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# An Improvement of the Triangular Inequality Elimination Algorithm for Vector Quantization

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**Abstract:** This study proposes an improvement of the triangular inequality elimination (TIE) algorithm for vector quantization (VQ). More than 26% additional computation saving is achieved. The proposed approach uses dynamic and intersection (DI) rules to recursively compensate and enhance the TIE algorithm. The dynamic rule changes the reference codeword dynamically and reaches the smallest candidate group. The intersection rule removes redundant codewords from these candidate groups. The DI-TIE approach avoids the over-reliance on continuity of input signal. The VQ-based line spectral pair (LSP) quantization in ITU-T G.729 standard and some standard test images are used to test the contribution of the DI-TIE. Experimental results confirm that the DI rules in the TIE algorithm have an excellent performance. Moreover, in comparison with the quasi-binary search (QBS) approach, both the QBS and the DI-TIE methods are independent on the continuity of input signal. Nevertheless, the DI-TIE approach proposed in the paper is superior to the QBS method in the computation saving issue.

Keywords: Vector quantization, Triangular inequality elimination, Speech coding, Image compression.

#### **1** Introduction

Vector quantization (VQ) [1–5] is a powerful method for image compression due to its excellent rate-distortion performance and its simple structure. Some efficient clustering algorithms [6–9] are developed based on the VQ-like approach. However, the VQ algorithm still employs a full search method to ensure the best-matched codeword and consequently results in the computational requirement is large. Therefore, many research efforts [10–20] were paid on simplifying the search complexity for the encoding process. These approaches are further classified into two types in terms of simplified technique. One is the tree-structured VQ (TSVQ) techniques [5, 10–14], and the other is the triangular inequality elimination (TIE) based approaches [15–20].

In the TSVQ based techniques, [10–13] were proposed to efficiently reduce the search complexity of VQ encoding. However, the losing on quantization quality is existed. In comparison with the full search approach, the accuracy rate of quantization on TSVQ approach is less than 47% in the speech case [14]. The naturalness test and mean opinion score (MOS) evaluation also demonstrated that the quantization quality of TSVQ approach is declined.

In the TIE based approaches, the previous research [15] applies the TIE algorithm to VQ-based image coding, achieving more than 90% and 42% computation saving in image and speech cases respectively. The accuracy rate of quantization is also equal to 100%. However, the TIE approach heavily depends on the continuous property of input signal. The definition of TIE approach, is referred to as the TIE-1, is defined as below:

$$TIE1 = \{c_t | d(c_j, c_t) < 2d(c_j, x)\}$$
(1)

where the reference codeword  $c_j$  who is more close to the input vector will make the search space  $\{c_j\}$  smaller.

However, the noise-like input vector will sharply reduce the performance of TIE approach. In the general TIE approach, the quantitative results or codeword index of previous input vector is employed as the reference codeword  $c_j$ . Because of the continuous property of input signal, the distance between two adjacent input vectors is small. Thus, in the case of smooth input signal, such as image 'Tomato', the computation saving of TIE approach

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is large than 93%. However, in case of the image 'Baboon', which the most of the image contents belong to high frequency components, the computation saving of TIE approach is less than 68%.

In addition, more sophisticated TIE studies [16, 17] were proposed to enhance the TIE approach. The definition of the sophisticated TIE approach, is referred to as the TIE-2, is defined as below:

$$TIE2 = \{c_t | d(c_j, c_t) < d(c_j, x) + d(c_i, x) \\ \wedge d(c_j, c_t) > d(c_j, x) - d(c_i, x)\}$$
(2)

In [16, 17], the TIE-1 and TIE-2 are combined to improve the computation saving. The additional reduction in computation was achieved. However, the computation saving is not good enough because the noise-like input vector will also cut down the performance of these new TIE approaches.

Other studies [18, 19], the multiple TIE (MTIE) algorithm, were employed to improve the TIE method by reducing the search space, achieving an additional reduction in computation. However, the selection rule of multiple anchors of MTIE is still indefinite. The noise-like input vector will also cut down the performance of the MTIE approach.

In [20], the fast searching algorithm based on features of vector and sub-vector was proposed. They claim that the performance outperformed the most of existing algorithms. However, the computation on the sqrt(.) function and the over-reliance on continuity of input signal still cannot be avoided.

Moreover, the quasi-binary search (QBS) algorithm was proposed in [14] to improve the computation of VQ algorithm. The performance of the QBS algorithm is better than the TIE-1, the TIE-2, and the MTIE methods. Especially on the noise-like input signal, the QBS algorithm is much better than TIE based approaches. However, the quantization accuracy of QBS is equal to 99.86% which is not perfect in comparison with the full search and the TIE approaches. The computation saving with 59.43% for the line spectral pair (LSP) coefficient quantization of ITU-T G.729 standard [21, 22] is also unsatisfactory.

It is concluded that the TIE algorithm fully depends on the continuous and invariant input vectors. The capacity of computation saving is powerful; nevertheless, three drawbacks can be founded. First, the computational reduction in the image 'Baboon' is insufficient because of the disordered image. Second, in the G.729's LSP quantization, a moving average (MA) filters the LSP coefficient beforehand and consequently destroys the continuous property of LSP's bias. Thus, the performance of TIE algorithm is decreased in the G.729 standard. Third, in the multi-stage VQ of G.729 standard, a small codebook size is given. Each codeword is closely neighbor and is tied up with each other, resulting in the TIE algorithm can't work efficiently. This article focuses on the improvement of the TIE-based VQ approach. The dynamic and intersection (DI) rules are proposed, called DI-TIE algorithm, to focus on the noise-like input signal. The speech and image experiments, i.e. the G.729's LSP quantization and the VQ-based image coding, are employed to verify the performance of proposed method.

This work is outlined as follows. The DI-TIE algorithm is described in Section 2. Experimental results are demonstrated and discussed in Section 3. This work is summarized at the end of this paper.

## 2 The DI-TIE Algorithm

This study uses DI rules to improve the performance of TIE algorithm. The dynamic rule is presented firstly. An example of the search table of TIE as well as the dynamic rule is illustrated in Fig.1 to describe the dynamic rule in detail.

To begin with, a selected codeword  $C_3$  in the TIE process is assigned to the reference codeword; meanwhile, the distance  $2 \times d(X,C_3)$  between the input vector X and the reference codeword  $C_3$  is calculated to determine the candidate codewords that are referred to the first nearest codeword group (NCG). In this example, there are 51 candidate codewords within the first NCG, which their distances are less than  $2 \times d(X,C_3)$ . It can reduce the search space from 128 to 51.

Sequentially, selecting a candidate codeword within the first NCG, which its distance is the nearest to the  $C_3$ , is assigned to replace the original reference codeword. In this example, the candidate codeword  $C_{24}$  is acquired as the new reference codeword in place of the  $C_3$ . Then, the distance  $2 \times d(X, C_{24})$  is also calculated to determine the candidate codewords and generate the second NCG. Thus, there are 40 candidate codewords, which their distances are less than  $2 \times d(X, C_{24})$ , within the second NCG in this example. The search space is decreased from 51 to 40, too. But an extra search is needed to find the new reference codeword.

Repeatedly, selecting a candidate codeword within the second NCG, which its distance is the nearest to the  $C_{24}$ , is assigned to replace the original reference codeword. In this example, the  $C_{127}$  is acquired as the new reference codeword in place of the  $C_{24}$ . Meanwhile, the third NCG can be generated and the search space is further reduced from 40 to 8. Therefore, the total number of searches become into 10 (8+1+1). The dynamic TIE reduces the search space obviously.

Besides, in case of the number of candidate codewords in the second NCG is not less than the number of candidate codewords in the first NCG, i.e. 50 (51-1) in this example, a new NCG is determined again. The second order in the first NCG,  $C_{29}$ , is selected and tested to generate a new NCG under the condition that the number of candidate codewords is less than 49 (51-2). Likewise, if the number of a new NCG's candidate





Fig. 1: An illustration of the architecture of dynamic rule.

codewords in the *k*-*th* order of original NCG is less than the number of original NCG minus k, the new NCG will be generated.

Moreover, the intersection rule is presented and applied to further reduce the search space of TIE. After the dynamic rule process, the intersection rule is employed to collect the best-matched candidate codewords that are determined by the intersection of all the NCGs. To continue the example in Fig. 1, the first intersection is computed between two NCGs,  $NCG(c_3)$ and  $NCG(c_{24})$ . There are 8 matched codewords are obtained. Then, the second intersection with  $NCG(c_{127})$ is further computed and consequently there are 4 best-matched codewords:  $C_{29}$ ,  $C_{44}$ ,  $C_{98}$  and  $C_{22}$ , are acquired. Thus, the final search space is equal to 6(4 + 2)for the DI-TIE approach which is much efficient in comparison with the general TIE. An illustration of the intersection rule is shown in Fig.2. Finally, presented in Algorithm 1 is the procedure of entire DI-TIE approach.

*Algorithm 1:* The procedure of entire DI-TIE approach. *Step-1:* 

Initialize the TIE Table, get input vector x, reference codeword  $C_i$ 

Candidate group:  $NCG(c_j) = \{c_t | d(c_j, c_t) < 2d(c_j, x)\}$ Codeword number of  $NCG(c_j) : N(c_j)$ Step-2:

Repeat each codeword  $C_k \in NCG(c_j), 1 \le k \le N(c_j)$  $if(N(c_k) < N(c_j) - k)$ 

 $NCG(c_j) = NCG(c_j) \cap NCG(c_k), k = 1$ eles

*continue until*  $k = N(c_j)$ 





Fig. 2: An illustration of the intersection rule.

## **3 Experimental Results**

In this section, experiments were performed to evaluate the efficiency of the proposed DI-TIE algorithm. The performance of the DI-TIE algorithm is carefully examined by speech and image experiments, i.e. the VQ-based LSP quantization in ITU-T G.729 and the VQ-based image coding. The LSP coefficients are quantized using a codebook with 10- dimension, 128 codewords, in G.729. In the speech experiments, all the speech data uttered in Mandarin were recorded at a sampling rate of 8 kHz and a resolution of 16 bits. These data contain over 30,000 input vectors in the duration of more than 300 seconds. In the image experiments, there are thirteen 512\*512 color images with 24-bit resolution are employed to train the VQ codebook and to test the DI-TIE approach. The VQ codebook is designed with 12-dimension, 1024 codewords, and is trained by the Linde-Buzo-Gray (LBG) algorithm [1].

First, tabulated in Table 1 is the computation saving rate of various methods, including the TIE-1, TIE-1+TIE-2, MTIE, QBS, and DI-TIE approaches, compared to the full search in the speech case. Meanwhile, experiments of the image case are tabulated in Table 2.

In the speech case, the computation savings of DI-TIE approach reach to a rate of 62.71%, 72.66%, 55.47%, 53.91%, and 96.09% for the male voice, female voice, music, noise, and silence, respectively. The DI-TIE approach is obviously superior to others approaches. Moreover, the additional contribution of DI rules tabulated in Table 1 means that the increment of computation saving between the DI-TIE and the TIE-1. For example, the additional contributions of DI rules reach to a rate of 26.45%, 29.69%, and 35.16% for the male voice, female voice, and noise, respectively. These

results demonstrate the DI rules proposed in this paper have an excellent efficiency and the DI-TIE is the best solution for the G.729's LSP quantization. Especially in the noise case, the efficiency of DI-TIE is highly outstanding.

Presented in the Fig.3(a) is the average search space vs. recursive step number for speech case. The initial step in Fig.3(a) is 128, which is identical to the search space of full search method. Each recursive step needs a cost of an extra search. Therefore, the best search space can be found in the valley of the curve. For example, the average search space of the first step in the female voice case is reduced to 76, and 35 of the sixth step. Thus, the computation saving is equal to 1-(35+6)/128=67.97% in the sixth step, and 72.66% in the optimal step. The curve of search space is obviously declined and converged according to the recursive step number.

Thus, the DI-TIE approach is independent on the continuous and invariant of input signal. Moreover, it can converge into the optimal state automatically for the target of computation saving.

In the image case, the computation savings of DI-TIE approach reach to a rate of 94.65%, 94.71%, and 98.08% for the image 'Baboon', 'Barbara', and 'Lena', respectively. Again, the DI-TIE approach is obviously superior to others approaches. Moreover, compared to the TIE-1, the additional contributions of DI rules reach to a rate of 26.92%, 18.67%, and 7.86% for the 'Baboon', 'Barbara', and 'Lena', respectively. These results demonstrate the DI rules proposed in this paper have an excellent efficiency.

In the image 'Baboon', which the most of contents belong to high frequency components, likes the noise signal. Therefore, the performance of traditional TIE approach is poor. However, the drawback of traditional







Table	1:	The com	putation a	saving	rate of	various	methods	comp	ared to	the ful	l search in	the s	speech c	ase.
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Computation Savings	TIE-1	TIE-1+TIE-2	MTIE	QBS	DI-TIE	Additional contribution from DI Rules
Voice (Male)	36.26%	48.56%	46.88%	59.43%	62.71%	26.45%
Voice (Female)	42.97%	51.56%	53.91%	59.43%	72.66%	29.69%
Music	29.69%	42.19%	42.97%	_	55.47%	25.78%
Noise	18.75%	28.91%	34.38%	_	53.91%	35.16%
Silence	94.53%	96.09%	94.53%	_	96.09%	1.56%

Table 2: The computation saving rate of various methods compared to the full search in the image case.

Computation Savings	TIE-1	TIE-1+TIE-2	MTIE	QBS	DI-TIE	Additional contribution from DI Rules
Baboon	67.73%	78.51%	73.48%	93.10%	94.65%	26.92%
Barbara	76.04%	84.12%	82.68%	_	94.71%	18.67%
Caster Stand	81.53%	88.21%	83.86%	_	95.06%	13.53%
House	82.98%	88.90%	85.37%	_	95.20%	12.22%
Monarch	87.81%	91.80%	88.51%	_	94.99%	7.18%
Boats	87.84%	92.33%	89.53%	_	96.97%	9.13%
Pills	88.35%	92.72%	88.77%	_	96.88%	8.53%
Strawberries	89.65%	93.18%	91.38%	_	96.48%	6.83%
Lena	90.22%	93.70%	91.46%	93.25%	98.08%	7.86%
Jet-F16	90.93%	94.04%	92.14%	_	98.21%	7.28%
Pepper	91.99%	94.71%	92.53%	_	97.73%	5.74%
Tiffany	93.15%	95.56%	94.33%	_	97.88%	4.73%
Tomato	93.91%	96.25%	94.20%	_	97.69%	3.78%

TIE can be compensated by the DI-TIE approach. On the other hand, in the image 'Tomato' case, the input signal is smooth. The performance of traditional TIE approach is good enough. Thus, the additional computation saving of DI-TIE is not obvious.

Presented in the Fig. 3(b) and Fig. 3(c) are the average search space vs. recursive step number for image case. The search space starts from 1024 and reaches to 70 with stable state for the image 'Baboon' case. The curve of search space is obviously declined and converged according to the recursive step number.

Those experimental results confirm that the performance of DI-TIE approach is outstanding. The DI-TIE approach not only improves the drawback of TIE method, but also outperforms the QBS approach as well as without any loss of quantization accuracy.

## **4** Conclusions

This paper presents a DI-TIE approach for improving the performance of the traditional TIE algorithm. The goal of the dynamic and intersection rules is to efficiently reduce the search space and achieve the computation saving, especially in the noise-like input signal. Through DI rules, the traditional TIE algorithm can be compensated and enhanced for the poor performance under noise-like input vector. Experimental results show that additional computation savings of the DI-TIE reach to a rate of 29.69% and 26.92% for the female voice case and the image 'Baboon' respectively.

These experimental results confirm that the DI rules proposed in this paper have an excellent efficiency as well as the performance of DI-TIE approach is outstanding. The DI-TIE approach not only improves the drawback of TIE method, but also outperforms the QBS approach as well as without any loss of quantization accuracy. The DI-TIE will be a good choice for the novel encoder system in the future.

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