Appl. Math. Inf. Sci. **6**, No. 2, 377-381 (2012)

Applied Mathematics & Information Sciences
An International Journal

The Triad Design of Order 19

T. M. Abu-Saa[1] and H. Ibrahim[2]

[1]*Ajman University of Science and Technology, Ajman, UAE.*
[2]*College of Arts and Sciences, Universiti Utara Malaysia, Sintok, Kedah Malaysia*
e-mail: [1]tareqabusaa@yahoo.com, [2]linda@uum.edu.my

ABSTRACT: In this paper we present a new method to construct the triad design of order 19, *TD*(19). First we construct the compatible factorization of order 19, *CF*(19). Then we build the starter of triad design of order 19, *STD*(19) using interval techniques of the number of triples in the design. Finally by using addition modular 19 to *STD*(19) we develop *TD*(19).

Keywords: Compatible factorization, Triad design, Starter.

1 INTRODUCTION

A *triad design* on *v* objects, denoted by *TD*(*v*), is a way of arranging the distinct triples of $\binom{v}{3}$ into *v* rows such that:

(i) Row *m* contains $\frac{v-1}{2}$ triples, among which object m meets every other object precisely once, and contains also other distinct triples;
(ii) Each triple occurs exactly once in the design;
(iii) No two elements (entries) occurs together in two or more triples in any row.

TD(*v*) is a set of distinct triples (3-element) of a *v*-set of points that deals with counting and listing triples from a graph of v points. It is associated with counting triangles in a graph which is called triangle problems that gained recently much practical importance since they are central in so-called complex network analysis [6, 7, 8].

The construction of *TD*(*v*) is based on the compatible factorization of a graph of order v, denoted by *CF(v)*, which is a $v \times \frac{v-1}{2}$ array that satisfies the following conditions: [5, 9]

(i) The entries in row m form a near-one-factor with focus m.
(ii) The triples associated with the rows contain no repetitions.

Example 1.1 If $v = 7$, that is a set of 7 points labeled 1, 2, 3, 4, 5, 6, 7, then the compatible factorization *CF(7)* is illustrated in Table 1. Append C_1 with C_2, C_1 with C_3 and C_1 with C_4, we obtain 21 distinct triples in *CF(7)*. Moreover, *TD*(7) = $CF(7) \cup \overline{CF(7)}$, where $\overline{CF(7)}$ is the completion of *CF(7)*. Clearly, *TD(7)* consists of $\binom{7}{3} = 35$ distinct triples. These triples are associated with the triangles that can be formed from K_7, the complete graph of order 7, see figure 1.

CF(7)				$\overline{CF(7)}$	
C_1	C_2	C_3	C_4		
1	2 7	3 6	4 5	2 3 5	7 6 4
2	3 1	4 7	5 6	3 4 6	1 7 5
3	4 2	5 1	6 7	4 5 7	2 1 6
4	5 3	6 2	7 1	5 6 1	3 2 7
5	6 4	7 3	1 2	6 7 2	4 3 1
6	7 5	1 4	2 3	7 1 3	5 4 2

7	1 6	2 3	3 4	1 2 4	6 5 3

Table 1.*TD*(7)

Table 1 shows that addition modular 7 to the first row, called starter, produces all the distinct triples. Once we construct the starter, listing all distinct triples can be done by addition modular *v*. Our aim in this paper is to present a new method to construct the starter of triad design of order 19, *STD*(19). This method depends on analysing the triples using interval techniques of the triples in the starter.

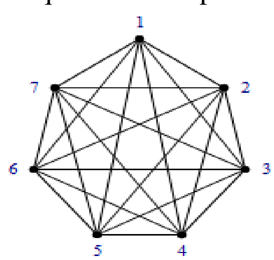


Figure 1. Plot of the complete graph K_7

It is known that *TD(v)* exists if $v \equiv$ 1or5(mod6)[2, 3]. The construction of *STD(7)* above is done by the brute force method (trial and error). Moreover, in [1],*STD*(13*)* was constructed by using the brute force method and a new method that depends on the interval techniques of the triples in the starter.

In section 2, we study some properties of thestarter of triad design on v objects, *STD*(*v*). In section 3, we again use the brute force method to construct *STD*(19*)* and then develop *TD*(19). In section 4, we extend the use of the interval techniques of the triples in the starter to build *STD*(19).

2 PROPERTIES OF THE STARTER

Definition 2.1. The*starter* of triad design on v objects, *STD(v),* is the set of triples on $v$that generates all the triples in *TD(v)* by addition modular v.

The following lemma provides the number of triples in *STD(v)* as well as in $\overline{SCF(v)}$, denoted by |*STD(v)*| and |$\overline{SCF(v)}$|respectively.

Lemma 2.1 If*v* =6*n*+1,then

|*STD*(*v*)| = *n* (6*n*-1*)* and|$\overline{SCF(v)}$| =2*n* (3*n* - 2*)*.

Proof: We prove the first and the proof of the second is the same. The number of triples of *TD(v)* is equal to $|TD(v)| = \binom{v}{3} = \binom{6n+1}{3} = n(6n+1)(6n-1)$. By the definition of $CF(v)$, the number of rows of*TD*(*v*) is equal to*v* = 6*n* + 1. Hence |*STD*(*v*)| = *n* (6*n*-1).

□

Each triple in *STD(v)* consists of three elements (numbers, objects). The first, second and the third element.

Definition 2.2 The *r-th* elements of $STD(v)$are the *r-th* numbers in each triple, denoted by $S_rTD(v)$, for 1≤ *r* ≤ 3.

SCF(7)			$\overline{SCF(7)}$	
1 2 7	1 3 6	1 4 5	2 3 5	7 6 4
± 1, $\pm 2, \pm 1$	± 2, $\pm 3, \pm 2$	± 3, $\pm 1, \pm 3$	± 1, $\pm 2, \pm 3$	± 1, $\pm 2, \pm 3$

Example 2.1 From Example 2.1,

STD(7) = {1 2 7; 1 3 6; 1 4 5; 2 3 5; 7 6 4}. Therefore

S_1TD *(7)*: 1, 1, 1, 2, 7. S_2TD *(7)*: 2, 3, 4, 3, 6. S_3TD *(7)*: 7, 6, 5, 5, 4.

Table 2. Difference sets for $STD(7) = SCF(7) \cup \overline{SCF(7)}$

Note that the difference sets for each triple in *SDT(7)* are listed in table 2 using the facts that $\pm 6 = \pm 1$, $\pm 5 = \pm 2$ and $\pm 4 = \pm 3$.

From Table 2, each difference occurs 3 times in $SCF(7)$ and 2 times in$\overline{SCF(7)}$, [4].

In the following section, the triples in *STD*(19) are analyzed in order to construct formulas to generate them and to develop *TD*(19).

3 ALGORITHM FOR*TD*(19)

In this section, addition modular 19 is used to produce the algorithm for*STD*(19). Obviously, *STD*(19)= *SCF*(19) ∪ $\overline{SCF(19)}$. By Lemma 2.1, with *n* = 3, | *STD*(19)| = 51 and$|\overline{SCF(19)}| = 42$. The algorithm for *TD*(19) is as follows:

Step 1. Generate$SCF(19)$ as shown below.
1 2 19 3 18 4 17 5 16
6 15 7 14 8 13 9 12
10 11.Therefore,
$SCF(19) = \{1\ 2\ 19,\ 1\ 3\ 18,\ 1\ 4\ 17,\ 1\ 516,\ 1\ 6\ 15,\ 1\ 7\ 14,\ 1\ 8\ 13,\ 1\ 9\ 12,\ 1\ 10\ 11\}$.

Note that the difference sets for the triples in$SCF(19)$ are listed in table 4 using the facts that$\pm18 = \pm1$, $\pm17 = \pm2$, $\pm16 = \pm3$, $\pm15 = \pm4$, $\pm14 = \pm5$, $\pm13 = \pm6$, $\pm12 = \pm7$, $\pm11 = \pm8$ and $\pm10 = \pm9$.

1	2 19	± 1	± 2	± 1
	3 18	± 2	± 4	± 2
	4 17	± 3	± 6	± 3
	5 16	± 4	± 8	± 4
	6 15	± 5	± 9	± 5
	7 14	± 6	± 7	± 6
	8 13	± 7	± 5	± 7
	9 12	± 8	± 3	± 8
	10 11	± 9	± 1	± 9

Table 3. Constructing $SCF(19)$

Step 2. Generate$\overline{SCF(19)}$ by using difference set method and addition modular 13.
Note that from Table 3 and 4, each difference occurs 3 times in $SCF(19)$ and 14 times in$\overline{SCF(19)}$.

Table 4. Constructing$SCF(19)$

$\overline{SCF(19)}$ Differences						$\overline{SCF(19)}$ Differences					
2	3	17	±1	±5	±4	19	18	4	±5	±1	±4
2	4	16	±2	±7	±5	19	17	5	±7	±2	±5
2	5	15	±3	±9	±6	19	16	6	±9	±3	±6
2	6	14	±4	±8	±7	19	15	7	±8	±4	±7
2	7	13	±5	±6	±8	19	14	8	±6	±5	±8
2	8	12	±6	±4	±9	19	13	9	±4	±6	±9
2	9	11	±7	±2	±9	19	12	10	±2	±7	±9
3	4	15	±1	±8	±7	18	17	6	±8	±1	±7
3	5	14	±2	±9	±8	18	16	7	±9	±2	±8
3	6	13	±3	±7	±9	18	15	8	±7	±3	±9
3	7	12	±4	±5	±9	18	14	9	±5	±4	±9
3	8	11	±5	±3	±8	18	13	10	±3	±5	±8
4	5	13	±1	±8	±9	17	16	8	±8	±1	±9
4	6	12	±2	±6	±8	17	15	9	±6	±2	±8
4	7	11	±3	±4	±7	17	14	10	±4	±3	±7
5	6	12	±1	±6	±7	16	15	9	±6	±1	±7
5	7	11	±2	±4	±6	16	14	10	±4	±2	±6
6	7	12	±1	±5	±6	15	14	9	±5	±1	±6
6	8	11	±2	±3	±5	15	13	10	±3	±2	±5
7	8	11	±1	±3	±4	14	13	10	±3	±1	±4
8	9	11	±1	±2	±3	13	12	10	±2	±1	±3

Therefore,$\overline{SCF(19)}$= {2 3 17; 19 18 4;2416; 19 17 5; …; 8 9 11, 13 12 10}.

Step 3: STD(19) = $SCF(19) \cup \overline{SCF(19)}$
={1 2 19, 13 18, 1 4 17, 15 16, 16 15, 1 7 14, 1 8 13, 1 9 12, 110 11}∪ {2 3 17, 19 18 4, 2 4 16, 19 17 5,…, 8 9 11,
13 12 10}.

Step 4: Using the starter and addition modular 19 to enumerate *TD*(19) as shown in table 5.

1	2	19	1	3	18	⋯	8	9	11	13	12	10
2	3	1	2	4	19	⋯	9	10	12	14	13	11
3	4	2	3	5	1	⋯	10	11	13	15	14	12
4	5	3	4	6	2	⋯	11	12	14	16	15	13
5	6	4	5	7	3	⋯	12	13	15	17	16	14
6	7	5	6	8	4	⋯	13	1	16	18	17	15
7	8	6	7	9	5	⋯	14	2	17	19	18	16
8	9	7	8	10	6	⋯	15	3	18	1	19	17
9	10	8	9	11	7	…	16	4	19	2	1	18
10	11	9	10	12	8	…	17	5	1	3	2	19
11	12	10	11	13	9	…	18	6	2	4	3	1
12	13	11	12	14	10	…	1	7	3	5	4	2
13	14	12	13	15	11	…	2	8	4	6	5	3
14	15	13	14	16	12	…	3	9	5	7	6	4
15	16	14	15	17	13	…	4	10	6	8	7	5
16	17	15	16	18	14	…	5	11	7	9	8	6
17	18	16	17	19	15	…	6	12	8	10	9	7
18	19	17	18	1	16	…	7	13	9	11	10	8
19	1	18	19	2	17	…	8	14	10	12	11	9

Table 5.*TD*(19)..

Our aim is to construct a new method for developing *TD*(19). This method depends on building the starter of *TD*(19) using interval techniques of the number of triples in the design.

4INTERVALS CONSTRUCTIONS $TD(19)$

In this section, we use analyse and divide$STD(19)$into intervals to construct formulas for$S_r TD(19)$,where$1 \le r \le 3$. From Step 3 of the previous section, weare able to summarize$STD(19)$ in terms of $S_r TD(19)$and the triple number k as shown in Table 6.

k	**1**	2	…	**9**	10	11	…	22	**23**	24	25	…	32	**33**	34	35
$S_1TD(19)$	1	1	…	1	2	19	…	2	19	3	18	…	3	18	4	17
$S_2TD(19)$	2	3	…	10	3	18	…	9	12	4	17	…	8	13	5	16
$S_3TD(19)$	19	18	…	11	17	4	…	11	10	15	6	…	11	10	13	8

k	…	38	**39**	40	41	42	**43**	44	45	46	**47**	48	49	50	**51**
$S_1TD(19)$	…	4	17	5	16	5	16	6	15	6	15	7	14	8	13
$S_2TD(19)$	…	7	14	6	15	7	14	7	14	8	13	8	13	9	12
$S_3TD(19)$		11	10	12	9	11	10	12	9	11	10	11	10	11	10

Table 6. $S_rTD(19)$, where $1 \le r \le 3$

By Lemma 2.1, $1 \le k \le 51$. Let $[S_rTD(19)]$ be the k-th element in $S_rTD(19)$, for $1 \le r \le 3$.. From Table 6, k can be divided into 7 intervals (periods). These intervals and the corresponding elements in $S_1TD(19)$ are illustrated in Table 7.

No. of Intervals	Intervals of k	Corresponding elements in $S_1TD(19)$
1	$1 \le k \le 9$	1, 1, 1, 1, 1, 1, 1, 1, 1
2	$10 \le k \le 23$	2, 19, 2, 19, 2, 19, 2, 19, 2, 19, 2, 19, 2, 19
3	$24 \le k \le 33$	3, 18, 3, 18, 3, 18, 3, 18, 3, 18
4	$34 \le k \le 39$	4, 17, 4, 17, 4, 17
5	$40 \le k \le 43$	5, 16, 5, 16
6	$44 \le k \le 47$	6, 15, 6, 15
7	$48 \le k \le 51$	7, 14, 8, 13

Table 7. Intervals of k and the corresponding elements in $S_1TD(19)$

It could be observed from Table 7 that the corresponding elements in $S_1TD(19)$ in the last interval of k need a special formula to produce them. Let f denotes the first number of the interval. The formula

$$y = \frac{1}{2}\left[k - f + \mathrm{mod}\left(\frac{k+3}{2}\right) + 1\right] \text{---------} \quad (1)$$

can be used to produce the corresponding elements in $S_1TD(19)$ in the indicated interval. From Tables 7, the construction of $S_1TD(19)$ is as follows.

$$[S_1TD(19)]_k = \begin{cases} 1 & & 1 \le k \le 9 \\ 2 & & 10 \le k \le 23, \; k \text{ iseven} \\ 19 & & 10 \le k \le 23, \; k \text{ isodd} \\ 3 & & 24 \le k \le 33, \; k \text{ iseven} \\ 18 & & 24 \le k \le 33, \; k \text{ isodd} \\ 4 & & 34 \le k \le 39, \; k \text{ iseven} \\ 17 & if & 34 \le k \le 39, \; k \text{ isodd} \\ 5 & & 40 \le k \le 43, \; k \text{ iseven} \\ 16 & & 40 \le k \le 43, \; k \text{ isodd} \\ 6 & & 44 \le k \le 47, \; k \text{ iseven} \\ 15 & & 44 \le k \le 47, \; k \text{ isodd} \\ 6+y & & 48 \le k \le 51, \; k \text{ iseven} \\ 15-y & & 48 \le k \le 51, \; k \text{ isodd} \end{cases}$$

Similarly, intervals of k and the corresponding elements in $S_2TD(13)$ are shown in Table 8.

No. of Intervals	Intervals of k	Corresponding elements in $S_2TD(13)$
1	$1 \le k \le 9$	2, 3, 4, 5, 6, 7, 8, 9, 10
2	$10 \le k \le 23$	3, 18, 4, 17, 5, 16, 6, 15, 7, 14, 8, 13, 9, 12
3	$24 \le k \le 33$	4, 17, 5, 16, 6, 15, 7, 14, 8, 13
4	$34 \le k \le 39$	5, 16, 6, 15, 7, 14
5	$40 \le k \le 43$	6, 15, 7, 14
6	$44 \le k \le 47$	7, 14, 8, 13
7	$48 \le k \le 51$	8, 13, 9, 12

Table 8. Intervals of k and the corresponding elements in $S_2TD(19)$

Using formula (1) above to produce the corresponding elements in $S_2TD(19)$ in all intervals of k except the first one. Therefore, the construction of $S_2TD(19)$ is the following.

$$[S_2TD(19)]_k = \begin{cases} 1+k & & 1 \le k \le 9 \\ 2+y & & 10 \le k \le 23, \; k \text{ iseven} \\ 19-y & & 10 \le k \le 23, \; k \text{ isodd} \\ 3+y & & 24 \le k \le 33, \; k \text{ iseven} \\ 18-y & & 24 \le k \le 33, \; k \text{ isodd} \\ 4+y & & 34 \le k \le 39, \; k \text{ iseven} \\ 17-y & if & 34 \le k \le 39, \; k \text{ isodd} \\ 5+y & & 40 \le k \le 43, \; k \text{ iseven} \\ 16-y & & 40 \le k \le 43, \; k \text{ isodd} \\ 6+y & & 44 \le k \le 47, \; k \text{ iseven} \\ 15-y & & 44 \le k \le 47, \; k \text{ isodd} \\ 7+y & & 48 \le k \le 51, \; k \text{ iseven} \\ 14-y & & 48 \le k \le 51, \; k \text{ isodd} \end{cases}$$

Finally, similar to the above discussion, intervals of k and the corresponding elements in $S_3TD(19)$ are shown in tables 9.

No. of Intervals	Intervals of k	Corresponding elements in $S_3TD(19)$
1	$1 \le k \le 9$	19, 18, 17, 16, 15, 14, 13, 12, 11
2	$10 \le k \le 23$	17, 4, 16, 5, 15, 6, 14, 7, 13, 8, 12, 9, 11, 10
3	$24 \le k \le 33$	15, 6, 14, 7, 13, 8, 12, 9, 11, 10
4	$34 \le k \le 39$	13, 8, 12, 9, 11, 10

5	$40 \le k \le 43$	12, 9, 11, 10
6	$44 \le k \le 47$	12, 9, 11, 10
7	$48 \le k \le 51$	11, 10, 11, 10

Table 9. Intervals of k and the corresponding elements in$S_3TD(19)$

Using the same formula (1), the construction of $S_2TD(19)$is the following.

$$[S_3TD(19)]_k = \begin{cases} 20-k & 1 \le k \le 9 \\ 18-y & 10 \le k \le 23,\ k \text{ iseven} \\ 3+y & 10 \le k \le 23,\ k \text{ isodd} \\ 16-y & 24 \le k \le 33,\ k \text{ iseven} \\ 5+y & 24 \le k \le 33,\ k \text{ isodd} \\ 14-y & 34 \le k \le 39,\ k \text{ iseven} \\ 7+y \quad if & 34 \le k \le 39,\ k \text{ isodd} \\ 13-y & 40 \le k \le 43,\ k \text{ iseven} \\ 8+y & 40 \le k \le 43,\ k \text{ isodd} \\ 13-y & 44 \le k \le 47,\ k \text{ iseven} \\ 8+y & 44 \le k \le 47,\ k \text{ isodd} \\ 11 & 48 \le k \le 51,\ k \text{ iseven} \\ 10 & 48 \le k \le 51,\ k \text{ isodd} \end{cases}$$

5 CONCLUSION

We have constructed a new method for developing the triad design oforder 19, $TD(19)$. This method analyses the pattern of triples using interval techniques of the number of triples in the design. One can use this method to construct the general cases of $TD(v)$, where $v = 6n + 1$ or $v = 6n + 5$.

Acknowledgement: We would like to acknowledge the financial supported from FRGS, Malaysia, grant No. 11772.

References

[1] Abu Saa, T. M., Ibrahim, H., Omer Z. & Rahim R. (2010). On particular enumeration for *TD(13)*. Proceeding of the 5th Social Economic & Information Technology (SEit), (pp. 33), Hatyai, Thailand.(23/11/2010-25/11/2010).

[2] Ibrahim H. (2003). "A Family of Designs for Triads and Related Factorizations" PhD thesis.

[3] Ibrahim, H., and Wallis, W. D., (2005). "An Enumeration of Triad Designs." J. Combinatorics Information and System Sciences: 5-17.

[4] Ibrahim, H., Abu Saa, T. M., Omar Z. and Rahim, R., (2009.). "Difference set method and its application in developing some triad designs." Proceeding of the 5th Asian Mathematical Conference, Malaysia.

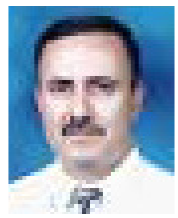
[5] Mendelssohn, E., and A. R., and (1976). "One-factorization of the complete graph-A survey." Journal of Combinatorial Theory 20B: 265-282.

[6] Schank, T., Wagner, D. (2005). Finding, Counting and Listing all Triangles in Large Graphs, An Experimental Study. WEA.

[7] M. Latapy. Practical algorithms for triangle computations in very large (sparse (power-law)) graphs. Submitted, 2007.

[8] Wallis, W. D., (1988). "Combinatorial Designs." Marcel Dekker, New York.

[9] Wallis, W. D., (1997). "One-Factorizations." Kluwer, Boston.



Tareq Abu Saa is currently employed as Mathematics Lecturer at AjmanUniversity, UAE. He obtained his PhD in Mathematics in 2011 fromUniversiti Utara Malaysia, Malaysia. He obtained his Master ofScience in 1984 from the Middle East Technical University, Turkey.His current research interests include Graph Theory, CombinatorialDesign Theory, and Enumerative Theory. He worked as a teachingassistant of Mathematics at the Middle East Technical University,Turkey. He also worked as a math supervisor assisting school mathdepartments with support in math instruction and assessments. He hasbeen awarded by the UAE-Ministry of Education Math Syllabi Award on27 September 1997.

Haslinda Ibrahim obtained her bachelor degree in Mathematics in1992(Michigan State University, USA), Master in Mathematics in1996(Indiana State University) and PhD in Mathematics in 2003(Southern Illinois University, USA). She is well known for herprofound contributions in combinatorics domain especially inCombinatorial Design Theory. Her current research interests includegraph theory, combinatorial design theory, enumerative theory andoptimization theory. Her work experiences include deputy dean ofpostgraduate, head of Mathematics Department, and Coordinator forPostgraduate program. She has been awarded several awards such asUniversity Best Research Award (Universiti Utara Malaysia),Dissertation Research Award (Southern Illinois University, USA),Outstanding PhD Teaching Assistant (Southern Illinois University,USA) and Bronze Medal Award, 9th Malaysia Technology Expo: Innovation Award (Malaysia). She is also a member for AmericanMathematical Society (AMS) and Malaysia Mathematical Science Society(PERSAMA). She is also an editorial board for Journal ofMathematics Research and has published numerous articles injournals and conference proceedings.