Self-focusing and Solitons in Photorefractive Media

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Abstract. We describe the basic physical mechanisms supporting the formation of spatial solitons in photorefractive crystals, and provide an up-to-date account of the developments in the field.

1 Introduction

Diversity and complexity, on one side, and extreme regularity and stability, on the other, are two faces of nonlinearity. Solitons are a paradigm of the second, deriving their scientific and technological importance from a remarkable universality and a specific amenability to application. The phenomenological trait of a soliton is a nonlinear wave that propagates without suffering distortion to the point that, when made to interact with other waves, it maintains its localized identity in a manner analogous to a particle. This striking stability and robustness is a consequence of the action of two counterbalancing effects: linear dispersion and nonlinear self-phase modulation, a dynamic feedback loop that locks the wave into a soliton.

Although soliton science dates more than a century back, the accessible generation and observation of solitons in Optics has in the past decade caused a revival of interest. A tassel in this revival is without doubt played by photorefractive spatial solitons, which are micron-size beams that propagate even tens of millimeters without diffracting. Discovered in 1992, they have become arguably the principal playground for soliton studies, pairing relative ease in experiments with a rich diversity of underlying physical mechanisms.

In this Chapter we describe how photorefraction can support optical spatial solitons, review some of the principal phenomenology, recall the main implications they have had on soliton science, and discuss some potential applications. Previous reviews can found in [1, 2, 3, 4, 5], whereas an up-to-date account can be found in [6].

2 Self-trapping in photorefractives

One of the principal characteristics of photorefraction is that optical self-action can build up in time, accumulating into a strong nonlinearity even for low power continuous-wave laser beams [7, 8]. In 1992, M. Segev, B. Crosignani, and A. Yariv, proposed the first scheme to use photorefraction to self-focus and ultimately self-trap a low power beam into a soliton [9]. The clue to their discovery was in how they were able to transform conventional photorefractive nonlinearity from a wave-mixing process to a mutual phase-modulation process. Photorefraction mediates self-action through the redistribution of photoexcited charge, which, forming a light-induced space-charge field, modulates the material index of refraction through electro-optic effects. In the absence of an external bias field, photorefraction is driven by diffusion of charge carriers. This leads to a light-induced change in the index of refraction that scatters light from the original beam into new optical modes in different directions, leading to a process known as beam fanning. Segev et al. suggested that, when the diffusion space-charge field could be neglected with respect to the external bias field, strong self-focusing occurs and self-trapping becomes possible [9, 10].

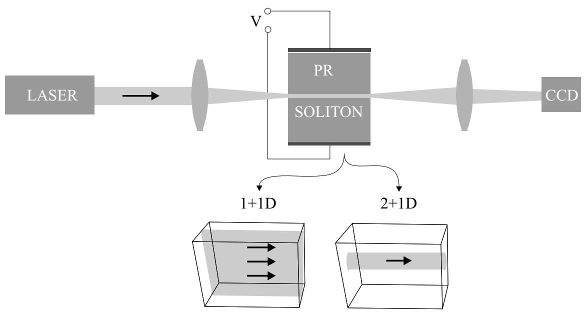

Fig. 1. Basic experimental scheme allowing the observation of 1+1D or 2+1D photorefractive solitons. For the 1+1D case, the laser beam is focused by a cylindrical lens onto the input face of the photorefractive crystal (PRχ). The beam is propagating in the crystal, exiting at the output face, and is imaged by a second lens onto a CCD camera. For the 2+1D case, the beam is focused by a standard spherical lens. The beam forms a soliton for an appropriate applied V and during a specific time window.

The first experiments, reported in [11] were carried out with the set-up shown in Fig.(1). The beam was launched in a zero-cut uniaxial photorefractive sample of rhodium-doped SBN (strontium-barium-niobate) along the ordinary axis a, with the external biasing field E_0 applied along the poling optical axis c, through two electrodes brought to a relative potential V. A characteristic electro-optic change in index of refraction $|\Delta n| \simeq (1/2)n^3 r_{33} E_0 \sim 2 - 5 \cdot 10^{-4}$ could be reached for fields of the order of $E_0 \sim$ 1-3kV/cm ($r_{33} \simeq 220pm/V$), a nonlinearity sufficient to produce the self-lensing to compensate the diffraction of a 10 μm wide beam in the visible wavelength range.

The basic result of first experiments was the self-trapping of visible continuous-wave beams into spatial solitons. A 15 μm wide continuous-wave 457nm μW beam was observed to propagate without spreading for external fields in the range of 400-500 V/cm. The effect was found to persist even under the influence of considerable noise and in conditions in which the launch was not of the shape thought to be optimal for soliton formation [12]. Within a short time a series of experiments confirmed this finding and provided the phenomenological basis for the field [12, 13, 14].

Although the results confirmed the basic qualitative predictions, they reflected a far richer and more complex phenomenology. Two principal and interesting features were observed. First, the self-trapping was transient, occurring during a specific temporal window. This has caused the self-trapping phenomenon to be termed a quasi-steady-state soliton. The second feature observed was that self-trapping could be obtained in both one and two transverse directions, suggesting the existence not only of 1+1D (one-transverse-plus-one-propagation-dimension) photorefractive solitons (envisaged by the first models) (see Fig.(4)), but also of 2+1D (two-transverse-plus-one-propagation-dimension) solitons (see Fig.(2)).

In a second set of experiments, it was found that illuminating the photorefractive sample with a second plane-wave background beam, effectively increasing the dark conductivity in the sample (see Fig.(3)), a particular set of experimental parameters transformed the transient (quasi-steady-state) nature of the solitons into stable steady-state effects [15, 16, 17, 18, 19]. These steady-state photorefractive solitons, which are termed screening solitons and form the most commonly studied photorefractive self-trapped beams, have since been observed in SBN, BSO, BGO, BTO, $BaTiO_3$, $LiNbO_3$, InP, CdZnTe, KLTN, $KNbO_3$, polymers and organic glass.

3 Nonlinear mechanism

3.1 Photorefraction

Photorefraction is observed in specific doped electro-optic crystals. In the most common case, impurities form deep donor sites and, in a lesser concentration, acceptor sites. At visible wavelengths the crystal is transparent but the donor site can be photoionized. Hence illuminated regions generate an out-of-equilibrium concentration of mobile electrons that rearranges into a space-charge distribution by diffusing to less illuminated regions and by drifting in an externally applied electric field. The charge distribution settles into the donor sites that, at equilibrium, are ionized by nearby acceptor sites, thus rendering the dislocation semi-permanent. Yet the space-charge creates a field, the space-charge field, that changes (locally) the index of refraction through the electro-optic effect. This causes the light beam (that originally generated the charge) to experience changes in its waveform, which again

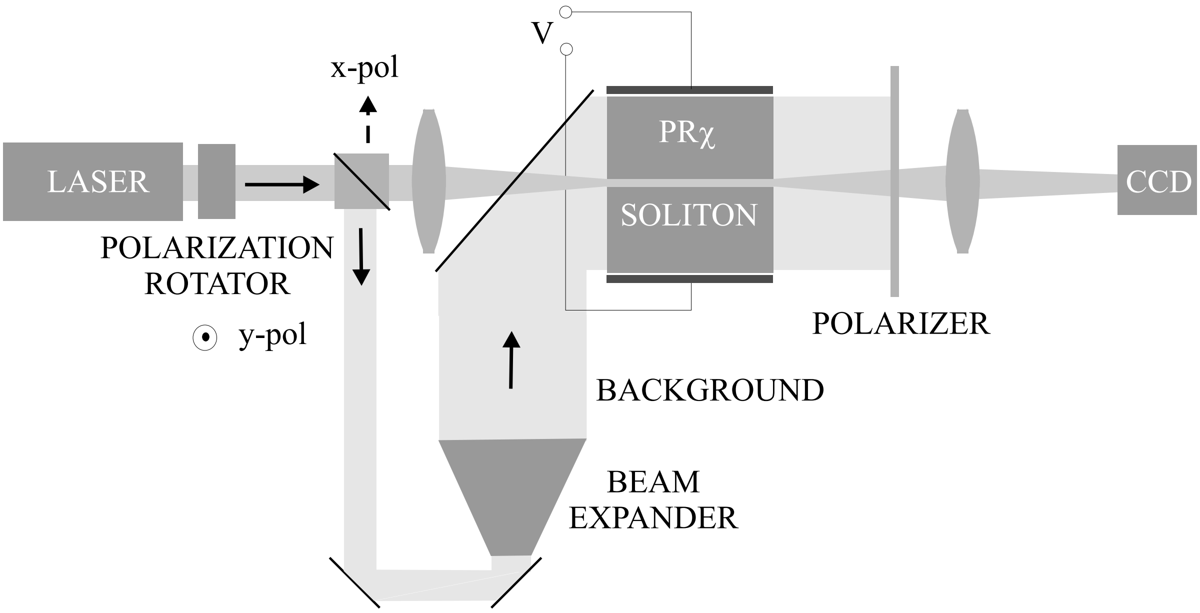

Fig. 2. Experimental two-dimensional soliton phenomenology compared to diffraction.

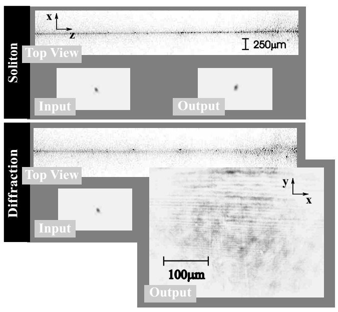

Fig. 3. A scheme to generate screening solitons [20, 21, 22]. The extraordinarily-polarized soliton-forming beam is co-propagating with an ordinarily-polarized beam of uniform intensity.

changes the charge distribution and hence the space charge field. The process eventually leads to the formation of a soliton when the light beam induces such a refractive index change that acts as a waveguide, while guiding the light beam itself in its own induced waveguide.

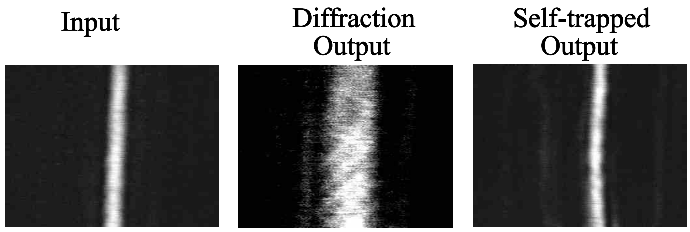

Fig. 4. A one-dimensional soliton observed in biased KLTN. From left, input 9 μm FWHM intensity distribution, output 29 μm linear diffraction (no bias), and output 9 μm self-trapped beam.

3.2 Light-induced space-charge field

To grasp how photorefraction can support solitons, the first step is to simplify the system to a one-dimensional condition, in which the beam depends only on one transverse direction x, i.e., the light intensity is $I(x,z)$, z being the propagation axis, and for conditions in which a time-independent steady-state regime has been reached. The corresponding solitons, for which I is moreover independent of z, are 1+1D solitons. In the conditions of interest, the optical intensity distribution I is such that the resulting concentration of photo-excited electrons N, the concentration of acceptor impurities N_a, and the concentration of donor impurities N_d follow the scaling $N << N_a << N_d$. In this case the x-directed space-charge field E is related to the optical intensity I through the nonlinear differential equation [23]

$$E(I_b+I)\frac{1}{1+\frac{\epsilon}{N_a q}\frac{dE}{dx}}+\frac{k_B T}{q}\frac{d}{dx}\left((I_b+I)\frac{1}{1+\frac{\epsilon}{N_a q}\frac{dE}{dx}}\right)=g, \qquad (1)$$

where ϵ is the sample dielectric constant, q is the electron charge, k_B is the Boltzmann constant, T is the temperature, and g is a constant related to the boundary conditions, i.e., to the voltage V applied on the x-facets L_x apart. I_b is the effective background illumination, the homogeneous optical intensity that allows a finite crystal conductivity.

The structure of Eq.(1) even in the one-dimensional case is complicated. Setting $Y=E/E_0$, $Q=(I_b+I)/I_b$, and $\xi=x/x_q=x/[\epsilon E_0/(N_a q)]$, Eq.(1) is

$$\frac{YQ}{1+Y'}+a\left[\frac{Q'}{1+Y'}-\frac{Q}{(1+Y')^2}Y''\right]=G, \qquad (2)$$

with $a=N_a k_b T/\epsilon E_0^2$ and $G=gE_0/I_b$. The prime stands for $(d/d\xi)$, or equally

$$Y=-a\frac{Q'}{Q}+\frac{G}{Q}+\frac{GY'}{Q}+a\frac{Y''}{1+Y'}. \qquad (3)$$

This form is rendered tractable by the fact that the greater part of spatial soliton study involves the trapping of beams with an intensity Full-Width-Half-Maximum (FWHM) $\Delta x \sim 10 \mu m$. For most configurations, $x_q \sim 0.1\ \mu m$, and $\eta = x_q/\Delta x \sim 0.01$ represents a smallness parameter, and the evaluation of the various terms indicates that

$$Y^{(0)} = \frac{G}{Q} + o(\eta), \tag{4}$$

since $a \sim 2.5$, and $G \simeq$-1 [19]. A first correction is obtained by iterating this solution into Eq.(3), and the resulting expression for Y is

$$Y^{(1)} = \frac{G}{Q} - a\frac{Q'}{Q} - \frac{Q'}{Q}\left(\frac{G}{Q}\right)^2 + o(\eta^2). \tag{5}$$

The first dominant term is generally referred to as the screening term and is the main agent leading to solitons. It is a local term, in the sense that the field (and hence the index change) at a given location depends on the optical intensity only at that (same) location. This "locality feature" is manifested in the fact that this leading term does not involve spatial derivatives or integration, has the same symmetry of the optical intensity Q, and represents a decrease in E with respect to E_0 as a consequence of charge rearrangement ($G \simeq$-1). The second term, of first order in η, involves a spatial derivative and can be identified with the diffusion field. The third, again of first order in η, is the coupling of the diffusion field with the screening field, a component sometimes referred to as deriving from charge-displacement [24]. Both these two last terms are nonlocal, in that they involve a spatial derivative, and thus provide an anti-symmetric contribution to the space charge field (Y) for a symmetric beam $I(x) = I(-x)$. That is, these last two terms lead to a beam self-action with symmetry opposite to that required to support solitons. Such terms lead to beam self-bending, which for most configurations amounts to a slight parabolic distortion of the preferentially z-oriented trajectory. The subject has attracted interest over the years [14, 21, 24] and has helped build an understanding into the limits of the local saturable nonlinearity model [25].

3.3 Nonlinear index change

In order to identify the nonlinearity, we must now translate the space-charge field E into an index modulation. Screening solitons are observed both in the noncentrosymmetric ferroelectric phase (for example, room temperature SBN) and in the centrosymmetric paraelectric phase (for example, room temperature KLTN). The standard configuration for generating screening solitons is such that a zero-cut crystal is positioned so that the x-axis is the direction along which E_0 is applied, parallel to the optical axis for ferroelectrics, the soliton beam of intensity I is extraordinarily-polarized and is propagating along z, while I_b is obtained through a co-propagating ordinarily-polarized plane-wave [20]. For a noncentrosymmetric photorefractive crystal, like SBN, $\Delta n = -\frac{1}{2}n^3 r_{33}E$, n being the unperturbed crystal index of refraction, and r_{ij} the linear electro-optic tensor of the sample. Consistent with our iterative scheme of Eq.(4), we obtain the nonlinearity [16]

$$\Delta n(I) = -\frac{1}{2}n^3 r_{33}\frac{V}{L_x}\frac{1}{1+I/I_b} = -\Delta n_0 \frac{1}{1+I/I_b}, \tag{6}$$

which constitutes a saturable nonlinearity. The nature of the self-action evidently depends on the sign of Δn_0, and is self-focusing, when $\Delta n_0 > 0$ and defocusing for $\Delta n_0 < 0$. The sign of Δn_0 is established by the orientation of the external bias with respect to the crystalline axes, having established the sign of r_{33} with respect to the chosen system of reference. For example, in SBN applying E_0 in the direction of the crystalline (ferroelectric) we observe a self-focusing nonlinearity. It is possible to apply E_0 in a direction opposite to ferroelectric axis, thus effectively changing the sign of Δn_0, then E_0 must be smaller than the coercive field, otherwise it may render the ferroelectric crystalline structure unstable and de-pole the crystal.

Analogously, for the centrosymmetric case of KLTN, the electro-optic response is quadratic $\Delta n = -(1/2)n^3 g_{eff}(\epsilon_r - 1)^2\epsilon_0^2 E^2$, where g_{eff} is the effective quadratic electro-optic coefficient, and ϵ_0 and ϵ_r are the vacuum and relative dielectric constants, and the zero order in η solution is [26, 27]

$$\Delta n(I) = -\Delta n_0 \frac{1}{(1+I/I_b)^2}, \tag{7}$$

where $\Delta n_0 = (1/2)n^3 g_{eff}(\epsilon_r - 1)^2\epsilon_0^2 E_0^2$. Here the nonlinearity is either focusing or defocusing, depending on the sign of g_{eff} and not evidently on the orientation of the applied field E_0.

3.4 The soliton-supporting nonlinear equation

Soliton formation is described by the nonlinear wave equation, representing the evolution of the beam in the light-induced index of refraction pattern Δn. Under scalar conditions (i.e. when no relevant polarization dynamics intervene), and for beam sizes much larger than the wavelength of the monochromatic beam, this evolution is described by the nonlinear monochromatic paraxial equation

$$\left[\frac{\partial}{\partial z} - \frac{i}{2k}\frac{\partial^2}{\partial x^2}\right] A(x,z) = -\frac{ik}{n}\Delta n A(x,z) \tag{8}$$

where $k = 2\pi n/\lambda$ is the wave-vector, A is the slowly varying optical field, i.e. $E_{opt}(x,z,t) = A(x,z)exp(ikz - i\omega t)$, $\omega = 2\pi c/n\lambda$, and $I = |A(x,z)|^2$.

Soliton solutions can now be identified through the self-consistency method. The balancing of diffraction (second term on the left hand side of Eq.(8)) by the nonlinearity (the term on the right hand side) leads to a solution A that has a stationary or non-evolving intensity distribution I, and hence must be of the form $A(x,z)=u(x)e^{i\Gamma z}\sqrt{I_b}$. The transverse spatial scale is normalized to the so-called nonlinear length scale $d=(\pm 2kb)^{-1/2}$, i.e., $\xi=x/d$. For photorefractive solitons in noncentrosymmetric crystals where the electro-optic response is linear in the field, i.e., $\Delta n=-\frac{1}{2}n^3 r_{33}E$, $b=(1/2)kn^2 r_{33}(V/L_x)$ and we obtain [16, 19]

$$\frac{d^2u(\xi)}{d\xi^2}=\pm\left(\frac{\Gamma}{b}-\frac{1}{1+u(\xi)^2}\right)u(\xi). \tag{9}$$

The plus sign is for $b>0$, the minus for $b<0$. The sign of b corresponds to the sign of Δn_0, and, as mentioned above, implies a self-focusing, for $b>0$, or a self-defocusing, for $b<0$, nonlinearity, having established that E decreases across the beam profile (see Eq.(6)). Both defocusing and focusing nonlinearities support solitons. A self-focusing nonlinearity traps a conventional bell-shaped beam into a bright soliton; a self-defocusing nonlinearity gives rise to a dark soliton: a non-broadening dark notch generated by a π phase jump upon an otherwise uniform amplitude wave.

A parallel formulation holds for paraelectrics, where Eq.(7) substitutes Eq.(6) [26].

3.5 Soliton waveforms and existence curve

The basic screening nonlinearity expressed by Eq.(6) indicates that, what plays a role in the attainment of self-trapping is the ratio I/I_b, but not the actual value of the intensity. This important result is the basis for low power solitons in photorefractives, the logical consequence of a cumulative effect brought to steady-state. Yet not any bell-shaped beam will necessarily self-trap. Equation (9) identifies the specific set of waveforms that can form solitons, and the observation of self-trapping is conditioned to launching a beam that reasonably approximates a given soliton waveform. Furthermore, given the saturable nature of Δn, the self-trapped waveforms of Eq.(9) not only do not have an explicit form, such as those of standard Kerr solitons, but more importantly, their shape changes for different values of saturation. The experimentally accessible parameters are evidently not the actual beam shape, but the nonlinear paramater b, the beam width, or Full-Width-at-Half-Maximum Δx, and the intensity, which, for bright solitons, is generally parametrized through the intensity ratio $u_0^2=I(0)/I_b$, i.e., the beam peak intensity at $x=0$ normalized to the background intensity. Similarly for dark solitons, the relevant parameter is $u_\infty^2=I(x\to\infty)/I_b$. The fundamental role of Eq.(9) is in providing, for each value of nonlinear response b, the values of u_0 and Δx (or $\Delta\xi$) of the bright soliton solution, the set of these points in the $(u_0,\Delta\xi)$ parameter space being termed the soliton existence curve. Analogously, the dark soliton existence curve will be in the $(u_\infty,\Delta\xi)$

plane. The experimental generation of, for example, a bright soliton will then be achieved by launching a bell-shaped waveform, such as a Gaussian beam from a laser, with a correct value of Δx and u_0, for the given b.

To construct the existence curve the first step is to reduce the number of relevant parameters in Eq.(9) by noting that it can be integrated once, giving the relationship $\Gamma/b = \log\left(1+u_0^2\right)/u_0^2$ for bright beams, and $\Gamma/b = 1/\left(1+u_\infty^2\right)$ for dark, where $u_\infty = u(\infty) = -u(-\infty)$. Thus, for example, for bright solitons Eq.(9) is

$$\frac{d^2u(\xi)}{d\xi^2} = \left(\frac{\log\left(1+u_0^2\right)}{u_0^2} - \frac{1}{1+u(\xi)^2}\right)u(\xi). \tag{10}$$

Next, Eq.(10) is integrated once by quadrature, and then the resultant first-order ordinary differential equation can be solved numerically [16, 19]. The values of $(u_0, \Delta\xi)$ that correspond to the soliton waveforms are obtained by solving a simple integral numerically, giving rise to the soliton existence curve.

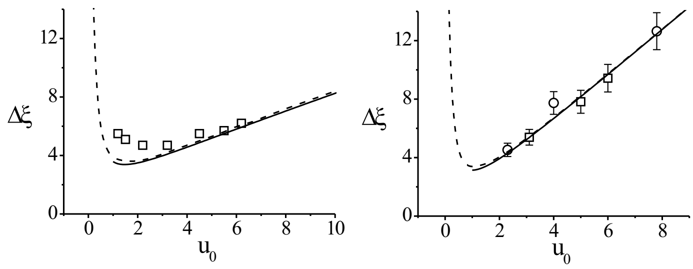

Fig. 5. Existence curves (dashed curve) for bright noncentrosymmetric (left) and centrosymmetric (right) solitons compared to results, from [22, 28] . Full line is the explicit asymptotic function describing the existence conditions from [28].

The usefulness of the notion of a soliton existence curve is evidently associated with experiments. As solutions of Eq.(10), the different points in the parameter space represent different levels of saturation and hence different waveforms, and u_0 and $\Delta\xi$ are not sufficient to characterize them. In experiments, the launch beam is a focused Gaussian beam from a laser, and for this family of beams, u_0 and FWHM unambiguously identify the waveform. The point is that since the soliton solutions are stable and robust with respect to perturbations, they attract the beam dynamics to the closest self-trapped solution by reshaping the initial launch beam in the first segments of propagation. In turn, this closest self-trapped solution will have, to a good approximation, the very same u_0 and $\Delta\xi$ of the launch.

3.6 Experiments and theory

The basic test for the validity of the approximations leading to Eq.(9) is in comparing the experimental conditions leading to solitons with those predicted through the existence curve. The best established method to carry out this test is to keep the actual Gaussian launch beam unchanged, and scan, for each fixed value of u_0, the value of E_0 that causes self-trapping. Since Δx is fixed, changing E_0 changes $\Delta\xi$ through d.

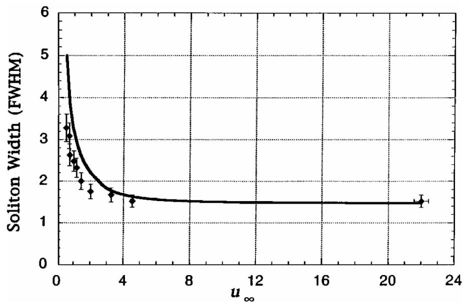

Fig. 6. Comparison between experiments and theory for (1+1)D dark screening solitons, from [29]

Experimental results compared to theory are shown in Fig.(5) for bright solitons and in Fig.(6) for dark [19, 26, 27, 29]. Whereas qualitative agreement is full, quantitative agreement in some experiments is weaker. The very small discrepancy in these cases is generally attributed to partial guiding of the background beam, for which also I_b depends on x, and in the evaluation of the actual value of the electro-optic coefficients, which depend on poling, clamping, and temperature.

4 Two-dimensional solitons

Photorefraction is able to support, both as quasi-steady-state, and as steady-state effects, also 2+1D solitons (see Fig.(2)), i.e., beams whose intensity $I(x, y)$ is well-localized in both transverse directions x and y [11, 20, 21], in the greater part of photorefractive materials, such as other ferroelectrics [30], semiconductors [31], paraelectrics [32], sillenites [33], and for most types of self-trapping: bright, photovoltaic [72], multimode [73, 34, 35], and incoherent [36, 37]. Furthermore, even (2+1)D dark solitons were observed in photorefractives, in quasi-steady-state [38] and in steady-state [39] under a bias field, as well as photovoltaic [40] and incoherent [41] dark "vortex" solitons. An example of a dark vortex screening soliton is shown in Fig.(7).

Since the Kerr nonlinearity cannot lead to stable 2+1D bright solitons, the effect, which appears similar to the formation of a self-induced optical fiber inside the bulk of the sample, represents an important achievement for optical soliton science, and the associated studies have contributed to the very understanding of higher-than-one-dimensional nonlinear waves.

The generalized relationship between the light-induced electric field **E** and the optical intensity of the beam I is

$$\nabla \cdot \left[\mathbf{E}(I_b + I) \frac{1}{1 + \frac{\epsilon}{N_a q} \nabla \cdot \mathbf{E}} + \frac{k_b T}{q} \nabla \left((I_b + I) \frac{1}{1 + \frac{\epsilon}{N_a q} \nabla \cdot \mathbf{E}} \right) \right] = 0, \quad (11)$$

and the irrotational condition for the dc space-charge field

$$\nabla \times \boldsymbol{E} = 0, \quad (12)$$

along with proper boundary conditions (the field at the electrodes). Although this two-dimensional situation can in general create components of the field **E** both in the x and in the y directions, in most conditions $E_x \gg E_y$ (there is an intrinsic asymmetry in the direction of the externally applied field E_0 along x), and hence the generally tensorial electro-optic response reduces to a scalar response $\Delta n = -(1/2) n^3 r_{eff} E_x$ analogous to the 1+1D case for an x-polarized beam. In this manner, the propagation equation is the scalar (i.e., the optical field remains uniformly x-polarized)

$$\left[\frac{\partial}{\partial z} - \frac{i}{2k} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] A(x, y, z) = -\frac{ik}{n} \Delta n A(x, y, z) \quad (13)$$

Finding **E** (and hence E_x) through Eq.(11) in itself involves a nontrivial three-dimensional, anisotropic, and spatially nonlocal nonlinear problem [42, 43, 44, 45, 46].

The first basic feature characteristic of the 2+1D process is that the photorefractive response does not follow the shape of the optical intensity (as for the 1+1D case of Eq.(6)). The combination of the x-oriented external field E_0 with the localized $I(x, y)$ gives rise to a central guiding index pattern, that to some extent recalls the 1+1D index pattern, and two lateral antiguiding lobes in the x direction [47]. These emerge as a basic feature of the response even at zero order in η. Using the very same normalization procedures described for the 1+1D case, at zero order in η the nonlinear problem becomes

$$\nabla \cdot (\boldsymbol{Y} Q) = 0 \quad (14)$$

and the irrotational condition

$$\nabla \times \boldsymbol{Y} = 0. \quad (15)$$

From these the lobular structure illustrated in the top right insert of Fig.(**??**) emerges, generated through numerical calculation of a specific solution.

The principal consequence of this index pattern that resembles the panda-fiber index distribution of polarization maintaining optical fiber, is that the nonlinearity is spatially-anisotropic, in the sense that the self-focusing is astigmatic, in most cases more pronounced in the x-direction with respect to the y, where the lobes are absent. The result is that the general response in conditions of small η supports elliptic soliton profiles [42]. Many experiments, among which the greater part of early discoveries, indicate that also circular solitons can form, i.e., with an approximately circular symmetric propagation invariant I [20, 21, 32]. The possibility of generating solitons with a round mode is particularly important for optoelectronic applications, since the circular symmetry provides optimal overlap with standard optical fiber. Without considering first corrections in η, circular symmetric 2+1D solitons cannot be explained, since the lateral lobes render the self-focusing in the x-direction stronger than in the y. In other words, in the absence of a finite contribution to the nonlinearity of the nonlocal mechanisms such as charge diffusion and displacement, round solitons are excluded by anisotropy. For relatively narrow launch beams η becomes finite and the lobular structure suffers an asymmetric distortion. In particular conditions, this distortion greatly decreases the effect of one of the two lobes, both bending the soliton trajectory and decreasing the x directed self-focusing power. When astigmatism is sufficiently decreased, round solitons emerge [48]. The apparently simple formation of round solitons from round Gaussian launch beams is in fact the consequence of a rather involved combination of anisotropy and response nonlocality.

The fact that 2+1D solitons are supported by this more complex nonlinearity does not substantially modify our soliton picture, other than the fact that we do not have a means to formulate in a straightforward manner an existence curve for (2+1)D solitons. Nevertheless, if we phenomenologically build the set of points in which it is possible to observe circular-symmetric self-trapping [21, 32], we find a single valued continuous curve that behaves and looks just like the existence curve of (1+1)D solitons (albeit at somewhat higher values of $\Delta\xi$).

5 Temporal effects and quasi-steady-state dynamics

Although the cumulative nature of photorefraction is the basis for strong nonlinear response, actual time dynamics play a negligible role in the physics behind steady-state 1+1D and 2+1D solitons. Temporal effects becomes relevant when we ask what happens to the beam during the transient from an initially diffracting Gaussian beam to a steady-state soliton, what occurs if the parameters, such as the external bias field E_0 or the light intensity distribution, are modulated in time, or, simply, what is the physical origin of quasi-steady-state or transient self-trapping.
The time dependent version of Eq.(2) truncated at zero order in η reads

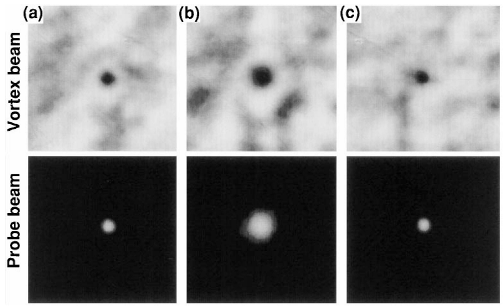

Fig. 7. A vortex screening soliton from [39]. (a) Input intensity distribution of the vortex; (b) Diffracting vortex after linear propagation to the output of the sample; (c) Self-trapped output intensity distribution in a biased sample. (bottom) Probe beam guided propagation.

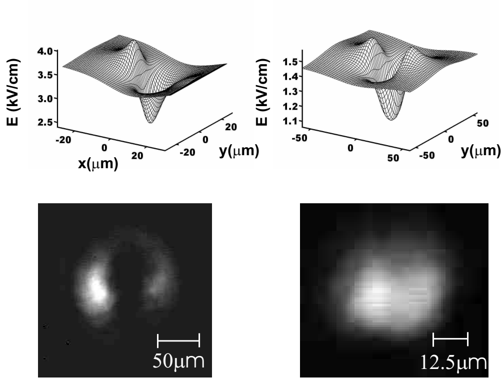

Fig. 8. (Top) Numerical evaluation of nonlinear index response for a narrow [6.5 μm] beam (left) and a wide [14 μm] beam (right). In the first case first order corrections in η play an important role, whereas in the second, these are negligible. (Bottom) Zero field electro-optic read-out of the index pattern underlying a 6.5 μm round soliton (left), and the read-out for a 14 μm soliton (elliptic) (right). Note how for the smaller beam one of the lateral lobes is almost absent (from [48]).

$$\frac{\partial Y}{\partial \tau} + QY = G, \tag{16}$$

where $\tau = t/\tau_d$, $\tau_d = \epsilon_0\epsilon_r\gamma N_a/(q\mu s(N_d - N_a)I_b)$ is the characteristic dielectric time constant, γ is the recombination rate, μ the electron mobility, s the donor impurity photoionization efficiency. As occurs for most configurations of interest to soliton dynamics, the charge recombination time τ_r=$1/(N_a\gamma)$ is much shorter than charge transport time, and no time dependence in the boundary conditions are considered (G=-1). If Q is almost constant in space and time, Eq.(16) gives an exponential build-up of Y with the time constant τ_d/Q. For solitons, Q is both space and time dependent, the continuum of different time constants leading to a highly time-nonlocal response, as can be appreciated by the formally equivalent integral version of Eq.(16)

$$Y = Ge^{-\int_0^\tau Qd\tau'}\left(1+\int_0^\tau d\tau' e^{\int_0^{\tau'} Qd\tau''}\right), \tag{17}$$

indicates [43].
The result is an evolution that presents a number of surprising phenomena.

5.1 The transition from a diffracting wave to a soliton

The full complexity of the nonlocality emerges when I undergoes relevant changes in time, i.e., during the very first collapsing stage from a diffracting to a self-trapped beam. This stage occurs for times $\tau \leq \tau_s = 1/(1+u_0^2)$ and is characterized by a stretched exponential dynamic. For example, if we consider the physically relevant time evolution of the output beam FWHM $\Delta x_{out}(t)$, we find that $\Delta x_{out}(t) = (\Delta x_{out}(0) - \Delta x_{in})e^{-(\tau/\tau_s)^\beta} + \Delta x_{in}$, where Δx_{in} is the input beam FWHM and $\beta < 1$ is the characteristic stretching parameter [49]. Intuitively, since the final size of the beam depends on the distributed self-focusing along the entire propagation axis z and each self-focusing process has a different time constant for each z (owing to the initial diffracting I), the stretching is a direct consequence of the superposition of a continuum of different time scales.

The situation is even more complicated for 2+1D solitons. Here there are generally two coupled dynamics $\Delta x_{out}(t)$ and $\Delta y_{out}(t)$, which lead also to an evolution in time of beam ellipticity [50].

5.2 External modulation of soliton parameters

Although specific experiments have been dedicated to the study of self-trapping with a time-dependent external bias E_0 [79, 52, 53, 54], or the spatial self-trapping of a single pulse [55, 56], the most widely studied case is when the transverse beam intensity is randomly modulated by having the beam pass through a rotating diffuser before being launched in the photorefractive sample. The effect gives rise to what are generally termed incoherent solitons. Consider Q to be a stochastic process with a characteristic time scale $\tau_r \ll \tau_s$. Defining $\overline{Q}(\tau) = \int_\tau^{\tau+\tau_r} Qd\tau'$, this will be a deterministic function of τ, and the entire space-charge formation process is described by Eq.(17) with Q substituted with $\overline{Q}$. In particular, the steady state solution (for $\tau \to \infty$) will simply be, in the tractable 1+1D case, $Y = G/\overline{Q}(\infty)$, and hence follows the case of a saturable nonlinearity.

5.3 Quasi-steady-state solitons

As described previously, in the absence of background illumination self-trapping can occur during a time window, known as the soliton plateau, after which the beam once again diffracts through the sample. The numerical solution of Eq.(17) coupled to the parabolic wave equation confirms experimental findings. To understand the process and the underlying mechanisms, a generalized spatio-temporal soliton self-consistent approach can be implemented, the resulting soliton supporting equation involves an exponential nonlinearity that allows the prediction of the size of the quasi-steady-state soliton Δx as a function of experimental conditions [57]. In particular,

$$\Delta x = \frac{\Delta\xi_{min}\lambda}{2\pi n^2 a_m} E_0^{-m/2}, \tag{18}$$

where $\Delta\xi_{min} \simeq 3.07$, $a_1 = (r_{eff})^{1/2}$ and $a_2 = \varepsilon_0\varepsilon_r(g_{eff})^{1/2}$, and m =1(2) for noncentrosymmetric (centrosymmetric) samples, and λ is the beam wavelength.

5.4 Response change in beams that approximately do not evolve in time

In very special cases in which the beam does not undergo time evolution, we can considerably simplify the prediction for the build of the space-charge field [58]. In this case Eq.(17) is simplified to give

$$Y = e^{-\tau Q}(1 + \frac{1}{Q}(e^{\tau Q} - 1)), \tag{19}$$

This approach can be meaningful and useful for conditions in which the beam initially suffers a negligible amount of diffraction [59, 60, 61, 62, 63].

6 Non-screening self-trapping mechanisms

In both the 1+1D and 2+1D self-trapping mechanisms described above, the driving process is the displacement of photoexcited charge so as to screen the externally applied field E_0, the mechanism being generally termed the screening nonlinearity. Further studies have uncovered other self-trapping mechanisms through different photorefractive effects, the main developments being summarized in Table 6.

Table 1. Principal non-screening self-trapping mechanisms.

Mechanism	Solitons	References
Photovoltaic	1+1D Dark, 2+1D Vortex	[64, 65, 66, 40]
Diffusion-driven	1+1D and 2+1D Self-focusing	[67, 68]
Resonance Enhancement	1+1D, 2+1D Bright	[69, 31, 70]
Spontaneous	1+1D, 2+1D Self-trapping	[71]

7 Materials

Photorefractive solitons can be observed in a number of different materials, these including most photorefractive crystals, polymers and organic gels. A map of main experimental findings associated to the different materials is contained in the Table 2.

Table 2. Principal experiments on materials supporting solitons.

Material	Soliton mechanism	References
SBN, $KNbO_3$, $BaTiO_3$, Polymers	Screening	[16, 30, 134, 135, 74]
$LiNbO_3$, KNSBN	Photovoltaic	[65, 64, 66, 72]
KLTN, Organic Gels, Unpoled SBN	Quadratic screening	[27, 32, 75, 76]
KLTN (near transition)	diffusion-driven, spontaneous	[68, 71]
BGO, BSO, BTO	Screening in optically-active media	[15, 77, 33]
InP, CdZnTe	Resonantly enhanced	[31, 70]

8 Soliton interaction-collisions

A soliton is not simply a propagation invariant wave, but the result of the balancing of diffraction by distributed nonlinear self-lensing. This "dynamic" equilibrium makes the phenomenon stable to perturbations and leads to a characteristic particle-like soliton-soliton phenomenology. Photorefractive crystals form an ideal setting for the observation and study of this multi-beam property, for a number of reasons. First, photorefractives offers a very strong nonlinearity at very low (microwatt) power levels. Second, launching and detecting different beams propagating through a bulk environment is relatively simple. Third, the saturable nature of the nonlinearity makes the collisional phenomenology richer, including events such as soliton fission and fusion. Last but not least, the possibility of experimenting with 2+1D solitons makes previously inaccessible soliton-soliton interaction scenarios observable, such as soliton spiraling and interactions between solitons carrying angular momentum. In fact, the system is so amenable to soliton propagation, that it can simultaneously support a 1+1D and a 2+1D soliton, allowing the singular study of collisions between solitons of different dimensionality. For all of these reasons, almost all pioneering experiments of soliton interactions in 2+1D settings were obtained first with photorefractive solitons, and only later (many years later) were followed up by similar experiments in other soliton-supporting saturable material systems.

The principal experiments on soliton interactions are summarized in Table 3. The * marks those cases where these experiments were the first in all soliton studies, including those beyond optics.

Table 3. Principal experimental results on photorefractive soliton collisions.

Phenomenon	Year	References
Incoherent collisions between 1+1D solitons and fusion *	1996	[78]
Incoherent collisions between 2+1D solitons and fusion	1996	[51]
Soliton annihilation and birth (fission) in coherent collisions *	1997-1998	[80, 81]
Coherent interaction between 1+1D and 2+1D solitons	1997-1998	[82, 83]
3D soliton spiraling *	1997	[84, 85]
Hybrid-dimensional collisions *	2000	[86]
Collisions between counter-propagating solitons *	1999-2004	[87, 88, 89]

9 Vector and composite solitons

Vector solitons are an important part of basic soliton phenomenology. Vector solitons are self-trapped beams composed of more than one (independent) optical field. In analogy to linear guiding terminology, a scalar (i.e., single component) soliton occupies the lowest mode of its self-induced waveguide. In turn, a vector soliton emerges when this waveguide is the result of the joint action of two (or more) independent optical fields (e.g., when the fields are not coherent with each other or when they are at two orthogonal polarizations), and all fields occupy the lowest mode. The vector soliton is said to be composite or multi-mode when one or more of the independent fields or components occupies higher modes. The components can be independent because they have orthogonal polarizations (Manakov-like solitons), different wavelengths, or simply are from mutually incoherent sources [90]. The principal experimental results on photorefractive vector soliton studies are summarized in Table 4. The * marks those cases where these experiments were the first in all soliton studies, including those beyond optics.

Table 4. Principal experimental results on photorefractive vector solitons.

Phenomenon	Year	References
Manakov-like soliton	1996	[91, 92]
Bright-dark vector soliton	1996	[93]
Multi-mode multi-hump solitons *	1998	[94]
Dipole-type composite solitons *	2000	[73, 35]
Propeller soliton *	2001	[34]
Collisions of Manakov-like solitons *	1999-2001	[95, 96]
Collisions of multi-mode solitons *	1999	[97]

10 Incoherent (random-phase) solitons

As briefly discussed previously, photorefraction is able to also trap beams that have a randomly varying Q, since the cumulative nature of the nonlinear response can in specific conditions be exclusively driven by $\overline{Q}$, which for a static stochastic process is well defined and deterministic. This allows the self-trapping of a spatially incoherent light beam, and even of a spatio-temporally incoherent beam. The result, an "incoherent soliton", is due to the simultaneous guiding of all the underlying independent light fields by a nonlinear index pattern that is generated by the time-average of the intensity resulting from the superposition of all the components. The phenomenon has attracted a considerable amount of interest and opened up a field in its own right. The principal achievements are summarized in Table 5. In this case, all the pioneering experimental work was performed in photorefractives.

Table 5. Principal experimental and theoretical results on incoherent solitons.

Achievement	Year	References
Self-trapping of a partially incoherent beam	1996	[36]
White-light soliton	1997	[37]
Coherent-density and modal theory	1997	[98, 99]
Dark incoherent solitons	1998	[41, 100]
Mutual coherence theory	1998	[101]
Anti-dark incoherent states	2000	[102]
Elliptic incoherent solitons	2000-2004	[103, 104]
Interaction of incoherent solitons	1998-2000	[103, 105, 106]
Incoherent modulation instability	2000-2004	[107, 108]
Arresting transverse instabilities via incoherence	2000	[109, 110]
White-light soliton theory	2003	[111, 112]
Modulation instability of white light	2002-2005	[113]
Modulation instability of white incoherent light	2004	[114]

11 Applications

Photorefractive solitons are not only the instruments for a substantial expansion of our understanding of nonlinear physics, but also for their role in developing new applicative designs, concepts, and devices. A soliton in itself is a beam that propagates through a bulk medium in a guided fashion, i.e., without losing its spatial definition. In turn, since photorefraction is wavelength dependent (long wavelengths are not able to photoactivate impurities) but the electro-optic response is much lesser so, a photorefractive soliton can guide passively infrared beams, these undergoing a purely linear propagation. In other words, in conditions in which the photorefractive charge does not redistribute, a photorefractive soliton imprints in the bulk a waveguide for non-photorefractively active light. The purely nonlinear nature of soliton interaction allows also an all-optical type of elaboration of light signals. Finally, the photorefractive crystal is both electro-optic and generally has strong electronic nonlinearity. The electro-optic response allows a fast, versatile, and multi-functional optical manipulation technique based on solitons, known as soliton electro-activation, whereas the nonlinear response for wavelength mixing and conversion can be strongly enhanced when combined with self-trapping. A brief list of principal experimental achievements is reported in Table 6

Table 6. Principal results on applications of photorefractive solitons.

Achievement	Year	References
Guiding a beam through a transient soliton	1995	[115]
Guiding a beam through a steady-state soliton	1996	[116]
Soliton-based Y junction	1996	[117, 118, 119]
Solitons at telecommunication wavelengths	1997	[31, 70, 120]
Soliton-based directional coupler	1999	[121]
Second-harmonic generation in a soliton	1999-2004	[122, 123, 124]
Soliton electro-optic effects	2000	[125]
Image transmission through waveguides induced by incoherent solitons	2001	[126]
Permanent fixing of multiple soliton-based devices	2001	[127]
Soliton electro-activation	2002	[128]
Optical parametric oscillation in solitons	2002	[129]
Coupling of fibers to soliton waveguides	2004	[130]
Low voltage solitons through top electrodes	2004	[131]
Soliton waveguides in organic glass	2005	[132]
Soliton-based fiber-slab couplers	2005	[133]

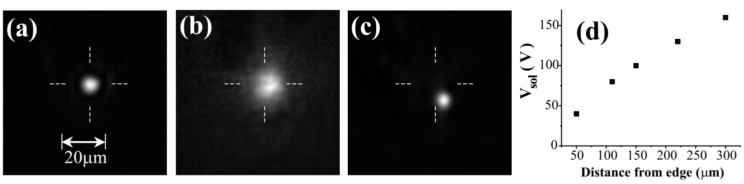

Fig. 9. 2+1D 7 μm photorefractive soliton at quasi-digital voltages generated through a top-sided electrode geometry. Input intensity distribution (a); diffraction at output face (b); and self-trapped soliton for 40 V (c). Measured values of required bias voltage to achieve self-trapping vs distance from the crystal edge is plotted in (d). From [131]

12 Concluding Remarks

The study of photorefractive solitons has, in the past decade, played a major role in the development of the present understanding of optical solitons, and has had an important impact on soliton science and nonlinear waves in general, in a variety of systems beyond optics. The drive continues to this day. The last few years (2002-2006) have witnessed another important breakthrough obtained with solitons in photorefractives: the invention of the optical induction method to make nonlinear photonic lattices [136, 137, 138], which has become the main experimental scheme to explore spatial soliton phenomena in periodic systems. The ability to induce 1D or 2D photonic lattices of any structure, and to tune the polarity and strength of the nonlinearity, have led to the observation of a series of soliton phenomena, many of which being the first observation in any system in nature. Examples include the observations of 2D lattice ("discrete") solitons [138], spatial gap solitons [137], vortex lattice solitons [139, 140], random-phase lattice solitons [141], solitons in quasi-crystals [142], as well as closely related phenomena of Brillouin-zone spectroscopy of photonic lattices [143], Zener tunneling in 2D photonic lattices [144], dynamics of polarons in photonic lattices [145], and much more. These are just a small sample group from the recent experiments with solitons in periodic structures, and they were all observed by employing the photorefractive screening nonlinearity while taking advantage of its inherent nonlinear anisotropy [136].

When coming to summarize this chapter, looking back and reviewing the progress and the impact photorefractive solitons have had on soliton science since their discovery in 1992, instead of sitting back and enjoy the accumulating progress, we chose to look forward with the wish that the best is yet to come.

13 Acknowledgements

The work of the authors has been supported over the years by a number of agencies. In particular, we wish to thank the Istituto Italiano di Fisica della Materia, the Ministero Italiano della Ricerca through the FIRB funding initiative, the US Army Research Office, the US National Science Foundation, and the Israeli Science Foundation.

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