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Research Article

POLYNOMIAL SOLUTION OF ONE- DIMENSIONAL ELASTIC WAVE EQUATION

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ABSTRACT
The present paper deals with the problem of one-dimensional elastic wave equation with certain boundary conditions. The polynomial solution of this problem is obtained as a polynomial using double interpolation.

Keywords: *One-dimensional Elastic Wave Equation, Polynomial Solution, Double Interpolation*

INTRODUCTION

In the study of wave propagation in different elastic solids it yields one-dimensional wave equation in some special cases. Usually the solution of these equations is given in Fourier series form (Iyengar *et al.*, 2007). In this paper, an attempt is made to find the solution of one-dimensional elastic wave equation with certain boundary conditions of a polynomial. Using the double interpolation (Scanborough, 1966) and Crank- Nicolson method (Stanley, 1982), the solution of one-dimensional wave equation obtained as an interpolating polynomial. The method is useful in case of simple harmonic motion.

Formulation of the Problem

We consider the following boundary value problem of one-dimensional wave equation.

$$25\ u_{xx} = u_{tt} \tag{1}$$

subject to the following boundary conditions

$$u(0,t) = 0 \tag{2}$$

$$u(4,t) = 0 \tag{3}$$

$$u(x,0) = x^2(x-4) \tag{4}$$

$$u_t(x,0) = 0 \tag{5}$$

where $u_t = \frac{\partial(x,t)}{\partial t}$

and $0 \le t \le 0.8$ and $0 \le x \le 4$

Solution of the Problem

A net of square mesher is formed by straight lines parallel to the coordinate axes for the domain of the problem. We take the interval of differencing of x as 1, i.e., h=1. Comparing the equation with $\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}$, we have *a*=5. Hence, as per Crank- Nicolson's method the interval of differencing *k=h/a=1/5=0.2*. Thus $x_0 = 0,\ x_1 = 1,\ x_2 = 2, x_3 = 3,\ x_4 = 4,$ and $t_0 = 0$, $t_1 = \frac{1}{5}$, $t_2 = \frac{2}{5}$, $t_3 = \frac{3}{5}$, $t_4 = \frac{4}{5}$,. We denote $u_{rs} = u(r, s/a)$, (r,s = 0, 1, 2, 3 , 4.)

The boundary condition (2) gives

$$u_{00} = u_{01} = u_{02} = u_{03} = u_{04} = 0 \tag{6}$$

The boundary condition (3) gives

$$u_{40} = u_{41} = u_{42} = u_{43} = u_{44} = 0 \tag{7}$$

The initial condition (4) gives

$$u_{00} = 0,\ u_{10} = -3, u_{20} = -8, u_{30} = -9, u_{40} = 0. \tag{8}$$

The initial condition (5) gives

$$u_{i1} = \frac{1}{2}\left[u_{(i-1)0} + u_{(i+1)0}\right].$$

This gives $u_{11} = -4\ \ u_{21} = -6,\ u_{31} = -4.$ (9)

The other values are obtained from the recurrence relation

$$u_{i(j+1)} = u_{(i+1)j} + u_{(i-1)j} - u_{i(j-1)}$$

and are shown in the Table 1.

Table 1

x **t**	**0**	**1**	**2**	**3**	**4**
0	0	-3	-8	-9	0
$\frac{1}{5}$	0	-4	-6	-4	0
$\frac{2}{5}$	0	-3	-2	3	0
$\frac{3}{5}$	0	2	6	2	0
$\frac{4}{5}$	0	3	2	3	0

The differences of different orders of $\Delta^{0+k}u_{oi}$; *(i, k=0,1,2,3,4))* are zero.

i.e., $\Delta^{0+k}u_{oi} = 0$ *(i, k=0,1,2,3, 4)* (10)

The general formula for the different order of differences are given by

$$\Delta^{m+n}u_{00} = \Delta^{m+n}u_{0n} + (-1)^1 n\Delta^{m+0}u_{0(n-1)} + (-1)^2\frac{n(n-1)}{2!}\Delta^{m+0}u_{0(n-2)} + (-1)^3\frac{n(n-1)(n-2)}{3!}\Delta^{m+0}u_{0(n-3)} + \ldots..$$

$$+\ (-1)^n\Delta^{m+0}u_{oo} \tag{11}$$

The differences of different order are given by in equations (12) to (21).

Table 2

u_{li}	$\Delta^{0+1}u_{1i}$	$\Delta^{0+2}u_{1i}$	$\Delta^{0+3}u_{1i}$	$\Delta^{0+4}u_{1i}$
-3	-1			
-4	1	2	2	
-3	5	4	-8	-10
2	1	-4		
3				

From Table 2 we have, $\Delta^{0+1}u_{10} = -1;\ \Delta^{0+2}u_{10} = 2\ \ \Delta^{0+3}u_{10} = 2;\ \Delta^{0+4}u_{10} = -10;$ (12)

Similarly we obtain,

$$\Delta^{0+1}u_{20} = 2; \Delta^{0+2}u_{20} = 2; \Delta^{0+3}u_{20} = 2 \; \Delta^{0+4}u_{20} = -18 \; . \tag{13}$$

$$\Delta^{0+1}u_{30} = 5; \Delta^{0+2}u_{30} = 2; \Delta^{0+3}u_{30} = -10; \Delta^{0+4}u_{30} = 20. \tag{14}$$

and

$$\Delta^{1+0}u_{00} = -3; \Delta^{2+0}u_{00} = -2; \Delta^{3+0}u_{00} = 2; \Delta^{4+0}u_{00} = 4 \; . \tag{15}$$

$$\Delta^{1+0}u_{01} = -4; \Delta^{2+0}u_{01} = 2; \Delta^{3+0}u_{01} = 2; \Delta^{4+0}u_{01} = -4. \tag{16}$$

$$\Delta^{1+0}u_{02} = -3; \Delta^{2+0}u_{02} = 4; \Delta^{3+0}u_{02} = 0; \Delta^{4+0}u_{02} = -12. \tag{17}$$

$$\Delta^{1+0}u_{03} = -2; \Delta^{2+0}u_{03} = 6; \Delta^{3+0}u_{03} = -14; \Delta^{4+0}u_{03} = 24. \tag{18}$$

$$\Delta^{1+0}u_{04} = 3; \Delta^{2+0}u_{04} = -4; \Delta^{3+0}u_{04} = 6; \Delta^{4+0}u_{04} = -12. \tag{19}$$

From equations (11) to (19) we obtain

$$\Delta^{1+1}u_{00} = -1 \;\; \Delta^{1+2}u_{00} = 2; \; \Delta^{2+2}u_{00} = -2; \; \Delta^{3+1}u_{00} = 0;$$

$$\Delta^{1+3}u_{00} = 2. \tag{20}$$

The formula for double interpolation (Scanborough, 1966) up to fifth order differences is

$$u(x,t) = u_{00} + \left[\frac{x-x_0}{h}\Delta^{1+0}_{u_{00}} + \frac{t-t_0}{k}\Delta^{0+1}u_{00}\right]$$
$$+\frac{1}{2!}\left[\frac{(x-x_0)(x-x_1)}{h^2}\Delta^{2+0}u_{00} + \frac{2(x-x_0)(t-t_0)}{hk}\Delta^{1+1}u_{00} + \frac{(t-t_0)(t-t_1)}{k^2}\Delta^{0+2}u_{00}\right]$$
$$+\frac{1}{3!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)}{h^3}\Delta^{3+0}u_{00} + \frac{3(x-x_0)(x-x_1)(t-t_1)}{h^2k}\Delta^{2+0}u_{00}\right.$$
$$\left.+\frac{3(x-x_0)(t-t_0)(t-t_1)}{hk^2}\Delta^{1+2}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)}{k^3}\Delta^{0+3}u_{00}\right]$$
$$+\frac{1}{4!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{h^4}\Delta^{4+0}u_{00} + \frac{(x-x_0)(x-x_1)(x-x_2)(t-t_0)}{h^3k}\Delta^{3+1}u_{00}\right.$$
$$\frac{+6(x-x_0)(x-x_1)(t-t_0)(t-t_1)}{h^2k^2}\Delta^{2+2}u_{00} + \frac{4(x-x_0)(t-t_0)(t-t_1)(t-t_2)}{hk^3}\Delta^{1+3}u_{00}$$
$$\left.+\frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{k^4}\Delta^{0+4}u_{00}\right] \tag{21}$$

Substituting the values of $\Delta^{m+n}u_{00}$ from equations (10), (15) and (20) in equation (21), and simplifying we obtain

$$u(x,t) = \frac{1}{6}\left(x^4 - 17x^2\right) - \frac{1}{3}\left(x^3 + x\right) + \frac{1}{3}\left(xT^3 + xT\right) - \frac{1}{2}\left(xT^2 + x^2T\right) \tag{22}$$

where $T = \frac{t}{k} = \frac{t}{1/5} = 5t$

The required polynomial solution *u(x, t)* of one- dimensional equation (1) with boundary conditions (2) to (5) is given by equation (22).

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