



## **Sensitivity analysis of optimal operation of irrigation supply systems with water quality considerations**

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**Abstract.** A model for optimal operation of water supply/irrigation systems of various water quality sources, with treatment plants, multiple water quality conservative factors, and dilution junctions is presented. The objective function includes water cost at the sources, water conveyance costs which account for the hydraulics of the network indirectly, water treatment cost, and yield reduction costs of irrigated crops due to irrigation with poor quality water. The model can be used for systems with supply by canals as well as pipes, which serve both drinking water demands of urban/rural consumers and field irrigation requirements. The general nonlinear optimization problem has been simplified by decomposing it to a problem with linear constraints and nonlinear objective function. This problem is solved using the projected gradient method. The method is demonstrated for a regional water supply system in southern Israel that contains 39 pipes, 37 nodes, 11 sources, 10 agricultural consumers, and 4 domestic consumers. The optimal operation solution is described by discharge and salinity values for all pipes of the network. Sensitivity of the optimal solution to changes in the parameters is examined. The solution was found to be sensitive to the upper limit on drinking water quality, with total cost being reduced by 5% as the upper limit increases from 260 to 600 mg Cl l<sup>-1</sup>. The effect of income from unit crop yield is more pronounced. An increase of income by a factor of 20 results in an increase of the total cost by a factor of 3, thus encouraging more use of fresh water as long as the marginal cost of water supply is smaller than the marginal decrease in yield loss. The effect of conveyance cost becomes more pronounced as its cost increases. An increase by a factor of 100 results in an increase of the total cost by about 14%. The network studied has a long pipe that connects two distinct parts of the network and permits the supply of fresh water from one part to the other. Increasing the maximum permitted discharge in this pipe from 0 to 200 m<sup>3</sup> h<sup>-1</sup> reduces the total cost by 11%. Increasing the maximum discharge at one of the sources from 90 to 300 m<sup>3</sup> h<sup>-1</sup> reduces the total cost by about 8%.

**Key words:** irrigation systems, network analysis, optimal operation of water supply systems, water quality, water supply systems

### **Introduction**

With the ever increasing use of high-quality water sources for domestic consumption, the use of brackish water for irrigation becomes more attractive, especially in arid regions. Water with different salinity levels can be used for

irrigation of a broad variety of crops (Oron 1987), several scientists have suggested that these variable requirements be met by diluting brackish (saline) water with high-quality water, while at the same time increasing the amount of water available for irrigation (Jury et al. 1980; Liang & Nahaji 1983; Males et al. 1985; Schwartz et al. 1985; Sinai et al. 1985a, 1985b; 1987; Shah & Sinai 1985; Pessen et al. 1986, 1989). Dilution occurs at the junctions of the network and can be automatically controlled. Pessen et al. (1986, 1989), and Reike et al. (1987) have studied the operation and control of dilution junctions in water supply systems. They first analyzed, simulated and tested the case of a single dilution junction in water distribution systems and suggested control schemes for simultaneous control of salinity and one or more hydraulic variables, e.g. outlet discharge, outlet pressure, or water level in an adjacent operational tank. Automated control of entire water supply systems with dilution junctions has been studied and successfully simulated by Reike et al. (1987). They suggested the use of hierarchical control concepts with two levels of control: i) At the lower level, junctions are controlled with a remote connection to an irrigation computer at the higher level; and ii) the higher level control, which employs a complete network model to search for an optimal operation and transmits set point control commands to the lower level. Automatically controlled dilution junctions have been operated successfully in southern Israel as a component of the irrigation/water supply systems in this region.

The optimal operation problem of such water systems considers the use of dilution junctions and water treatment plants that enable changes of water quality within the supply network during operation. Diverse water quality at the sources can be changed by dilution and treatment to meet variable demand (with time and location) of consumers. Skillful management of the water distribution system may permit simultaneous supply of water subject to water quality and hydraulic constraints.

Cohen (1991), Cohen et al. (2000a, 2000b, 2000c, 2003), Ostfeld & Shamir (1993a, 1993b), Mehrez et al. (1992) and Percia et al. (1997) suggested models for optimal operation of multiquality water systems of the type mentioned above. Ostfeld and Shamir (1993a, 1993b) considered change of water quality by dilution and treatment plants. Their objective function includes two terms: (1) water cost, which is the sum of water cost at the sources and the treatment cost needed to improve water quality; and (2) energy cost needed to operate pumping stations. They do not consider agricultural crop consumers and yield loss due to irrigation with brackish water. Percia et al. (1997) simulated regional network in southern Israel. Their concern was with the short time operation problem, attempting to include hourly changes in the electricity tariff for pumping. They did not consider long-term effects, such as agricultural crop yield losses due to use of irrigation with low quality water.

Cohen (1991) and Cohen et al. (2000a, 2000b, 2000c, 2003) suggested a more general model for optimal operation of multiquality networks. They include in the objective function the cost of water at the sources (depending on water quality), cost of hydraulic operation of the system (pump stations, booster pumps), yield loss of agricultural crops due to irrigation with poor quality water, and cost of water treatment. Water quality was expressed by multiple conservative water quality parameters (dependent and independent). Typical independent (primary) water quality parameters are ions, e.g. magnesium, sulfur, chloride, whose concentration in the irrigation water determines its water quality. Dependent water quality factors are those that are expressed as functional relations between constituent concentrations. For example, the sodium adsorption ratio (SAR), where  $SAR = [Na] / \sqrt{[Mg] + [Ca]}$  and  $[Na]$ ,  $[Mg]$ , and  $[Ca]$  are the concentrations of sodium, magnesium and calcium, respectively. They also suggested the decomposition of the general solute transport process into two submodels: (i) a water quality model,  $Q-C$ , and (ii) a hydraulic model,  $Q-H$  ( $Q$  is water discharge,  $C$  is conservative water quality parameters, and  $H$  is hydraulic head parameters). Two submodels were developed for the  $Q-C$  problem (Cohen et al. 2000a) and for the  $Q-H$  problem (Cohen et al. 2000b). The two submodels were then merged into a more general model,  $Q-C-H$  (Cohen et al. 2000c, 2003), which deals with both water quality and hydraulic aspects of multiquality networks.

Application of the water quality submodel ( $Q-C$ ) to realistic rural regional system is demonstrated in this paper. The model is general enough to deal with either pipeline or canal irrigation systems, and can be used as a simplified model for optimal operation of multiquality water supply or irrigation conveyance systems. The hydraulics of the system is taken into account indirectly by adopting nonlinear functions for water conveyance in the pipes/canals, with specified maximum discharges in all pipes/canals. This simplification overcomes some of the complexities associated with the simultaneous solution of multiple water quality factors, treatment plants, and complete hydraulic constraints for given loading conditions in the network.

Questions concerning the sensitivity of the solution to various parameter values of the problem are considered in this paper. These parameters include changes in income from unit crop yield, upper quality limits for drinking water, conveyance costs, network topology, and supply capacity of the source. A single water quality parameter (salinity) is considered in a network without treatment plants (they are included in the mathematical description of the model, but not in the example analyzed).

Additional treatment is sometimes required for the irrigation of more sensitive crops or for domestic use. Within the context of water supply system operation, a treatment plant can be regarded as a black box where the relation of the outflow concentration ( $c_{out}$ ) of a water quality parameter to its inflow

concentration ( $c_{in}$ ) is defined by a removal ratio  $r$ , with  $r = 1 - (c_{out}/c_{in})$ , which is the operational decision variable. (Small amounts of water that are removed from the system together with the quality constituents are disregarded in the current treatment of the problem).

### Mathematical model

A short description of the water quality ( $Q-C$ ) model is presented here. For a more detailed description, see Cohen et al. (2000a, 2000c). The topology of the network is represented by the connectivity matrix  $\mathbf{D}$ , and the adjacency between nodes and pipes is given by matrix  $\mathbf{A}$ . The cyclic structure of the network is defined by loops and pseudo-loops (paths between nodes at which the heads are fixed) and are represented by a cyclic matrix  $\mathbf{L}$ . The connectivity of the treatment plants to the pipes are represented by a matrix  $\mathbf{B}_t$ . Each of these topological matrices elements are equal +1 or -1 or zero.

The decision variables are water flows (discharges) and water quality concentration in all pipes, and the removal ratios in the treatment plants. Use of a loop-flow formulation of the pipe discharges obviates the need for explicit water flow continuity equations at the nodes. For this formulation, an initial flow distribution which satisfies water continuity at all the nodes is specified, and is then modified as the solution process progresses in a way such that flow continuity at the nodes is maintained at all times. This is done by considering the circular flows,  $\mathbf{q}$ , in loops and pseudo-loops as the decision variables, rather than pipe flow around every node which is an equivalent formulation.

The relationships between pipe discharges,  $\mathbf{q}_a$ , and circular the discharge vector  $\mathbf{q}$  is given by:

$$\mathbf{q}_a = \mathbf{q}_a^0 + \mathbf{L}^T \mathbf{q} \quad (1)$$

where  $\mathbf{q}_a^0$  is the initial pipe discharge,  $\mathbf{L}$  the cyclic matrix, and T the transpose operator.

The relationship between the discharges from the sources,  $\mathbf{q}_s$ , and the pipe discharges is:

$$\mathbf{q}_s = \hat{\mathbf{A}}[\mathbf{q}_a^0 + \mathbf{L}^T \mathbf{q}] \quad (2)$$

where  $\hat{\mathbf{A}}$  is a submatrix of  $\mathbf{A}$ , obtained from the rows of  $\mathbf{A}$  which are related to sources.

The relationship between the treatment plant discharges,  $\mathbf{q}_t$ , and the circular discharges is

$$\mathbf{q}_t = \mathbf{B}_t[\mathbf{q}_a^0 + \mathbf{L}^T \mathbf{q}] \quad (3)$$

Water quality can be described by two matrices: matrix  $\mathbf{C}$  for the primary water quality parameters, and matrix  $\mathbf{C}^d$  for the dependent quality parameters (e.g. SAR). The removal ratios in the treatment plants are given by the matrix  $\mathbf{R}$ , whose components are the removal ratios,  $r$ , in the treatment plants.

### Constraints

#### *Mass conservation law for the quality parameters*

This constraint is expressed by  $n_3$  (the number of primary quality parameters) sets of equations, one set for each primary quality parameter. Each set of equations includes an equation for each non-source node.

$$\sum_{(ij) \in E_1} a_{ij} Q_{ij} C_{ijm} + \sum_{(ij) \in E_2} a_{ij} Q_{ij} C_{ijm} - \left\{ \sum_{(ij) \in E_2} a_{ij} Q_{ij} C_{ijm} R_{ijm} \right\} - d_j C_{jjm} = 0 \quad \forall i \notin N_1 \quad \text{and} \quad \forall m \in M_1 \quad (4)$$

where  $a_{ij}$  is the element  $ij$  of the adjacency matrix,  $d_j$  the consumption at node  $j$ ,  $M_1$  the set of primary quality parameters,  $N_1$  the set of source nodes,  $N_2$  the set of consumer nodes,  $Q_{ij}$  the flow from node  $i$  to  $j$ ,  $C_{ijm}$  the concentration of primary quality parameter  $m$  in the pipe between nodes  $i$  and  $j$ ,  $C_{jjm}$  the concentration of primary quality parameter  $m$  at node  $j$ ,  $R_{ijm}$  the removal ratio of primary quality parameter  $m$ , at the treatment plant located on the pipe between nodes  $i$  and  $j$ ,  $E_1$  the set of pipes that do not contain treatment plants and  $E_2$  the set of pipes that contain treatment plants.

The term in curly brackets is an approximation of the amount of the quality parameter removed by the treatment plant (assume the removed discharge is negligible compared to the flow through the treatment plant  $Q_{ij}$ ). This term is only included in the equations for the downstream end of the pipe that contains the treatment plant. Since the flow direction in these pipes is fixed *a priori* (it is regarded as part of the design of the treatment plant), which of its endpoints is the upstream end and which the downstream will always be known, so that computer codes will have no problem in setting up the equations.

*Note:* The discharge of the removed quality parameter (brine flow) is negligible compared with the flow through the treatment plant. It is, therefore, convenient to neglect it, and to assume equality between the inflow and outflow values through the treatment plant.

An approximation to the global balance of the quality parameters is therefore given by

$$\sum_{i \in N_1} Q_{ii} C_{iim} - \sum_{j \in N_2} Q_{jj} C_{jjm} - \sum_{(ij) \in E_2} Q_{ij} C_{ijm} R_{ijm} = 0 \quad \forall m \in M_1 \quad (5)$$

where  $Q_{ii}$  and  $C_{ii}$  are the sources discharges and concentration,  $Q_{jj}$ ,  $C_{jj}$  are that of the consumers, and  $N_2$  is the set of consumer nodes. Note,  $Q_{ij}$  and  $C_{ij}$  in Equation (4) refer to pipes. Equation (4) is written to all the nodes in the network and Equation (5) is a global mass balance of the network inlets and outlets. However, this relation is *automatically* met if the balance equations at the nodes, Equation (4), are met, so it is not necessary to include it explicitly in the model.

Dilution conditions are obtained by assuming total mixing at all the nodes resulting in the concentration in all pipes leaving a node being equal. However, since the flow direction in pipes is not known in advance, the dilution condition is written as:

$$C_{ijm} = \frac{C_{iim} \exp\{\varpi(a_{ij} Q_{ij})\} + C_{jjm} \exp\{-\varpi(a_{ij} Q_{ij})\}}{\exp\{\varpi(a_{ij} Q_{ij})\} + \exp\{-\varpi(a_{ij} Q_{ij})\}} \quad \forall (ij) \text{ and } \forall m \in M_1 \quad (6)$$

$\varpi(x) = k_p(x/\sqrt{x^2 + \varepsilon})$ ,  $k_p$  is a gain coefficient, and  $\varepsilon$  a small arbitrary number.

This formulation overcomes the difficulties in specifying the dilution conditions by allowing the flow directions to change during the solution process (Cohen et al. 2000a).

#### *Dependent quality parameter function*

According to the definition of conservative dependent quality parameters, each dependent water quality parameter has a function that defines its relationship with primary parameters. An example is sodium absorption ratio, which depends on the concentrations of Ca, Na and Mg. These functions are incorporated as constraints in the following manner:

$$C_{jjm}^d = \xi_{jm}(C_{jj1}, C_{jj2}, \dots, C_{jjn}) \quad \forall j \in N \quad \text{and} \quad \forall m \in M_2 \quad (7)$$

where  $C_{jjm}^d$  is the value of dependent parameter,  $m$ , in node  $j$  ( $d$  is annotation for dependent),  $M_2$  the set of dependent quality parameters,  $N$  the set of all the nodes in a network,  $\xi_{jm}(\cdot)$  is a function defining the value of the dependent parameter  $m$  at node  $j$  with respect to the values of the primary parameters at the node.

*Pipe discharge limits*

As indicated earlier, the current formulation of the problem assumes a wide feasibility domain from the hydraulic point of view. However, in order to prevent infeasibilities and unreasonable hydraulic conditions, limits are imposed on the flow in each pipe:

$$\mathbf{q}'_a \leq \mathbf{q}_a \leq \mathbf{q}''_a \quad (8)$$

where  $\mathbf{q}''_a$  and  $\mathbf{q}'_a$  are upper and lower discharge limits, respectively. If the flow direction in pipe  $i$  is restricted then  $(\mathbf{q}'_a)_i = 0$ , otherwise  $(\mathbf{q}'_a)_i = -(\mathbf{q}''_a)_i$ . Equation (8) can be transformed, using Equation (1), into constraints on  $\mathbf{q}$ :

$$\mathbf{q}'_a - \mathbf{q}_a^0 \leq \mathbf{L}^T \mathbf{q} \leq \mathbf{q}''_a - \mathbf{q}_a^0 \quad (9)$$

Velocity limits in a pipe can be expressed as an equivalent limit on the discharge, since the diameter of the pipes are given.

*Source discharge limits*

The discharge supplied from each source may be restricted by an upper limit  $\mathbf{q}''_s$  and a non-negativity limit:

$$0 \leq \mathbf{q}_s \leq \mathbf{q}''_s \quad (10)$$

Using Equation (3), Equation (10) can be expressed as a constraint on  $\mathbf{q}$ :

$$-\hat{\mathbf{A}}\mathbf{q}_a^0 \leq \hat{\mathbf{A}}\mathbf{L}^T \mathbf{q} \leq \mathbf{q}''_s - \hat{\mathbf{A}}\mathbf{q}_a^0 \quad (11)$$

*Consumer quality limits*

These constraints are introduced for consumers who require that the quality of delivered water be within specified limits, and for whom no cost or benefit function for quality is specified. With respect to the primary parameters, the constraints are:

$$C'_{jjm} \leq C_{jjm} \leq C''_{jjm} \quad \forall j \in N_2 \quad \text{and} \quad \forall m \in M_1 \quad (12)$$

where  $C''_{jjm}$  and  $C'_{jjm}$  are upper and lower limits on the quality parameter  $m$  at node  $j$ , respectively. Similarly, for dependent parameters:

$$C^d_{jjm} \leq C^d_{jjm} \leq C''^d_{jjm} \quad \forall j \in N_2 \quad \text{and} \quad \forall m \in M_2 \quad (13)$$

*Treatment limits*

The removal ratio,  $r_{ijm}$  should lie between limits

$$R'_{ijm} \leq r_{ijm} \leq R''_{ijm} \quad \forall (ij) \in E_2 \quad \text{and} \quad \forall m \in M_1 \quad (14)$$

where  $R'_{ijm}$  and  $R''_{ijm}$  are upper and lower limits on the removal ratio with respect to primary water quality parameter  $m$  of the treatment plant located between nodes  $i$  and  $j$ . Note that according to the definition of the removal ratio  $R''_{ijm} \leq 1$ .

*Objective function*

The objective of the optimization is to minimize the total cost of operation over the planning horizon,  $t$ . It includes the following components.

*Water supply cost  $\phi_s$* 

$$\phi_s = t \mathbf{w}_s(\mathbf{q}_s)^T \mathbf{q}_s \quad (15)$$

where  $\mathbf{w}_s(\mathbf{q}_s)$  is the specific cost of water at the sources,  $t$  the duration of operation, and  $\mathbf{q}_s$  the vector of sources discharge. Maximum supply amounts can be imposed for each supplier.

*Treatment cost,  $\phi_t$* 

$$\phi_t = t \mathbf{w}_t(\mathbf{r})^T \mathbf{q}_t \quad (16)$$

where  $\mathbf{w}_t(\mathbf{r})$  is a non-linear cost function vector for treatment, which depends on  $\mathbf{r}$ , which is a vector of removal ratio (relative concentration reduction parameters) for treatment plants (a decision variable). In principle,  $r$  is in the range 0–1, but for practical purposes, upper and lower bounds within this range are imposed.

*Conveyance cost over all pipes  $\phi_p$* 

$$\phi_p = t \mathbf{w}_p(\mathbf{q}_a)^T \mathbf{q}_a \quad (17)$$

where  $\mathbf{w}_p(\mathbf{q}_a)$  is a non-linear specific conveyance cost depends of  $\mathbf{q}_a$ , which is the vector of discharges in the pipes of the network. This function expresses the hydraulics of the network indirectly, since we assumed maximum discharge limits in the pipes. This assumption increases the chance that the solution will be hydraulically feasible even without expressing the hydraulics explicitly. Upper limits for supply discharge at the source nodes should also be given.



The conveyance cost function (Equation (17)) is not smooth, since flow direction is not known in advance, and its derivative is not defined at zero flow. To overcome this difficulty, an exponential smoothening procedure (similar to Equation (6)) is used to define the absolute value of the discharge.

#### *Yield reduction cost*

Agricultural consumers have a relative yield function which depends on water quality (salinity). Maas & Hoffman (1977) proposed a bilinear function to describe crop yield reduction due to salinity of irrigation water for an extensive list of crops. This bilinear function can be approximated by a quadratic function to produce the following yield loss function. Denote the relative yield function vector by  $\mathbf{y}$ , the yield achieved under ideal conditions by  $\mathbf{y}_0$ , and the income matrix by  $\mathbf{B}_0$ . The total loss due to yield reduction  $\phi_y$  is:

$$\phi_y = \mathbf{y}_0^T \mathbf{B}_0 [\mathbf{1}_y - \mathbf{y}] \quad (18)$$

where  $\mathbf{1}_y$  is a unit vector, and  $\mathbf{B}_0$  a diagonal square income matrix for unit yield for each agricultural consumer node.

#### *Total-cost (objective) function*

Combining all the cost components into a total objective function yields:

$$\begin{aligned} \min f = & t\mathbf{w}_s(\mathbf{q}_s)^T \mathbf{q}_s + t\mathbf{w}_t(\mathbf{r})^T \mathbf{q}_t + t\mathbf{w}_p(\mathbf{q}_a)^T \mathbf{q}_a + y_0^T \mathbf{B}_0 [\mathbf{1}_y - \mathbf{y}] \\ & + \sum_m \phi_L^m + \sum_m \phi_D^m \end{aligned} \quad (19)$$

where the last two terms are for penalty cost of water quality (primarily and dependent) parameters, respectively. The constraints are given in Equations (1)–(14).

#### *Optimization strategy*

The model used is a general non-linear optimization model. Using existing nonlinear programming packages is not practical for problems of the size of regional networks and for multiple water quality factors, since the number of decision variables and constraints may exceed hundreds or even thousands. It is therefore necessary to exploit special properties of the problem to develop an optimization method which is efficient and practical. Consequently, the problem is divided into an “internal problem” and an “external problem.” If the flows  $\mathbf{q}$  and removal ratios  $\mathbf{r}$  are fixed, the resulting optimization problem is “the internal problem,” in which the decision variables are the water quality values in the network pipes and nodes. The problem of finding optimal  $\mathbf{q}$  and  $\mathbf{r}$  is the “external problem.” Division of the decision variables into two groups:

(i) control variables,  $\mathbf{q}$  and  $\mathbf{r}$ , and (ii) state variables, e.g. quality concentrations in the network enables the formulation of a simpler optimization problem  $p_o$  (see Cohen et al. 2000a, 2003 for details). Problem  $p_o$  can be transformed into an equivalent problem  $p_1$  by introducing the constraints on the water quality values into the objective function as a penalty term. A further simplification can be obtained (transforming problem  $p_1$  to  $p_2$ ) by solving the quality mass conservation equations to obtain the state variable values (quality concentrations,  $C$ ) for given control variables  $\mathbf{q}$  and  $\mathbf{r}$ . The remaining constraints are functions of the circular flows,  $\mathbf{q}$ , and removal ratios,  $\mathbf{r}$ , which are constrained between bounds. These are linear constraints, and consequently problem  $p_2$  has linear constraints and a nonlinear objective function, which makes it more tractable. The projected gradient method is used for its solution.

### **Parametric study of the optimal operation of a multiquality regional network**

The effect of various model parameters on the optimal solution and total cost of operation for a regional water supply system has been studied. These results are presented, following a description of the example network.

#### *Data for the example network*

The network of the Central Arava region in southern Israel has been used to study the sensitivity of the optimal operation solution to changes in parameters values (see Figure 1). The network supplies irrigation water to agricultural consumers and drinking water to domestic consumers in this rural region (see Figure 1a).

There are 39 pipes in the network, 37 nodes, of which 11 are sources, 10 agricultural consumers, and 4 drinking water consumers. The network is operated 4000 h a year and cost per unit energy is  $0.22 \text{ NIS kWh}^{-1}$  ( $\$1 \cong 4.5 \text{ NIS}$  (New Israeli Shekel), presently). The water quality of the 11 sources is defined by salinity and the network does not contain treatment plants. Source salinities, water specific cost, and maximal discharge ( $\text{m}^3 \text{ h}^{-1}$ ) are summarized in Table 1.

Data for the 39 pipes are given in Table 2. This data includes pipe diameter, length, and specific cost coefficient, which is related to the hydraulics of flow in each pipe. The maximum flow limit is related to pipe diameter and maximal hydraulic gradient. Initial flow values, which satisfy water continuity at the nodes, are also given.

Two real loops and 10 pseudo-loops (paths connecting sources) are given in Table 3, for the initial flow distribution of Figure 1a. The table gives the pipes

in every loop including the direction of flow where positive pipe number indicates flow in the direction of the initial distribution and negative pipe number indicates opposite direction. The two sources connected by a pseudo-loop (flow path) are also given for every loop.

There are 10 agricultural consumers. A quadratic yield function which is an approximation to Maas & Hoffman (1977) models of the following form was defined with respect to salinity

$$y = a_0 + a_1C + a_2C^2$$

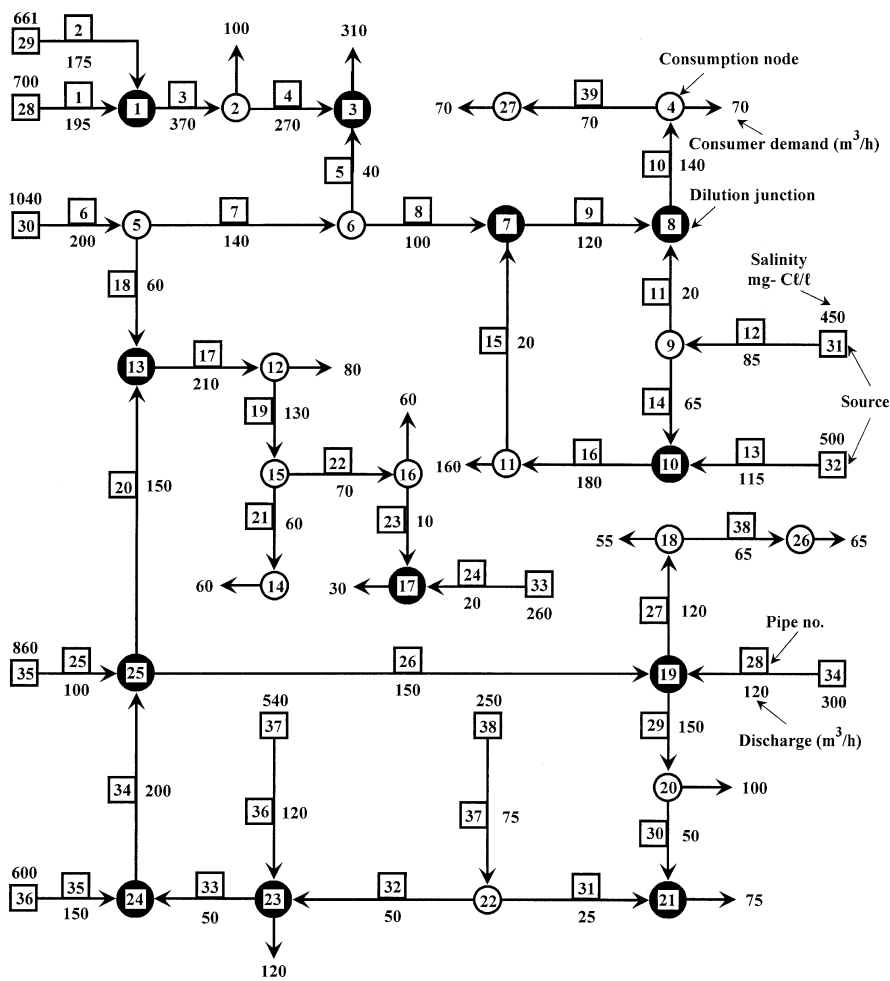


Figure 1. (a) Network layout of the Central Arava region, Israel, with initial flow distribution and salinity of water in the sources. (b) Optimal flow and salinity in the Central Arava Network (all numbers were rounded to the nearest integer number).

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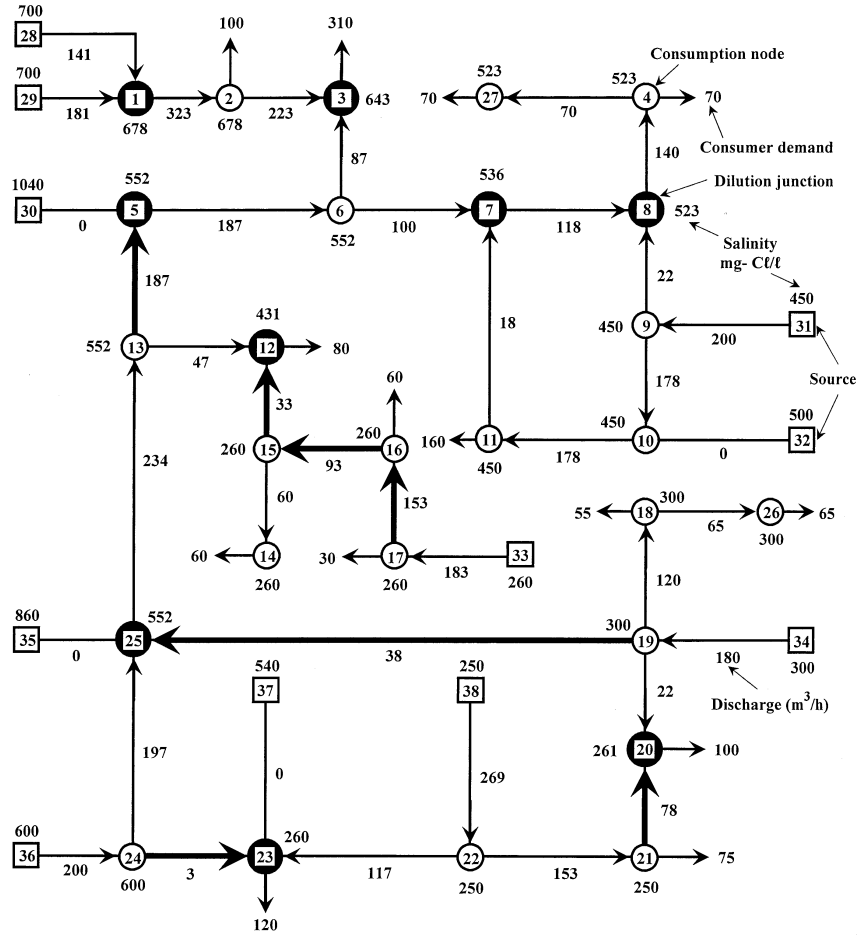


Figure 1. (Continued)

where  $a_0, a_1, a_2$  are coefficients shown in Table 4 for the various crops,  $y$  is the relative yield and  $C$  salinity in  $\text{mg Cl l}^{-1}$ . Their consumption demands, crop types and salinity tolerance levels, coefficients of the yield function and expected income in ideal conditions are presented in Table 4.

The four drinking water consumers are at nodes 14, 16, 17 and 23. Their demands are  $60 \text{ m}^3 \text{ h}^{-1}$  at nodes 14 and 16,  $30 \text{ m}^3 \text{ h}^{-1}$  at node 17, and  $120 \text{ m}^3 \text{ h}^{-1}$  at node 23. Maximum salinity for all drinking water consumers is  $260 \text{ mg Cl l}^{-1}$ .

*Optimal solution*

Salinity is the only water quality factor and no treatment plants exist in this example. Other cases where treatment plants exist and salinity, magnesium and

Table 1. Data for sources of the Central Arava network (CAN).

Source node	Maximal discharge (m <sup>3</sup> h <sup>-1</sup> )	Specific cost (NIS m <sup>-3</sup> )	Salinity (mg Cl l <sup>-1</sup> )
28	210	0.386	700
29	220	0.408	661
30	280	0.109	1040
31	200	0.638	450
32	200	0.554	500
33	200	0.735	260
34	180	0.713	300
35	150	0.256	860
36	200	0.458	600
37	150	0.535	540
38	300	0.723	250

sulphur concentration determine water quality are simulated and discussed in Cohen et al. (2000a, 2000c, 2003).

The layout of the network and an initial solution is shown in Figure 1a, in which water discharges (m<sup>3</sup> h<sup>-1</sup>) are shown along the pipes. An optimal solution is shown in Figure 1b, in which water discharges and salinities in mg Cl l<sup>-1</sup> are shown above each node. Circles with black filling designate dilution junctions.

The model has the capability to reverse flow directions from that of the initial solution during the process of optimization by using Equation (6). Flow directions in pipes 18, 19, 22, 23, 26, 30, 33 were changed from that of the initial solution (compare Figure 1a with 1b). These pipes are marked bold in Figure 1b. These changes in flow directions were required to meet consumer demands in the optimal solution. The calculated optimal cost of operation is 7,271,581 NIS per year, which is the result of the proposed model. The contributions of the four cost components are summarized in Table 5.

#### *Effect of maximum allowed salinity at drinking water consumers on the total cost of operation*

The allowable salinity value for drinking water is 260 mg Cl l<sup>-1</sup> and was taken from the Israel Standard for Drinking Water, 1985. Sensitivity of the total cost to this standard was tested by assuming higher salinity from the base value 260 to 600 mg Cl l<sup>-1</sup> incrementally. Results are given in Table 6 and include optimal cost of operation and number of iterations required for the algorithm to reach a solution.

Table 2. Data for pipes of the Central Arava network.

Pipe	Diameter (mm)	Length (m)	Specific cost coefficient ( $\times 10^{-7}$ NIS $m^{-3} h^{-1}$ )	Maximal flow ( $m^3 h^{-1}$ )	Initial flow	
					( $m^3 h^{-1}$ ) (Figure 1)	For test of pipe 20
1	300	2500	5.57	763	175	175
2	300	2000	4.46	763	195	195
3	400	1500	.082	1357	370	370
4	400	2000	1.10	1357	270	270
5	300	500	1.11	763	40	40
6	300	1600	3.57	763	200	280*
7	300	1000	2.23	763	140	140
8	300	2300	5.13	763	100	100
9	250	3500	19.0	530	120	120
10	300	4300	9.59	763	140	140
11	250	1000	5.42	530	20	20
12	250	500	2.71	530	85	85
13	250	1300	7.05	530	115	115
14	300	3700	8.25	763	65	65
15	250	5000	27.1	530	20	20
16	250	800	4.34	530	180	180
17	300	1200	5.58	763	210	140*
18	300	4200	9.37	763	60	140*
19	250	1000	5.42	530	130	60*
20	300	8000	17.8	763	150	0*
21	150	8000	522.0	190	60	60
22	250	2600	14.1	530	70	0*
23	250	2000	10.8	530	10	60*
24	300	2500	5.60	763	20	90*
25	250	2600	14.1	530	100	0*
26	400	4500	2.48	1357	150	150
27	300	1500	3.35	763	120	120
28	250	500	2.71	530	120	120
29	300	3000	6.69	763	150	150
30	250	500	2.71	530	50	50
31	250	800	4.34	530	25	25
32	300	2300	5.13	763	50	50
33	300	1800	4.02	763	50	50
34	300	3900	8.70	763	200	150*
35	300	500	1.11	763	150	100*
36	200	1000	16.1	339	120	120
37	250	2800	15.2	530	75	75
38	150	1300	84.8	190	65	65
39	250	2600	14.1	530	70	70

\*Change in initial flow values in network topology sensitivity analysis (pipe 20).

Table 3. Real loops and pseudo-loops in the network.

Loop	Type	Between sources (from, to)	Pipes							
1	B	28, 29	-2	1						
2	B	35, 30	-6	-18	20	25				
3	B	35, 36	-35	-34	25					
4	B	37, 36	-35	33	36					
5	B	38, 37	-36	32	37					
6	B	34, 38	-37	-31	30	29	28			
7	B	35, 34	-28	26	25					
8	B	30, 33	-24	23	22	19	17	18	6	
9	B	31, 32	-13	14	12					
10	B	29, 30	-6	-7	-5	4	3	2		
11	A	+	16	15	14	-11	9			
12	A	-	-30	-29	-26	31	-32	-33	-34	

Note: a – real loops; b – pseudo-loops (path between two sources). Directions of loops and pseudo-loops correspond to the initial flow directions assumed, see Figure 1. “+”: clockwise; “-”: counter clockwise circular flows in real loops.

Table 4. Data for agricultural field consumption nodes.

Node	Discharge ( $\text{m}^3 \text{h}^{-1}$ )	Crop type	Coefficients of the yield function			Income at ideal conditions NIS ( $\times 10^7$ )
			$a_0$	$a_1 (\times 10^5)$	$a_2 (\times 10^8)$	
2	100	Dates (T)	1	1.25	-2.71	2.5
3	310	Dates (T)	1	1.25	-2.71	7.75
4	70	Vegetables (S)	1	-6.78	-7.52	0.70
11	160	Vegetables (MS)	1	-5.19	-1.22	1.02
12	80	Vegetables (MS)	1	-3.06	-3.90	0.51
18	55	Vegetables (MS)	1	-3.06	-3.90	0.35
20	100	Vegetables (S)	1	-6.78	-7.52	1.02
21	75	Vegetables (MS)	1	-3.06	-3.90	0.48
26	65	Vegetables (S)	1	-6.78	-7.52	0.65
27	70	Vegetables (MS)	1	-3.06	-3.90	0.45

Note: T, S, MS refer to salt tolerance level of crops. T: tolerant, S: sensitive, MS: moderately sensitive.

### Discussion

The total cost of operation decreases as allowable salinity increases from a cost of 7,271,600 NIS at a salinity of  $260 \text{ mg Cl l}^{-1}$  to 6,655,990 NIS at a salinity of  $600 \text{ mg Cl l}^{-1}$ . The change is not linear. In the range  $260\text{--}500 \text{ mg Cl l}^{-1}$ , the decrease in the total cost is negligible, while a more significant

Table 5. Values of the objective function components in  $10^6$  NIS per year.

Component	Cost (NIS $\times 10^6$ )	% of total
Supply	3.2	44
Conveyance	0.2	3
Yield reduction	3.8	53
Penalty	0	0
Total	7.2	100

Note: There is no water treatment in this example, hence there is no treatment cost in the objective function.

Table 6. Operational cost for various allowable salinity values at drinking water consumer nodes (nodes 14, 16, 17 and 23) in  $10^6$  NIS per year.

Threshold salinity (mg Cl l <sup>-1</sup> )	Objective function cost ( $\times 10^6$ NIS per year)	Iterations	Relative cost (%)*
260	7.27	17	100
300	7.23	15	99.4
400	7.14	15	98.2
500	7.17	16	98.6
600	6.66	25	91.5

\*Relative cost to 260 mg Cl l<sup>-1</sup>.

decrease (9.5%) of the total cost is obtained when the allowable salinity is in the 500–600 mg Cl l<sup>-1</sup> range.

The demand for drinking water is only 20% of the total water supply of this network, and this by itself may explain the relatively small effect of changes in the allowable salinity on the total cost of operation.

#### *The effect of unit income from crop yield for the agricultural consumers on the total cost of operation*

The effect of unit income from crop yield was studied by multiplying the income coefficients by a factor,  $F_y$  in the range  $0.1 \leq F_y \leq 10$ . A maximal allowable salinity of 600 mg Cl l<sup>-1</sup> was arbitrarily assumed for the drinking water consumer to minimize their effect on the total cost. Table 7 summarizes the total cost and relative cost for various values of  $F_y$ , the



Table 7. Effect of unit crop yield factor ( $F_y$ ) on total operating costs.

$F_y$	Objective function cost ( $\times 10^6$ NIS per year)	Relative cost (%)
0.1	3.41	51.3
0.25	4.15	62.3
0.5	5.3	79.6
0.75	6.13	92.1
1.00	6.66	100
1.25	7.42	111.5
1.50	8.21	123.4
2.00	9.70	145.8
10.00	37.02	556.1

Note:  $F_y$  = a multiplier of income coefficients for sensitivity analysis of the effect of unit income from crop yield.

Table 8. Effect of unit crop yield factor ( $F_y$ ) on optimal circular discharges in  $\text{m}^3 \text{h}^{-1}$ .

Loop	$F_y$								
	0.1	0.25	0.5	0.75	1	1.25	1.5	2	10
1	2.1	0.8	2.1	15.1	-39.9	-62.6	-43.10	-70.0	-70.0
2	100.5	104.8	97.8	95.2	65.0	65.0	65.0	65.0	65.0
3	-36.6	-32.0	-42.2	-53.7	33.3	33.3	33.3	33.3	33.3
4	9.9	14.9	20.8	59.8	116.7	116.7	116.7	116.7	116.7
5	-3.3	2.7	7.0	29.8	86.7	86.7	86.7	86.7	86.7
6	0.50	3.2	3.1	-81.5	-138.3	-138.3	-138.3	-138.3	-138.3
7	-13.9	-22.8	-56.9	-141.5	-198.3	-198.3	-198.3	-198.3	-198.3
8	-78.9	-78.6	-75.2	-64.7	-149.9	-172.6	-153.1	-180.0	-180.0
9	-5.1	1.7	19.5	115.0	115.0	115.0	115.0	115.0	115.0
10	20.6	16.62	27.1	40.1	-14.9	-37.6	-18.1	-45.0	-45.0
11	-5.7	-2.8	1.6	-19.9	-140.5	-155.1	-150.9	-149.3	-140.7
12	-29.9	-30.0	-16.31	79.8	166.7	166.7	166.7	166.7	166.7

corresponding circular flows of the 12 loops and pseudo-loops of the network are given in Table 8, and discharges from the sources are given in Table 9.

### Discussion

Increase in the unit income from crop yield causes an increase in the total cost (see Table 7) because the solution uses more fresh water to increase the

Table 9. Effect of unit crop yield factor ( $F_y$ ) on source discharges (in  $\text{m}^3 \text{h}^{-1}$ ).

Source	$F_y$								
	0.1	0.25	0.5	0.75	1.00	1.25	1.50	2.00	10.00
28	177	176	177	190	135	112	132	105	105
29	213	211	220	220	220	220	220	220	220
30	0	0	0	0	0	0	0	0	0
31	80	87	104	200	200	200	200	200	200
32	120	113	95	0	0	0	0	0	0
33	99	98	95	84	170	193	174	200	200
34	134	146	180	180	180	180	180	180	180
35	150	150	98	0	0	0	0	0	0
36	177	167	171	143	0	0	0	0	0
37	133	133	134	150	150	150	150	150	150
38	71	74	79	186	300	300	300	300	300

income from agriculture. This increases the operating cost as the unit costs of fresh water at the sources is higher for brackish water. The supply from fresh water sources will increase as long as the incremental marginal cost of supply is smaller or equal to the incremental decrease of the yield cost. For example, the solution uses the entire available supply from sources 29, 31, 33, 34, 37 and 38, when the income from agriculture is high. When the income decrease, the solution reduces the supply from the sources as long as the incremental marginal yield loss at the agricultural consumers is lower than the incremental increase in cost of supply from the sources. It can be shown, similarly, that the solution does not use the (more saline) inexpensive sources as long as the marginal yield loss is greater than the marginal increase in supply cost. For example, the use of source 30 is not profitable and its supply for the entire range of unit incomes examined is zero. This is due to its high salinity, which causes a high yield decrease despite its low cost.

Use of source 31 demonstrates the influence of connectivity of the network. Node 10 is a dilution junction between sources 31 and 32 and consumer 11. The crop at consumer 11 is sensitive to salinity and consequently, the solution compensates for the yield loss at the node by supply from the fresh water source at node 31. The flow directions in pipes 13 and 16 remain unchanged from source 32 to consumer 11. The supply from source 32 increases the salinity at consumer 11 and yield loss there is greater than the benefit from reducing the supply from source 31. If the income from agriculture at consumer 11 decreases, the supply from source 31 becomes more expensive than the saving from the reduction of the yield loss and saline water from source 32 is thus diluted at node 10 with water from source 31. It can be seen in Table 9

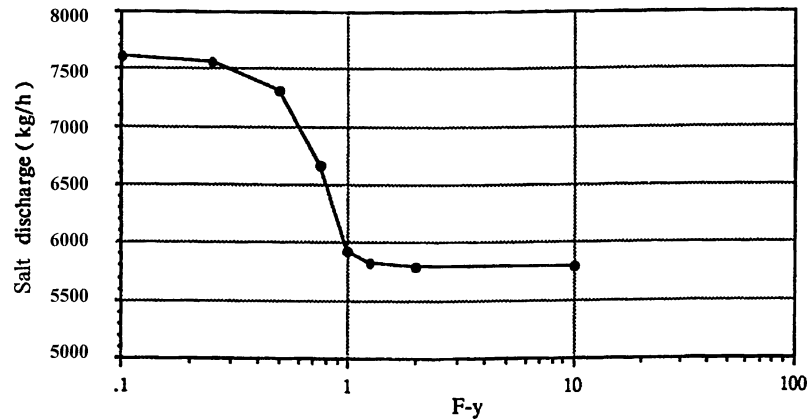


Figure 2. Salt discharge from the sources as a function of income from agriculture ( $F_y$ ).

(sources 31 and 32) how the dilution ratio between sources 32 and 31 increases as the income from agriculture decreases.

Salt supply (salinity  $\times$  discharge) from all the sources is a measure of irrigation water quality. The change in total salt supply from all sources with income level from agriculture is shown in Figure 2. Salt supply decreases as income from agriculture increases (as  $F_y$  increases).

#### *Effect of conveyance costs*

The effect of conveyance cost on the total cost of operation is relatively small and the solutions were, therefore, not affected by varying cost of conveyance. The reason may be that the flows at the optimal solution were much smaller than the maximum carrying capacities of the pipes in the network (which were derived from the pipe diameters and maximum allowed flow). Note that the conveyance costs represent the hydraulics of the network. The model is, therefore, not sensitive to the distances between the sources and consumers. To test this point, the specific cost of conveyance was changed by multiplying the original cost by a factor,  $F-O$ . The allowable limit for drinking water arbitrarily assumed  $600 \text{ mg Cl}^{-1}$  and other data remain unchanged. Changes in yield loss, cost of supply at the sources, and their sum as a function of the increase in conveyance cost (increase in  $F-O$ ) are shown in Figure 3.

#### *Discussion – Effect of conveyance cost*

The sum of yield loss and supply cost increases as the specific cost of conveyance increases (see Figure 3) because the increase in conveyance cost reduces supply from fresh water sources that are distant from the consumer nodes. Yield loss increases at these nodes and the supply from the sources decreases, see Figure 3. This process can further be demonstrated by changing

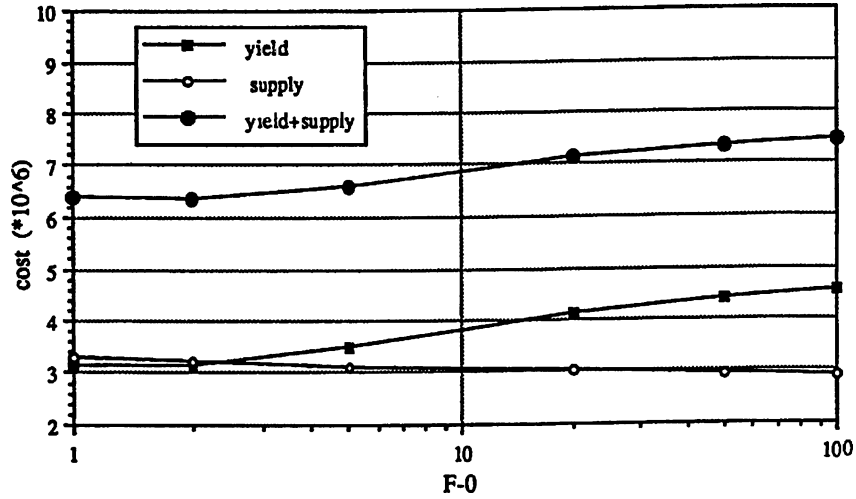


Figure 3. Optimal supply cost, yield losses, and their sum in  $10^6$  NIS per year as a function of transportation cost ( $F-O$ ).

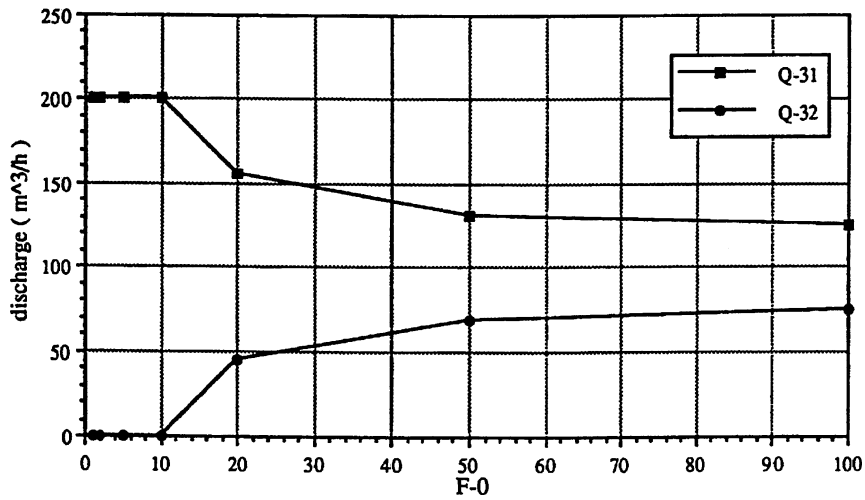


Figure 4. Optimal supply from sources 31 and 32 as a function of increase in the conveyance cost ( $F-O$ ).

the supply from sources 31 and 32 as a function of conveyance cost ( $F-O$ ), as shown in Figure 4.

At low conveyance costs ( $F-O < 10$ ), the demand at consumer 11 is supplied entirely from source 31, even though the distance of this source is greater than that of source 32. This is due to the greater incremental increase in the benefit from reducing yield loss at consumer 11, which is higher than the increase in conveyance cost. On the contrary, where cost of conveyance

increases, ( $F-O > 10$ ), the difference in the distance from sources 31 and 32 to consumer 11 causes a greater differences in conveyance cost, so the use of source 32 becomes more profitable (although its salinity is higher than that of source 31).

#### *Network topology – Separation of the network*

Examination of the regional network (see Figure 1), shows that the long pipe (8 km) 20, conveys water from the lower part of the network (sources 34–38) to the upper part (sources 28–32) of the network. The importance of this pipe and the connection between the two parts of the network was studied by considering the sensitivity of the cost of operation of the system to the value of the upper limit on the discharge of pipe 20 in the range 0–500  $\text{m}^3 \text{h}^{-1}$ . The initial discharges in pipes 6, 17–20, 22–25, 34 and 35 were changed to enable examination of the selected range. The new initial discharges are presented in Table 2 in the far right column. The cost of operation of the network as a function of the maximal discharge at pipe 20 is shown in Figure 5.

The salinity at node 8 (Figure 1) is strongly affected by the flow of fresh water from the lower part of the network *via* pipe 20 to the upper part. Figure 6 shows the effect of maximum allowed flow in pipe 20 on the salinity at node 8.

#### *Discussion*

Cost of operation increases as the maximal discharge in pipe 20 decreases. The two parts of the network are practically disconnected when the maximum discharge in pipe 20 is zero. Total consumption discharge of the upper part of the network is  $810 \text{ m}^3 \text{h}^{-1}$ , while the supply capacity of all the source in this part is  $1112 \text{ m}^3 \text{h}^{-1}$ . Sources 28 and 29 should logically have been used to

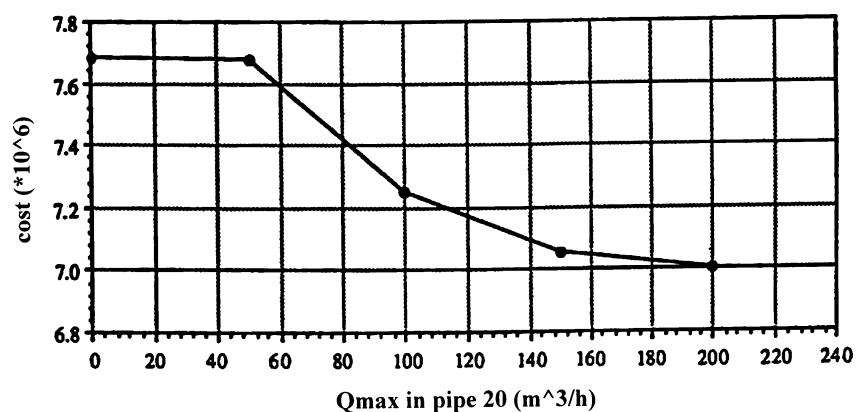


Figure 5. Optimal operational cost in  $10^6$  NIS per year for various values of maximum allowed discharge in pipe 20.

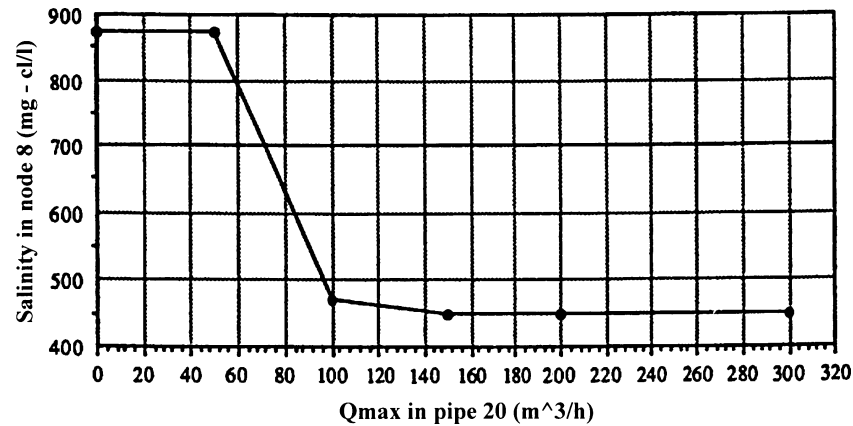


Figure 6. Optimal salinity at node 8 for various values of maximum allowed discharge in pipe 20.

supply consumers 2 and 3. The residual discharge from sources 28 and 29 is only  $10 \text{ m}^3 \text{ h}^{-1}$  and the demand of consumers 4, 27 and 11 should therefore be supplied from sources 30–32. Yield loss and salt tolerance of the crop at consumer 11 show that the demand decreases at consumer 11 as the supply from source 31 increases. The demand for water discharge at consumer 11 could not be met by supply from source 31 alone, and other sources should be used. Supply from the saline source (32) causes further yield loss at consumer 11, so the salinity level at consumers 4, 27 and 11 depends on the salinity of pipe 8. The remaining discharge from sources 28 and 29 *via* pipe 5 is very small. The salinity of source 30 is very high, so the only solution is to convey fresh water from the left-hand part of the network *via* pipe 20. The salinity of water supply to consumers 4 and 27 decreases as the discharge of pipe 20 increases, and total cost of operation therefore decreases as maximum discharge at pipe 20 increases up to a value of  $200 \text{ m}^3 \text{ h}^{-1}$ . A value higher than  $200 \text{ m}^3 \text{ h}^{-1}$  does not change the total cost (Figure 5). A similar situation is shown in Figure 6, where the salinity at node 8 drops from 870 to about  $450 \text{ mg Cl l}^{-1}$  as the maximum discharge in pipe 20 increases from 0 to about  $200 \text{ m}^3 \text{ h}^{-1}$ . For higher values of maximum discharge, the salinity of node 8 remains unchanged.

#### *Effect of supply capacity of the sources*

Examination of the sensitivity of the optimal solution to supply capacity of the sources is complicated due to the large number of combinations that can be considered. In the following case, the effect of the maximum available supply from source 38 in the range  $95\text{--}500 \text{ m}^3 \text{ h}^{-1}$  on the optimal solution is considered. The allowable salinity for drinking water was assumed to be

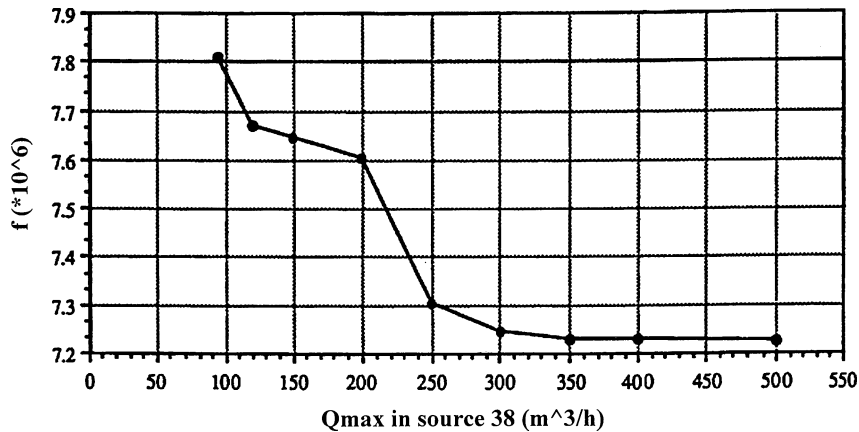


Figure 7. Effect of maximal available supply from source 38 on optimal operation cost in  $10^6$  NIS per year.

300 mg Cl l<sup>-1</sup> while other data remained unchanged. Change of total optimal cost of operation as a function of maximum supply from source 38 is shown in Figure 7.

#### Discussion

Total optimal cost decreases as maximum discharge from source 38 increases, up to a level of 300 m<sup>3</sup> h<sup>-1</sup>, after which the solution is not affected. Analysis reveals that for discharges smaller than 120 m<sup>3</sup> h<sup>-1</sup> the demand at node 23 cannot be met unless additional water flows through pipes 29 and 30. The salinity level at nodes 18 and 26 increases as use of the more saline sources 35 and 36 is required to meet the demands at nodes 18 and 26. Yield loss in these consumers as well as that of consumers 4 and 27, which are affected by the flow from the left-hand side *via* pipe 20, increases as previously mentioned. The effect of maximum available supply from source 38 on the salinity of nodes 8, 25 and 26 is shown in Figure 8.

Salinity at node 8 changed in a similar fashion to that of node 28. Both are affected by the flow from source 38 *via* pipes 32–34, 20, 18, 7–9. Supply to consumer 26 flows through a different path (37, 29–31, 27 and 38) and the response of salinity at consumer 26 to change in supply levels at source 38 is different from that of nodes 25 and 8 (Figure 8). Increase of supply from source 38 to a level of more than 300 m<sup>3</sup> h<sup>-1</sup> does not affect salinity at either of the nodes because for supply discharges greater than this value, the solution can reverse the flow direction in pipe 26. Node 19 then becomes a dilution junction between the two fresh water sources at nodes 34 and 38 such that the salinity at consumers 18 and 26 and the discharges in pipes 26, 20, etc., decrease. The incremental reduction in yield loss at these consumers is

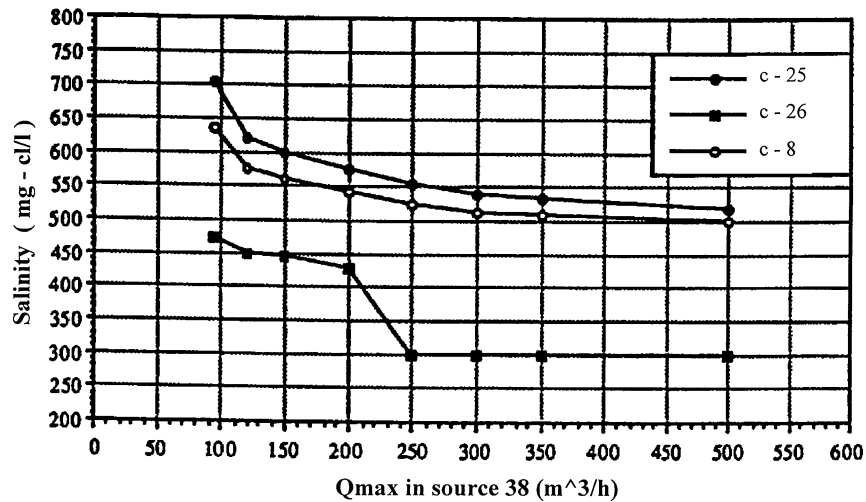


Figure 8. Optimal salinity at nodes 25, 26, 8 as a function of maximum available supply from source 38.

smaller (for supply of source 38 greater than  $300 \text{ m}^3 \text{ h}^{-1}$ ) than the additional cost of the relative expensive supply from source 38.

## Conclusions

A method has been developed to find the optimal operation of multiquality networks that considers the special properties of the problem. Use of several decompositions transforms the optimization to a problem with nonlinear objective function and linear constraints that is solved by the projected gradient method. A simplified problem considering pipe discharges ( $Q$ ) and water quality factors ( $C$ ), the  $Q$ - $C$  problem, has been presented and applied to an example of a regional irrigation supply network. The network includes 39 pipes and 37 nodes (among them 11 sources of variable water salinity, 10 agricultural crop consumers with different salt tolerance levels, and 4 drinking water consumers). Water quality is defined by salinity in the example. The objective function includes water cost at the sources, a nonlinear water transportation function (which indirectly represents the hydraulics of the network), yield loss function (which expresses crop tolerance level to salinity in term of yield loss), and treatment cost.

Sensitivity analysis of the optimal solution to various parameters show that for the example network the solution is very sensitive to changes in unit income from crop yield, less sensitive to changes in maximum allowable drinking water salinity, changes in conveyance cost, and maximum available source supply in one of the sources.



The suggested model can be applied to find optimal management of water supply for irrigation and drinking water demand within the inherent conflicts between water quality aspects and hydraulic requirements. It presents an integrated approach to this problem and makes use of treatment plants and dilution junction to control supplied water quality in addition to the conventional hydraulic control devices (pumps, valves, etc.).

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### List of symbols

<b>A</b>	adjacency matrix, with elements $a_{ij}$
<b>B</b> <sub>0</sub>	income matrix with respect to yield
<b>B</b> <sub>t</sub>	location matrices for treatment plants
$c_m$	concentration of quality parameter $m$
<b>C</b>	matrix of concentrations at nodes of the primary quality parameters
<b>C</b> <sub>d</sub>	matrix of concentrations at nodes of the dependent quality parameters
$d$	demand at a node
<b>D</b>	connectivity matrix
$E_1$	pipes which do not contain treatment plants
$E_2$	set of pipes that contain treatment plants
$f$	objective function representing the total cost of operation
$k_p$	a gain factor
$l_p$	vector whose elements are 1
<b>L</b>	cyclic matrix
$M_1$	set of primary quality parameters
$N_1$	set of sources
$q$	circular flows
$q_{a,s,t}$	pipe flows, source flows, and treatment plant flows, respectively
<b>Q</b>	matrix of flows between nodes
<b>R</b>	matrix of removal ratios in treatment plants
$t$	a time period over which the optimization is performed
$y$	relative yield function with respect to water quality

$y_0$	yields achieved under ideal conditions
$w_{s,t,p}(\cdot)$	specific cost of water at sources, treatment, and conveyance, respectively
$\varepsilon$	arbitrary small number
$\phi_{s,t,p}$	water supply, treatment, and conveyance cost, respectively
$\phi_y$	cost of yield loss
$\phi_p$	total conveyance cost (of the hydraulic solution)
$\varpi(\cdot)$	normalized direction function