=\frac{1}{k+1} p_{k, \text { max }}\left(p_{1}-2 k\right), \quad k \geq 3-$ a priori found maximum values of $p_{k}$ derived from inequalities (6).
5. Find all partitions of $Z$ into $[n / 2]+1$ integers $p_{k}, k \geq 0$ (see (1)), where $p_{0}=1, p_{1}$ and $p_{2}$ are found above, and $p_{k}$ satisfies the conditions (6).
6. For the sets $\left\{p_{k}\right\}$ check the following necessary conditions for the existence of graphs with given $p_{k}, k \geq 0$ [25]:
a. All roots of the acyclic polynomial $M(x)=\sum_{k=0}^{[n / 2]}(-1)^{k} p_{k} x^{n-2 k}$ are real numbers.
$b$. For the maximal root $\mu_{1}$ of $M(x)$ the following inequalities are true:

$$
\frac{1}{2} \sqrt{\Delta_{\max }+1+\sqrt{\Delta_{\max }^{2}-2 \Delta_{\max }+5}} \leq \mu_{1} \leq 2 \sqrt{\Delta_{\max }-1}
$$

where $\Delta_{\max }=\max \left\{i: n_{i}>0\right\}$.
7. Construct all molecular graphs with obtained sets $\left\{n_{i}\right\}$ and $\left\{p_{k}\right\}$, using, for example, the structure generator SMOG [26]. This program allows one to construct all chemical structures with given molecular formula and lists of required and forbidden fragments. The number of the required fragments can be also given. The generated graphs yield a solution of the IP.

Suppose that molecular structures include heteroatoms and multiple bonds. In this case simple molecular graphs are generated by the algorithm described above and then labeled graphs are constructed by arranging labels on edges and vertices in all non-equivalent ways.

## 4 EXAMPLES

Let us consider the examples of solving the IP for Eq. (2). Below $r$ denotes a correlation coefficient, $r_{\text {LOO }}$ - a leave-one-out cross-validation correlation coefficient, $s$ - a standard deviation, $F$ - a Fisher test, $N$ - the number of structures in the training set.

Example 1. For the boiling points of alkylsulfides [27] we have obtained the following equation:

$$
\begin{gathered}
\text { B.p. }=-17.61+52.744 \ln Z, \\
r=0.995, r_{\text {LOO }}=0.994, s=2.87, F=1886, N=21,
\end{gathered}
$$

where B.p. is the boiling point $\left({ }^{\circ} \mathrm{C}\right)$, the training set consists of structures $1-21$ (see Table 1 ).

Consider the problem of finding all sulfides with the number of carbon atoms from 2 to 8 and the boiling point from $113^{\circ} \mathrm{C}$ to $126^{\circ} \mathrm{C}$. As a result of using the described algorithm structures $7,9,19$, 20, 22, 23 from Table 1 were obtained. These structures are shown in Figure 1. The calculated values of the boiling point are in a good agreement with the experimental values. The experimental value for compound 23 is absent.


7


20


9


22


19


23

Figure 1. Structures generated in the example 1.

Table 1. Experimental and predicted values of boiling point for alkyl sulfides used in the Example 1.

| No | Sulfide | B.p. exp., ${ }^{\circ} \mathrm{C}$ | B.p. calc., ${ }^{\circ} \mathrm{C}$ | Res. |
| :---: | :--- | :---: | :---: | :---: |
| 1 | dimethyl sulfide | 37.3 | 40.3 | -3.0 |
| 2 | methyl ethyl sulfide | 66.6 | 67.2 | -0.6 |
| 3 | methyl propyl sulfide | 95.5 | 92.1 | 3.4 |
| 4 | diethyl sulfide | 92.0 | 92.1 | -0.1 |
| 5 | methyl isopropyl sulfide | 84.4 | 85.0 | -0.6 |
| 6 | ethyl isopropyl sulfide | 107.4 | 108.9 | -1.5 |
| 7 | methyl butyl sulfide | 123.2 | 117.7 | 5.5 |
| 8 | methyl isobutyl sulfide | 112.5 | 108.9 | 3.6 |
| 9 | ethyl propyl sulfide | 118.5 | 117.7 | 0.8 |
| 10 | methyl t-butyl sulfide | 101.5 | 98.3 | 3.2 |
| 11 | methyl amyl sulfide | 145.0 | 143.0 | 2.0 |
| 12 | ethyl butyl sulfide | 144.2 | 143.0 | 1.2 |
| 13 | dipropyl sulfide | 142.8 | 143.0 | -0.2 |
| 14 | propyl isopropyl sulfide | 132.0 | 134.9 | -2.9 |
| 15 | ethyl isobutyl sulfide | 134.2 | 134.8 | -0.6 |
| 16 | methyl isoamyl sulfide | 137.0 | 134.8 | 2.2 |
| 17 | methyl 2-methylbutyl sulfide | 139.0 | 137.7 | 1.3 |
| 18 | ethyl s-butyl sulfide | 133.6 | 137.7 | -4.1 |
| 19 | ethyl t-butyl sulfide | 120.4 | 121.6 | -1.2 |
| 20 | diisopropyl sulfide | 120.0 | 125.2 | -5.2 |
| 21 | methyl 1-ethylpropyl sulfide | 137.0 | 140.4 | -3.4 |
| 22 | methyl s-butyl sulfide | 114.5 | 113.5 | 1.0 |
| 23 | methyl 2,2-dimethylpropyl sulfide | - | 121.59 | - |

Example 2. For the solubility of saturated alcohols in water [28] we have obtained the following correlation:

$$
\begin{gathered}
-\log X=-0.8+1.186 \ln Z, \\
r=0.976, r_{\mathrm{LOO}}=0.975, s=0.21, F=966, N=50,
\end{gathered}
$$

where $X$ denotes the mole fraction solubility of alcohols in water. Let us find all such compounds with $2.6 \leq-\log X \leq 3.0$. Structural formulas of 20 generated alcohols are shown in Figure 2. Experimental and calculated values of $-\log X$ for the training set (structures $1-50$ ) and calculated values for the generated structures $(51-59)$ not belonging to the training set are given in Table 2.


12


24


31


52


56


13


26


32


53


57


14


29


33


54


58


22


30


51


55


59

Figure 2. Structures of alcohols generated in the example 2.

Table 2. Experimental and predicted values of the mole fraction solubility in water for alcohols (example 2).

| No | Alcohols | $-\log X, \exp$. | $-\log X$, calc. | Res. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1-Butanol | 1.750 | 1.666 | 0.084 |
| 2 | 2-Methylpropanol | 1.743 | 1.508 | 0.235 |
| 3 | 2-Butanol | 1.724 | 1.508 | 0.216 |
| 4 | 1-Pentanol | 2.332 | 2.242 | 0.090 |
| 5 | 3-Methylbutanol | 2.254 | 2.044 | 0.210 |
| 6 | 2-Methylbutanol | 2.207 | 2.147 | 0.060 |
| 7 | 2-Pentanol | 2.025 | 2.044 | -0.019 |
| 8 | 3-Pentanol | 1.961 | 2.147 | -0.186 |
| 9 | 3-Methyl-2-butanol | 1.926 | 1.931 | $-0.005$ |
| 10 | 2-Methyl-2-butanol | 1.608 | 1.806 | -0.198 |
| 11 | 2,2-Dimethyl-1-propanol | 2.030 | 1.806 | 0.224 |
| 12 | 1-Hexanol | 2.957 | 2.811 | 0.146 |
| 13 | 2-Hexanol | 2.612 | 2.628 | -0.016 |
| 14 | 3-Hexanol | 2.542 | 2.692 | $-0.150$ |
| 15 | 3-Methyl-3-pentanol | 2.109 | 2.488 | -0.379 |
| 16 | 2-Methyl-2-pentanol | 2.233 | 2.330 | -0.097 |
| 17 | 2-Methyl-3-pentanol | 2.445 | 2.560 | -0.115 |
| 18 | 3-Methyl-2-pentanol | 2.458 | 2.560 | -0.102 |
| 19 | 2,3-Dimethyl-2-butanol | 2.118 | 2.242 | $-0.124$ |
| 20 | 3,3-Dimethyl-1-butanol | 2.870 | 2.330 | 0.540 |
| 21 | 3,3-Dimethyl-2-butanol | 2.359 | 2.242 | 0.117 |
| 22 | 4-Methylpentanol | 2.737 | 2.628 | 0.109 |
| 23 | 4-Methyl-2-pentanol | 2.534 | 2.412 | 0.122 |
| 24 | 2-Ethyl-1-butanol | 2.956 | 2.753 | 0.203 |
| 25 | 1-Heptanol | 3.554 | 3.383 | 0.171 |
| 26 | 2-Methyl-2-hexanol | 2.820 | 2.919 | -0.099 |
| 27 | 3-Methyl-3-hexanol | 2.729 | 3.018 | -0.289 |
| 28 | 3-Ethyl-3-pentanol | 2.579 | 3.152 | $-0.573$ |
| 29 | 2,3-Dimethyl-2-pentanol | 2.615 | 2.866 | -0.251 |
| 30 | 2,3-Dimethyl-3-pentanol | 2.588 | 2.919 | $-0.331$ |
| 31 | 2,4-Dimethyl-2-pentanol | 2.678 | 2.692 | -0.014 |
| 32 | 2,4-Dimethyl-3-pentanol | 2.962 | 2.969 | -0.007 |
| 33 | 2,2-Dimethyl-3-pentanol | 2.893 | 2.866 | 0.027 |
| 34 | 3-Heptanol | 3.132 | 3.273 | $-0.141$ |
| 35 | 4-Heptanol | 3.133 | 3.234 | -0.101 |
| 36 | 1-Octanol | 4.091 | 3.953 | 0.138 |
| 37 | 2,2,3-Trimethyl-3-pentanol | 3.018 | 3.234 | -0.216 |
| 38 | 2-Octanol | 3.811 | 3.767 | 0.044 |
| 39 | 2-Ethylhexanol | 3.915 | 3.886 | 0.029 |
| 40 | 1-Nonanol | 4.745 | 4.524 | 0.221 |
| 41 | 2-Nonanol | 4.490 | 4.337 | 0.153 |
| 42 | 3-Nonanol | 4.402 | 4.412 | -0.010 |
| 43 | 4-Nonanol | 4.330 | 4.383 | -0.053 |
| 44 | 5-Nonanol | 4.240 | 4.397 | -0.157 |
| 45 | 2,6-Dimethyl-3-heptanol | 4.253 | 4.076 | 0.177 |
| 46 | 3,5-Dimethyl-4-heptanol | 4.046 | 4.239 | $-0.193$ |
| 47 | 1,1-Diethylpentanol | 4.165 | 4.273 | -0.108 |
| 48 | 7-Methyloctanol | 4.240 | 4.337 | -0.097 |
| 49 | 3,5,5-Trimethylhexanol | 4.251 | 3.886 | 0.365 |
| 50 | 1 -Decanol | 5.444 | 5.095 | 0.349 |
| 51 | 2-Methyl-1-pentanol | - | 2.692 | - |
| 52 | 3-Methyl-1-pentanol | - | 2.692 | - |
| 53 | 4,4-Dimethyl-1-pentanol | - | 2.919 | - |
| 54 | 3,3-Dimethyl-2-pentanol | - | 2.919 | - |
| 55 | 3,4-Dimethyl-2-pentanol | - | 2.969 | - |
| 56 | 4,4-Dimethyl-2-pentanol | - | 2.692 | - |
| 57 | 2,2,3-Trimethyl-1-butanol | - | 2.919 | - |
| 58 | 2,3,3-Trimethyl-1-butanol | - | 2.866 | - |
| 59 | 2,4,4-Trimethyl-2-pentanol | - | 2.969 | - |

Example 3. For a training set of hydrocarbons with the number of carbon atoms $n$ from 6 to 8 and known values of the boiling point $t_{b}$ [29], we obtained an equation

$$
\begin{aligned}
& \text { B.p. }=-98.2+41.81 \ln Z+11.1 n_{1}+9.22 n_{2}+5.1 n_{3}, \\
& r=0.983, r_{\text {LOO }}=0.981, s=4.28, F=525, N=78
\end{aligned}
$$

Experimental data and calculated values of the boiling point for the training set are given in Table 3 (structures 1-78). Further a problem of generating hydrocarbons with $6 \leq n \leq 8$ and the boiling point $100 \pm 1.5^{\circ} \mathrm{C}$ was considered. Using the described algorithm, 77 structures were generated. Then 57 tetrahedranes, pyramidanes, propellanes, and other extremely strained and unrealistic structures were removed manually. The remained 20 structures (no. 18, 33, 34, 55, 74, 79-93) are shown in Table 3. The structures 18, 33, 34, 55, 74 were present in the training set, structures 79 and 80 were present in the database [29] while the other generated structures were not present in that database.

Table 3. Experimental and predicted values of boiling point for hydrocarbons considered in example 3.
Baccer

Table 3. (Continued).

| No | Structure | Name | B.p. exp., ${ }^{\circ} \mathrm{C}$ | B.p. calc., ${ }^{\circ} \mathrm{C}$ | Res. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 |  | $2 \mathrm{mn6}$ | 90.0 | 88.7 | 1.3 |
| 15 |  | 24 mn 5 | 80.5 | 78.9 | 1.6 |
| 16 |  | 23 mn 5 | 89.8 | 84.1 | 5.7 |
| 17 |  | 223 mn 4 | 80.9 | 70.0 | 11.2 |
| 18 |  | 1 bc 3 | 98.0 | 100.3 | -2.3 |
| 19 | $\triangle$ | 12ec3 | 90.0 | 96.4 | -6.4 |
| 20 |  | 1 m 1 pc 3 | 84.9 | 90.1 | -5.2 |
| 21 | $\Delta$ | 11 ec 3 | 88.6 | 93.8 | -5.2 |
| 22 |  | 1 mlipc 3 | 81.5 | 83.9 | -2.4 |
| 23 |  | 12 mlec 3 | 85.2 | 85.9 | -0.7 |
| 24 |  | 1123 mc 3 | 78.0 | 79.5 | -1.5 |
| 25 |  | 1ipc4 | 92.7 | 94.7 | -2.0 |
| 26 |  | 1 e 3 mc 4 | 89.5 | 94.7 | -5.2 |
| 27 |  | 13 mc 5 | 91.3 | 93.0 | -1.7 |
| 28 |  | 12 mc 5 | 95.6 | 94.7 | 0.9 |
| 29 |  | c7 | 118.4 | 107.1 | 11.3 |
| 30 |  | dcprm | 102.0 | 103.0 | -1.0 |

Table 3. (Continued).
Bers.

Table 3. (Continued).
(124.0

Table 3. (Continued).

| No | Structure | Name | B.p. exp., ${ }^{\circ} \mathrm{C}$ | B.p. calc., ${ }^{\circ} \mathrm{C}$ | Res. |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 29 | 2 mbc 221 h | 125.0 | 128.6 | -3.6 |  |

70


2 mbc 320 h


1 mbc 221 h
130.5
117.0


1 mbc 410 h
125.0
115.0
104.0


1223 mbcb
75


33mbc310hx

2244 mbcb


76

tc 510035 o

tc3210o


3 mtc 2210 h

1 me6


223 mn 5


1cpbc110b

2scpbc210p
$3 m t c 310026 h x$

3mtc220026hx
130.3
0.3 124.2 $-7.2$
125.1 $-0.1$
$115.0 \quad 0.0$
101.3
2.7
$101.6 \quad 3.3$
$132.9 \quad 9.1$
$134.2 \quad 1.7$
129.4
$-8.9$
$100.3 \quad 0.7$
$100.9 \quad 8.9$
101.0
101.0
99.7
100.8

Table 3. (Continued).

| No | Structure | Name | B.p. exp., ${ }^{\circ} \mathrm{C}$ | B.p. calc., ${ }^{\circ} \mathrm{C}$ | Res. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 85 | - | 3 mtc 310024 hx |  | 99.7 |  |
| 86 |  | 1etc210025p |  | 99.8 |  |
| 87 |  | 2 etc 210025 p |  | 98.7 |  |
| 88 | $<$ | 2 pbc 110 b |  | 100.8 |  |
| 89 | $\alpha$ | 2 ebc 210 p |  | 100.8 |  |
| 90 |  | 2mbc220hx |  | 100.8 |  |
| 91 |  | 5mbc112hx |  | 100.8 |  |
| 92 |  | 2 mbc 112 hx |  | 99.4 |  |
| 93 |  | 1 m 2 tbc 3 |  | 100.9 |  |

## 5 CONCLUSIONS

An algorithm has been developed for solving the inverse problem for correlation equation containing the Hosoya index and some other simple topological descriptors. A method has been tested for a number of correlation equations of described type. The results obtained show a sufficiently high efficiency of the suggested approach for searching the structures of some simple classes of organic compounds with prescribed property values.

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[^0]:    \# Dedicated to Professor Haruo Hosoya on the occasion of the $65^{\text {th }}$ birthday.

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