

Proposal and testing for a fiber-optic-based measurement of flow vorticity

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A fiber-optic arrangement is devised to measure the velocity difference, $\delta\mathbf{v}(l)$, down to small separation l . With two sets of optical fibers and couplers the new technique becomes capable of measuring one component of the time- and space-resolved vorticity vector $\boldsymbol{\omega}(\mathbf{r}, t)$. The technique is tested in a steady laminar flow, in which the velocity gradient (or flow vorticity) is known. The experiment verifies the working principle of the technique and demonstrates its applications. It is found that the new technique measures the velocity difference (and hence the velocity gradient when l is known) with the same high accuracy and high sampling rate as laser Doppler velocimetry does for the local velocity measurement. It is nonintrusive and capable of measuring the velocity gradient with a spatial resolution as low as ~ 50 μm . The successful test of the fiber-optic technique in the laminar flow with one optical channel is an important first step for the development of a two-channel fiber-optic vorticity probe, which has wide use in the general area of fluid dynamics, especially in the study of turbulent flows. © 2001 Optical Society of America

OCIS codes: 120.7250, 060.2420, 060.1810, 030.7060.

1. Introduction

A quantity of fundamental interest in the study of fluid turbulence is the time- and space-resolved vorticity vector $\boldsymbol{\omega}(\mathbf{r}, t) = \nabla \times \mathbf{v}(\mathbf{r}, t)$, where $\mathbf{v}(\mathbf{r}, t)$ is the local flow velocity. However, the determination of vorticity, i.e., the measurement of rotation, is extremely difficult because of its severe requirements on spatial resolution and accuracy during simultaneous measurement of velocity differences. One method of probing flow vorticity is to measure velocity components and their gradients simultaneously, with hot-wire anemometry.^{1,2} Multiple sensors (four or more) must be used in a small region in order to realize a central difference approximation. The multisensor probe is intrusive, however, and requires a delicate calibration procedure. Various image techniques, such as particle imaging, fluorescence imaging, and holographic imaging, have also been used to visualize the vortex structures in turbulent flows.^{1,2} These techniques display a two-dimensional (2D) vorticity map, but their applications are often limited to

low-speed flows with limited spatial and temporal resolution. Some of them provide only qualitative information.

The technique of particle image velocimetry^{3,4} (PIV) has been used increasingly in recent years to obtain the velocity field map, from which one calculates the vorticity field of the area of interest. With the PIV technique one captures two consecutive 2D images of the seed particles, using a CCD camera situated normal to an illuminating light sheet. Spatial correlations between the two images are then calculated to obtain information about the displacement of each particle, from which one obtains the velocity map. Whereas the current PIV technique is adequate for the study of vortex structures in low-speed flows, its spatial and temporal resolutions are rather limited to further resolve the small vortex structures in turbulent flows at high Reynolds numbers. In fact, the accuracy of the vorticity measurement by PIV has not been examined in great detail, because of the lack of an alternative vorticity probe. Other methods of measuring flow vorticity are cited in two recent reviews by Foss and Wallace.^{1,2} Although many attempts have been made to measure one or more components of the time- and space-resolved vorticity vector $\boldsymbol{\omega}(\mathbf{r}, t)$, an accurate and reliable vorticity probe that can be used widely in the general area of fluid dynamics is not yet available.

Recently, a new fiber-optic technique^{5,6} capable of measuring the velocity difference, $\delta\mathbf{v}(l) = \mathbf{v}(\mathbf{r} + l) -$

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Received 13 October 2000; revised manuscript received 20 February 2001.

0003-6935/01/244022-06\$15.00/0

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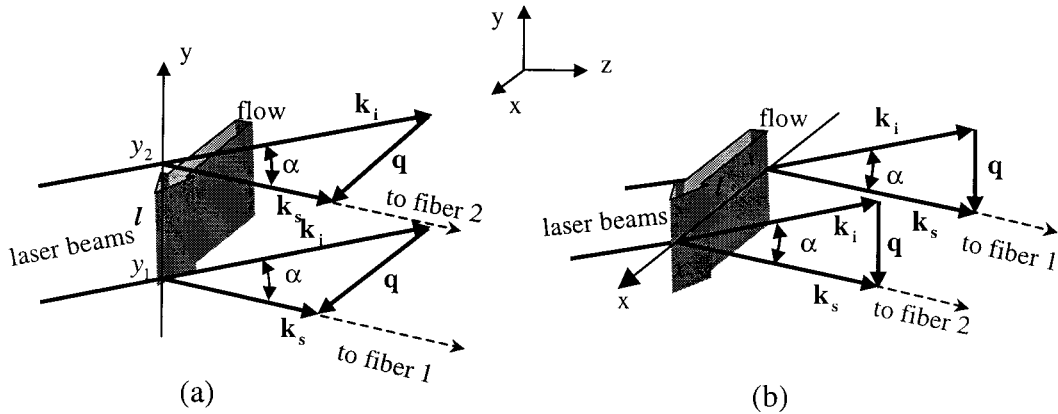


Fig. 1. Laser light scattering arrangements for the measurement of the velocity difference $\delta\mathbf{v}(l)$ in two different directions. \mathbf{k}_i , incident wave vector; \mathbf{k}_s , scattered wave vector; α , scattering angle; and $\mathbf{q} = \mathbf{k}_s - \mathbf{k}_i$.

$\mathbf{v}(\mathbf{r})$, down to small separation l , was developed. In the experiment two single-mode polarization-preserving fibers collect the scattered light with the same polarization and momentum transfer vector \mathbf{q} by seed particles but from two spatially separated locations in the flow. The collected signals from these two regions interfere when combined by use of a fiber-optic coupler such that the resultant light detected by a photomultiplier becomes modulated at a frequency equal to the *difference* in Doppler shifts: $\Delta\omega_2 = \mathbf{q} \cdot \mathbf{v}_1 - \mathbf{q} \cdot \mathbf{v}_2 = \mathbf{q} \cdot \delta\mathbf{v}(l)$. Because the magnitude of \mathbf{q} is known for a given scattering geometry, the measured $\Delta\omega_2$ becomes directly proportional to the velocity difference, $\delta\mathbf{v}(l)$. A similar method was also developed independently by Otugen *et al.*⁷ and by Zimmerli *et al.*⁸

In this paper we show that, with a different optical arrangement and signal-processing scheme, the fiber-optic technique can be used to measure two velocity-gradient components at four closely spaced positions in the x - y plane, and the difference of the two velocity gradients gives the z component of $\boldsymbol{\omega}(\mathbf{r}, t)$. With the new technique one will be able to devise a compact and robust vorticity probe consisting of two sets of optical fibers and receiving optics. This probe will be much like a commercial fiber-optic laser Doppler velocimetry (LDV) probe and has all the advantages of the LDV probe. The new technique has the advantages of high spatial resolution, fast temporal response, and ease of use. It is nonintrusive and capable of measuring the flow vorticity with a spatial resolution as low as $\sim 50 \mu\text{m}$.

2. Working Principle and Experimental Setup

Figure 1 shows the scattering geometry for the measurement of $\delta\mathbf{v}(l)$ in two different directions. In the experiment we use two single-mode polarization-preserving fibers to collect light scattered by the seed particles. Both fibers collect the scattered light with the same polarization and momentum-transfer vector, $\mathbf{q} = \mathbf{k}_s - \mathbf{k}_i$ (i.e., at the same scattering angle α), but from two spatially separated locations in the flow. The collected signals from these two regions (i.e.,

from two different particles with a separation l) interfere when combined by use of a fiber-optic coupler such that the resultant light detected by a photomultiplier becomes modulated by a frequency $\Delta\omega_2$ equal to the *difference* in Doppler shifts. As shown in Fig. 1(a), \mathbf{q} is in the x direction, and $\Delta\omega_2 = \mathbf{q} \cdot \mathbf{v}(y_2) - \mathbf{q} \cdot \mathbf{v}(y_1) = q[v_x(y_2) - v_x(y_1)]$. The magnitude of \mathbf{q} is given by $q = (4\pi n/\lambda)\sin(\alpha/2)$, where n is the refractive index of the fluid, λ is the wavelength of the incident light, and α is the same scattering angle for each receiving fiber. From the measured $\Delta\omega_2$, one obtains the velocity gradient $\partial v_x/\partial y \approx [v_x(y_2) - v_x(y_1)]/l$ when the beam separation $l = y_2 - y_1$ is known.

Similarly, the scattering geometry shown in Fig. 1(b), which can be obtained by rotation of the scattering geometry shown in Fig. 1(a) about the z axis by 90° , gives the velocity gradient $\partial v_y/\partial x \approx [v_y(x_2) - v_y(x_1)]/l$. With a complete arrangement of four optical fibers and two couplers, one can simultaneously measure the two velocity gradients at four closely spaced locations in the x - y plane and obtain the z component of the local vorticity

$$\begin{aligned} \omega_z\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \\ &\approx \frac{[v_y(x_2) - v_y(x_1)]}{l} \\ &\quad - \frac{[v_x(y_2) - v_x(y_1)]}{l}. \end{aligned} \quad (1)$$

It should be mentioned that Fig. 1 shows only the forward-scattering mode ($\alpha < 90^\circ$) for the vorticity measurement. With a slightly different optical arrangement one can also use the backward-scattering mode to measure the velocity gradients.

To test the technique, we conduct vorticity measurements in a laminar tunnel flow, in which the flow vorticity is known. Because the flow field is quasi 2D, the local vorticity becomes equal to the velocity gradient, $\omega_z = -\partial v_x/\partial y = -\delta v_x(l)/l(\partial v_y/\partial x = 0)$. In

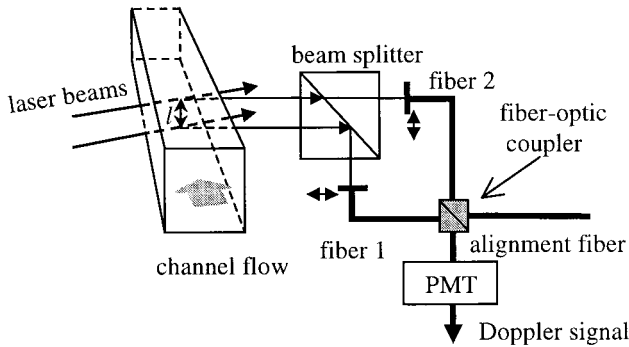


Fig. 2. Experimental setup for the measurement of the velocity gradient in the horizontal direction over a vertical distance l .

the experiment, water seeded with a small amount of polymer latex spheres circulates through a 1-m-long segment of square tubing with an inner width $D = 2.48$ cm. The square tubing, made of transparent Plexiglas, admits the incident laser beams and allows for observation of the scattered light. The measuring point is in the midplane of the square tubing and 82 cm downstream from the entrance of the long tubing. Undesirable velocity fluctuations are dampened in a $5\text{ cm} \times 5\text{ cm} \times 10\text{ cm}$ chamber filled with straws of 5.5-mm diameter, and then the flow is fed through a square contraction section (4:1 by area) of 16 cm in length. The Reynolds number used in the experiment is defined as $\text{Re} = DU/\nu$, where U is the average flow velocity across the square tunnel and ν is the fluid viscosity. Local velocity measurements show that velocity fluctuations at the center of the tunnel are less than 3% of the mean flow. Because the seed particles are small and their density matches well with that of water, they follow the local flow.

Figure 2 shows the experimental setup. We generate the two incident laser beams by passing an incident beam from a Nd:YVO_4 laser with a power range of 0.5–2 W and wavelength $\lambda = 532$ nm through a Bragg cell (not shown). Only the zeroth-order (unshifted) and the first-order (40-MHz frequency-shifted) outgoing beams are used in the experiment, and all the higher-order beams are blocked. The two laser beams are adjusted to have the same intensity and are directed to the measuring point by a lens ($f = 7$ cm), which is positioned such that the two outgoing beams become parallel at the measuring point. Beam profile measurements at the measuring point show that the two beams are separated by a distance $l = 0.42$ mm and that the diameter of each beam is $\sigma \approx 38\ \mu\text{m}$. The value of l is fixed in the experiment, because it is sufficient for the vorticity measurement in the laminar flow.

It is evident that the spatial resolution of the velocity-gradient (or vorticity) measurement is dependent on the separation l between the two incident beams. An incident laser beam becomes two diverging beams (one of them is frequency shifted) when passing through the Bragg cell, which acts like an optical grating. If the distance between the beam-

splitting point (inside the Bragg cell) and the lens is set to be equal to the focal length f , the two diverging beams become parallel after passing through the lens. We find that the value of l , which is proportional to f , can be readily reduced to 0.1 mm by use of a low-magnification objective ($f \approx 2$ cm) as the collimating lens. The value of l can be further reduced to $50\ \mu\text{m}$ by use of a $25\times$ objective. Clearly, this spatial resolution will be adequate for most turbulent vorticity measurements.

A graded-index lens is installed at the front end of each input fiber for better light collection. With an extra focusing lens ($f = 7$ cm), the two input fibers collect the scattered light from two very small scattering spots of size 0.12 mm .⁵ In the experiment the scattering angle α is fixed at 47° . One of the output fibers is connected to a photomultiplier tube, whose analog output is fed into a LDV signal processor (Model IFA655, TSI Incorporated). An oscilloscope is also connected to the photomultiplier tube for direct viewing of the output signals. The other output fiber is connected to a He–Ne laser, which is used for optical alignment. With the reversed He–Ne light coming out of the input fibers, we can directly observe the scattering volume viewed by each input fiber.

For the new technique to measure $\Delta\omega_2$ accurately, one requirement is that the residence time of the seed particles in each measuring volume, i.e., the beam crossing time $\tau_r = \sigma/\bar{v}$, should be kept much longer than the cross-beat time $(\Delta\omega_2)^{-1}$. This will allow the LDV signal processor to detect enough oscillations (say, ten) in each burst signal (see Fig. 3 below). As in the standard LDV, this condition is satisfied by means of shifting the frequency of an incident beam. When the shift frequency Ω_0 is set to be much larger than $\Delta\omega_2$, the above requirement becomes $\tau_r\Omega_0 \gg 1$ or $\sigma\Omega_0 \gg \bar{v}$. With a maximum frequency shift of 40 MHz available for many Bragg cells used in LDV, this condition is satisfied for most turbulent flows. The frequency shift also allows us to measure small fluctuations of the velocity difference near zero and to determine the sign of the velocity gradient, which is *essential* for the vorticity measurement in turbulent flows. This frequency shift discriminates the self-beat signals (between the particles in the same measuring volume or laser beam) from the desired signal. Because the cross-beat frequency between the particles in the two measuring volumes is shifted to a frequency range much higher than the self-beat frequency, we can readily remove the self-beat signals by using a high-pass filter.

3. Results and Discussion

Figure 3 shows the typical beat signal from the analog output of the photomultiplier tube. The signal strongly resembles the burst signal in the standard LDV. The only difference is that the signal shown in Fig. 3 results from the beating of the scattered light with the same \mathbf{q} but from two different particles separated by a distance l . In the standard LDV, however, the Doppler signal is produced by the beating of the scattered light from the same particle but with

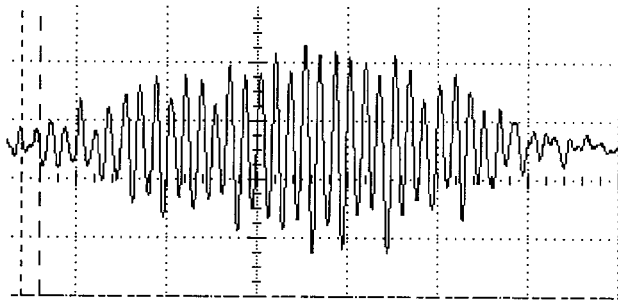


Fig. 3. Oscilloscope trace of a typical beat burst between two moving particles. The signal was obtained in the near-wall region at $Re = 1020$.

two different values of \mathbf{q} . Figure 3 reveals that the cross-beat signal is large enough that the LDV signal processor can be used to obtain the instantaneous velocity gradient $\delta v_x(l, t)/l$ in real time.

Figure 4(a) shows the time-averaged velocity-gradient (or vorticity) profiles across the channel height y at $Re = 510$ and 1020 . The solid and the

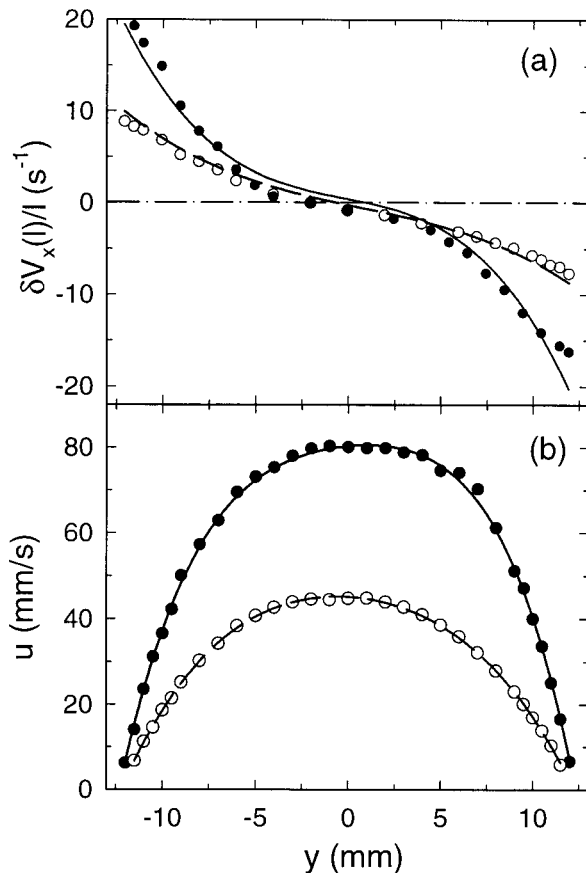


Fig. 4. (a) Measured velocity-gradient profile across the channel height y at $Re = 1020$ (filled circles) and $Re = 510$ (open circles). The solid and the dashed curves are the calculated velocity gradients from the measured velocity profiles shown in (b). (b) Measured velocity profile at $Re = 1020$ (filled circles) and $Re = 510$ (open circles). The solid and the dashed curves are the polynomial fits (up to y^4) to the data points.

dashed curves are the calculated velocity gradients using the polynomial fits (up to y^4) of the measured local velocity profiles shown in Fig. 4(b). Both the velocity measurement and the velocity-gradient measurement are conducted with the LDV signal processor. In Fig. 4 each data point is averaged over a time interval of 200 s with more than 3000 readings. It is seen from Fig. 4(a) that the measured velocity-gradient profiles agree well with the calculated ones both in magnitude and in sign. Figure 4 thus demonstrates that the new fiber-optic technique works well. The small deviations shown in Fig. 4(b) between the solid curve and the measured velocity gradient at $Re = 1020$ arise from the poor accuracy of the polynomial fit to the velocity profile in the central region of the flow. The velocity gradient in this region is small when compared with the local velocity fluctuations. This is especially true at high Reynolds numbers.

The measurements shown in Fig. 4 reveal an important advantage of the new technique. Because it measures the velocity difference directly from the cross-beat between two moving particles, the accuracy of the velocity gradient (and hence vorticity) measurements becomes as high as the local velocity measurement with LDV. By using the best LDV signal processor available on the market, one can resolve the beat frequency up to 1 out of 2^{16} , which is the highest accuracy among all the velocimetry methods.⁹ Another way of obtaining the velocity gradient is to measure the local velocities at two closely spaced locations individually and then find the velocity difference by data subtraction. This is the usual approach employed by many vorticity measurement methods, such as the multisensor hot-wire anemometry, PIV, and the multipoint LDV.^{1,2} With these methods, however, the measured velocity differences may incur relatively large uncertainties in certain turbulent flows, in which the mean velocity \bar{v} is much larger than the velocity difference δv . Under this circumstance the relative error is magnified by a factor proportional to $2\bar{v}/\delta v$. The error bar may become even larger for higher-order statistics. Because the optics of the new technique is similar to that of LDV, we expect its cost to be approximately the same as that for LDV.

Since the measurement of the velocity gradient requires two particles for optical mixing, particle seeding becomes important in determining the data rate. Figure 5 shows the measured data rate S as a function of the particle-number density n for three different particle sizes. The measurements are performed in the near-wall region (2 mm from the top wall), where the mean velocity is $\bar{v} \approx 3.3$ cm/s. It is seen that the measured S increases rapidly with n for small values of n and then reaches a maximum value S_m at a concentration n_m , which depends on the particle size. The maximum data rate achieved with the current experimental condition is in the range of 500–640 counts/s, varying slightly with particle size. The data rate reaches its maximum when there is

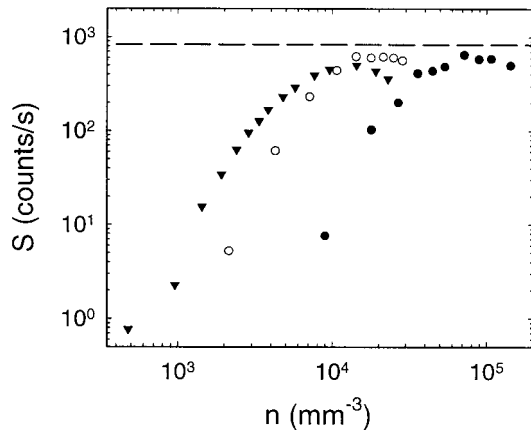


Fig. 5. Measured data rate S as a function of the particle-number density n . The diameters of the particles used are $1.0\ \mu\text{m}$ (triangles), $0.55\ \mu\text{m}$ (open circles), and $0.4\ \mu\text{m}$ (filled circles). The dashed line indicates the calculated maximum data rate S_m .

always at least one pair of particles in the measuring volume at any given time. Further increase in particle concentration introduces the possibility of having more than one pair of particles in the measuring volume at a given time. Evidently, signals from more than one pair of particles cause additional interference, resulting in a decrease in the signal-to-noise level (and hence the data rate S).

In fact, one can estimate S_m and n_m by assuming that the seed particles are uniformly distributed in the fluid. Given that the measuring volume has a cylindrical shape of $\sigma \approx 40\ \mu\text{m}$ diameter and $L = 120\ \mu\text{m}$ length, we find $S_m = \bar{v}/\sigma \approx 825\ \text{counts/s}$ and $n_m = 4/(\pi\sigma^2L) \approx 6.6 \times 10^3\ \text{mm}^{-3}$. It is seen from Fig. 5 that the measured S_m is close to the calculated value (dashed line). Figure 5 thus demonstrates that, under a proper seeding condition, the data rate for the velocity-gradient measurement is comparable with that for the LDV measurement. It is also found from Fig. 5 that the measured S_m for the particles of $1.0\ \mu\text{m}$ in diameter is located at $n_m \approx 1.1 \times 10^4\ \text{mm}^{-3}$, which is 67% larger than the estimated value. This suggests that the scattering from some of the particles is not counted as valid signal. The effect becomes more pronounced for smaller particles and needs to be studied further.

From the above measurements we find that the seed particles of $0.55\ \mu\text{m}$ in diameter are more effective than the other particles. They produce a larger value of $S_m \approx 620\ \text{counts/s}$ at $n_m \approx 1.42 \times 10^4\ \text{mm}^{-3}$, which corresponds to a particle volume fraction of 1.2×10^{-6} . This is a very small particle seeding, and the solution looks clear. For the particles of $1.0\ \mu\text{m}$ in diameter, however, the solution becomes somewhat turbid at the larger particle concentrations. It should be noted that, to increase the particle number density and at the same time avoid the multiple scattering problem, we have used the seed particles that are 5–10 times smaller than those used in LDV (typically $5\ \mu\text{m}$ in diameter).

4. Summary

We have devised a dual-beam, two-fiber scheme to measure the velocity difference, $\delta\mathbf{v}(l)$, over a small separation l . With two sets of optical fibers and couplers, the new technique becomes capable of measuring two velocity-gradient components simultaneously at four closely spaced positions in the x - y plane, and the difference of the two velocity gradients gives the z component of the time- and space-resolved vorticity vector $\boldsymbol{\omega}(\mathbf{r}, t)$. Because it is based on the same Doppler beating principle, the new fiber-optic technique operates in a manner similar to that of the standard LDV. The only difference is that the beat signal in the new scheme, which is proportional to $\delta\mathbf{v}(l)$, results from two different particles separated by a distance l (cross beat). In the standard LDV, however, the signal is proportional to the local velocity of the seed particle, because it is produced by the beating of the scattered light from the same particle (self-beat).

To find optimal experimental conditions for the measurement of the velocity gradient (and hence the flow vorticity), we test the technique in a steady laminar flow, in which the velocity gradient (or flow vorticity) can be calculated from the independent measurement of the mean velocity profile. The experiment verifies the working principle of the fiber-optic technique and demonstrates its applications. It is found that the new technique measures the velocity difference (and hence the velocity gradient when l is known) with the same high accuracy and high sampling rate as LDV does for the local velocity measurement. It is nonintrusive and capable of measuring the velocity gradient (or flow vorticity) with a spatial resolution as low as $\sim 50\ \mu\text{m}$.

The successful test of the fiber-optic technique in the laminar flow with one optical channel is an important first step for the development of a two-channel fiber-optic vorticity probe, which has wide use in the general area of fluid mechanics, especially in the study of turbulent flows. Because it is based on the Doppler beating principle, the new technique can be applied to both laminar and turbulent flows without any need for recalibration (or any calibration at all). With the tested optical design and the optimal seeding conditions found in the experiment, it becomes straightforward to add an additional optical channel and build a compact fiber-optic vorticity probe. The commercial LDV signal processor usually has at least two independent input channels and thus is capable of processing two velocity-gradient signals simultaneously. The new fiber-optic technique combined with the LDV electronics, therefore, can be used to perform time-series measurements of $\omega_z(\mathbf{r}, t)$. Information concerning the vorticity field can be obtained by means of scanning the vorticity probe over an area of interest. We have recently set up a new turbulent tunnel flow system, and further testing of the technique in the turbulent flow is under way.

The new fiber-optic technique should be compared with PIV, which has been used increasingly in recent

years. PIV can provide a simultaneous 2D vorticity map but with relatively low spatial and temporal resolution. Because the laser sheet used in PIV has to be aligned along the direction of the mean flow to prevent the seed particles from moving out of the illumination plane, PIV is unable to measure the streamwise vorticity component. The new vorticity probe, however, measures only a local vorticity component but with high precision and much better spatial and temporal resolution. It is capable of measuring both the streamwise and the cross-stream vorticity components. With a high data rate, the new vorticity probe can be used to measure the vorticity statistics and power spectra in various turbulent flows.

In a typical PIV setup one needs approximately ten particles to construct a velocity vector and ten more velocity vectors to calculate a vorticity vector.⁴ As a result, approximately 100 particles are needed for resolving an average vorticity vector in the occupied region. The new vorticity probe requires only four particles for resolving a local vorticity component. These four particles, however, are required to be present at four closely spaced fixed locations simultaneously. To increase the data rate, one may perform independent time-series measurements for each velocity-gradient component (instead of synchronous measurements) and then apply a temporal smoothing algorithm to the respective data sets. Clearly, the two types of technique are complementary and cannot be substituted by each other.

We thank F. W. Chambers for useful discussions and for loaning us the LDV parts. The assistance of M. Lucas and his team in fabricating the experimental apparatus is gratefully acknowledged. This re-

search was supported by NASA grant NAG3-1852 and by National Science Foundation (NSF) grant DMR-9981285.

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9. This frequency resolution is obtained when a simple sinusoidal wave is used as the input signal. The actual frequency resolution of LDV for real burst signals is somewhat less.