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A SINGLE AND BATCH SERVICE QUEUEING SYSTEM WITH ADDITIONAL SERVICE STATION

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ABSTRACT

A single and batch service queueing system with two servers is considered in this paper. The arrival process is assumed to be Poisson with parameter λ and the service rule is as follows: when the number of customers is less than or equal to c, then server-I takes a customer one by one for service according to First Come First Served (FCFS) basis and mean service rate follows an exponential distribution with parameter μ_1 and if the number of customers is more than c and less than d then server-I serves the customers altogether in a batch and mean service rate follows an exponential distribution with parameter μ_2 and if the number of customers is more than d then server II is activated and both servers are busy with batch service, where server-I serves (d-1) customers in a batch and the remaining customers are served by server-II. The mean service rate of server-II follows an exponential distribution with parameter μ_3. The Laplace transform of the transient and steady state behavior of the model is considered and we obtained the expressions for the expected queue length.

Keywords : Single and batch service queueing system, Laplace Transform ,Transient behavior and Steady state.

1. INTRODUCTION

Queueing systems with multi servers have been studied by [1,6,8,9,12,4]. Queueing systems with accessible batches were considered by serveral researchers Gross and Harris (1985), Klei7,nrock (1975), Sivasamy (1986,1990) and Baburaj and Rema(2002) [5,7,10,11,3]. Baburaj [2] has considered a single server queueing system with a single and batch service. Here we consider a two server queueing system with a single and batch service, and allow the later arriving customers to join the batches of ongoing service in server -I when the system size is less than or equal to control limit.

The special feature of operating policy of M/M/2 queueing system is that, the number of operating service stations can always be adjusted based on the customers in the system. It is monitored at every service completion epochs and he may find the system size (n) in any one of the following three categories.

(i) When the number of customers n in the system is less than or equal to c ($n \le c$) then server-I serves a customer one by one and the mean service rate follows an exponential distribution with parameter μ_1 , according to FCFS discipline.

(ii) If the number of customers in the system is more than c and less than d ($c+1 \le n \le d-1$) then server-I serves the customers altogether in a batch and admits the subsequent arrivals to join the batch till the size of the queue reaches (d-1). Suppose the service is done before the system size reaches (d-1) then it may find the system size in any one of the following three categories $n \le c, c+1 \le n \le d-1,\ n \ge d$ and the mean service rate follows an exponential distribution with parameter μ_2

(iii) If the number of customers in the system is more than d ($n \ge d$) then server-II is activated and the first (d-1) customers are served by server-I and the remaining customers are served by server II in a batch and mean service rate follows an exponential distribution with parameter μ_3

If server-I becomes idle first and he finds the system size is less than or equal to c then he serves one by one and if system size is greater than c and less than d then he serves altogether in a batch. If server-II becomes idle first, he serves the customer altogether in a batch. However, if the number of customers in the system decreases to 1 and both the servers operate simultaneously, then the service station just finishing service is closed at that time. The Markovian model is analyzed and the Laplace transform of the transient probabilities are obtained in Section 2. The steady state probabilities are obtained in Section 3 and the expression for the expected queue length is obtained in Section 4. Some Numerical illustrations are provided and discussions are made in Section 5. Finally Section 6 concludes the paper.

2. MARKOVIAN MODEL AND ITS ANALYSIS

Let P(0,n,t), n=0,1,2,…,c represents that the probability of the server-I is idle or busy with one by one service and there are n customers in the system at time t. P(1,n,t), n=c+1,c+2,…,d-1 represents that the probability of the server-I is busy with batch service and there are n units in the system at time t and P(2,n,t), $n \ge 0$ represents that the probability of both the servers are busy with batch service and there are n,($n \ge 0$) units in the waiting line at time t. Then state space $S=S_1US_2US_3,$ where S_1={(0,n),n=0,1,2,…,c}, S_2={(1,n),n=c+1,c+2,…,d-1}and S_3={(2,n), $n \ge 0$ }.

The transient distribution of the system P(i, j, t) satisfies the following system of difference differential equations with the initial condition $P\left(0,0,0\right)=1$

$$P'(0,0,t) = -\lambda P(0,0,t) + \mu_1 P(0,1,t) + \mu_2 \sum_{n=c+1}^{d-1} P(1,n,t) + (\mu_2+\mu_3)P(2,0,t) \quad (1)$$

$$P'(0,1,t) = -(\lambda+\mu_1)P(0,1,t) + \lambda P(0,0,t) + \mu_1 P(0,2,t) + (\mu_2+\mu_3)P(2,1,t) \quad (2)$$

$$P'(0,n,t) = -(\lambda+\mu_1)P(0,n,t) + \lambda P(0,n-1,t) + \mu_1 P(0,n+1,t)$$
$$+(\mu_2+\mu_3)P(2,n,t) \qquad \text{for, } 2 \le n \le c-1 \quad (3)$$

$$P'(0,c,t) = -(\lambda+\mu_1)P(0,c,t) + \lambda P(0,c-1,t) + (\mu_2+\mu_3)P(2,c,t) \quad (4)$$

$$P'(1,c+1,t) = -(\lambda+\mu_2)P(1,c+1,t) + \lambda P(0,c,t) + (\mu_2+\mu_3)P(2,c+1,t) \quad (5)$$

$$P'(1,n,t) = -(\lambda+\mu_2)P(1,n,t) + \lambda P(1,n-1,t) + (\mu_2+\mu_3)P(2,n,t)\,,$$
$$\text{For, } c+2 \le n \le d-1 \quad (6)$$

$$P'(2,0,t) = -(\lambda+\mu_2+\mu_3)P(2,0,t) + \lambda P(1,d-1,t) + (\mu_2+\mu_3)\sum_{n=d}^{\infty} P(2,n,t) \quad (7)$$

$$P'(2,n,t) = -(\lambda+\mu_2+\mu_3)P(2,n,t) + \lambda P(2,d-1,t), n>0 \quad (8)$$

Let P*(i,n,s) denote the Laplace transform of P(i,n,t) and taking Laplace transform from (1) to (8) we obtain the following system of equations of the transient probabilities

$$(s+\lambda)P^*(0,0,s) - 1 = \mu_1 P^*(0,1,s) + \mu_2 \sum_{n=c+1}^{d-1} P^*(1,n,s) + (\mu_2+\mu_3)P^*(2,0,s) \quad (9)$$

$$(s+\lambda+\mu_1)P^*(0,1,s) = \lambda P^*(0,0,s) + \mu_1 P^*(0,2,s) + (\mu_2+\mu_3)P^*(2,1,s) \quad (10)$$

$$(s+\lambda+\mu_1)P^*(0,n,s) = \lambda P^*(0,n-1,s) + \mu_1 P^*(0,n+1,s)$$
$$+(\mu_2+\mu_3)P^*(2,n,s), \quad 2 \le n \le c-1 \quad (11)$$

$$(s+\lambda+\mu_1)P^*(0,c,s) = \lambda P^*(0,c-1,s) + (\mu_2+\mu_3)P^*(2,c,s) \quad (12)$$

$$(s+\lambda+\mu_2)P^*(1,c+1,s) = \lambda P^*(0,c,s) + (\mu_2+\mu_3)P^*(2,c+1,s) \quad (13)$$

$$(s+\lambda+\mu_2)P^*(1,n,s) = \lambda P^*(1,n-1,s) + (\mu_2+\mu_3)P^*(2,n,s) \quad c+2 \le n \le d-1 \quad (14)$$

$$(s+\lambda+\mu_2+\mu_3)P^*(2,0,s) = \lambda P^*(1,d-1,s) + (\mu_2+\mu_3)\sum_{n=d}^{\infty} P^*(2,n,s) \quad (15)$$

$$(s+\lambda+\mu_2+\mu_3)P^*(2,n,s)=\lambda P^*(2,d-1,t), n>0 \tag{16}$$

Solving (16) recursively, we get

$$P^*(2,n,s)=P^*(2,0,s)\Upsilon_1^n \tag{17}$$

Where, $\Upsilon_1=\dfrac{\lambda}{s+\lambda+\mu_2+\mu_3}$

Using (17) in (15), we get

$$P^*(1,d-1,s)=P^*(2,0,s)\left[\frac{1}{\Upsilon_1}-\frac{(\mu_2+\mu_3)\Upsilon_1^d}{\lambda(1-\Upsilon_1)}\right] \tag{18}$$

Solving (11) as a difference equation in $P^*(0,n,s)$, we have

$$P^*(0,n,s)=A(s)R^n-\frac{(\mu_2+\mu_3)\Upsilon_1^n}{k(\Upsilon_1)}P^*(2,0,s), \text{ for } 2\le n\le c-1 \tag{19}$$

where A(s) is a constant depending on s and $R\equiv R(s)$ is the positive real root less than unity of the equation $K(z)=(\mu_2+\mu_3)z^2-(s+\lambda+\mu_1)z+\lambda=0$

Put $n=c-1$ in (19) and using this result in (12), we have

$$P^*(0,c,s)=\Upsilon_2A(s)R^{c-1}+\left[\Upsilon_3-\frac{(\mu_2+\mu_3)\Upsilon_2\Upsilon_1^{-1}}{k(\Upsilon_1)}\right]\Upsilon_1^cP^*(2,0,s) \tag{20}$$

where $\Upsilon_2=\dfrac{\lambda}{s+\lambda+\mu_1}$ and $\Upsilon_3=\dfrac{\mu_2+\mu_3}{s+\lambda+\mu_1}$

using (20) and (17) in (13), we get

$$P^*(1,c+1,s)=\Upsilon_4\Upsilon_2A(s)R^{c-1}+\left[\left\{\Upsilon_3-\frac{(\mu_2+\mu_3)\Upsilon_2\Upsilon_1^{-1}}{k(\Upsilon_1)}\right\}\Upsilon_1^c\Upsilon_4+\Upsilon_5\Upsilon_1^{c+1}\right]P^*(2,0,s) \tag{21}$$

where $\Upsilon_4=\dfrac{\lambda}{s+\lambda+\mu_2}$ and $\Upsilon_5=\dfrac{(\mu_2+\mu_3)}{s+\lambda+\mu_2}$

using (21) and (17) in (14), we get

$$P^*(1,n,s)=\Upsilon_4^2\Upsilon_2A(s)R^{n-3}+\left[\begin{array}{l}\left\{\Upsilon_3-\dfrac{(\mu_2+\mu_3)\Upsilon_2\Upsilon_1^{-1}}{k(\Upsilon_1)}\right\}\Upsilon_1^{n-2}\Upsilon_4^2\\+\left(1+\Upsilon_4\Upsilon_1^{-1}\right)\Upsilon_5\Upsilon_1^n\end{array}\right]P^*(2,0,s)$$

where, $c+2\le n\le d-1$ (22)

Put $n=d-1$ in (22) and equating this result with (18) we get

$$A(s)=D_1P^*(2,0,s) \tag{23}$$

where $D_1 = \frac{1}{\Upsilon_2 \Upsilon_4^2 R^{d-4}} \left[\begin{array}{l} \left\{ \frac{1}{\Upsilon_1} - \frac{(\mu_2+\mu_3)\Upsilon_1^d}{\lambda(1-\Upsilon_1)} \right\} - \left\{ \Upsilon_3 - \frac{(\mu_2+\mu_3)\Upsilon_2\Upsilon_1^{-1}}{k(\Upsilon_1)} \right\} \Upsilon_1^{d-3}\Upsilon_4^2 \\ +\left(1+\Upsilon_4\Upsilon_1^{-1}\right)\Upsilon_5\Upsilon_1^{d-1} \end{array} \right]$

Using this in (10), we get

$$p^*(0,1,s) = \Upsilon_2 p^*(0,0,s) + \left[\Upsilon_6 D_1 R^2 - \frac{(\mu_2+\mu_3)\Upsilon_6\Upsilon_1^2}{k(\Upsilon_1)} + \Upsilon_3\Upsilon_1 \right] p^*(2,0,s) \tag{24}$$

Where, $\Upsilon_6 = \frac{\mu_1}{s+\lambda+\mu_1}$

Using (22), (23) and (24) in (9), we get,

$$p^*(0,0,s) = \frac{s+\lambda+\mu_1}{(s+\lambda)^2 + s\mu_1} + D_2 p^*(2,0,s) \tag{25}$$

where

$$D_2 = \left(\frac{s+\lambda+\mu_1}{(s+\lambda)^2+s\mu_1} \right) \left[\begin{array}{l} \mu_1 \left\{ \Upsilon_6 D_1 R^2 - \frac{(\mu_2+\mu_3)\Upsilon_6\Upsilon_1^2}{k(\Upsilon_1)} + \Upsilon_3\Upsilon_1 \right\} + \\ \mu_2 \left[\sum_{n=c+2}^{d-1} \left[\Upsilon_4^2\Upsilon_2 D_1 R^{n-3} + \left\{ \Upsilon_3 - \frac{(\mu_2+\mu_3)\Upsilon_2\Upsilon_1^{-1}}{k(\Upsilon_1)} \right\} \Upsilon_1^{n-2}\Upsilon_4^2 \right. \right. \\ \left. \left. +\left(1+\Upsilon_4\Upsilon_1^{-1}\right)\Upsilon_5\Upsilon_1^n \right] \right] + (\mu_2+\mu_3) \end{array} \right]$$

Hence, we get the Laplace transform of the transient probabilities as

$$p^*(0,0,s) = \frac{s+\lambda+\mu_1}{(s+\lambda)^2} + D_2 p^*(2,0,s) \tag{26}$$

$$p^*(0,1,s) = \frac{\lambda}{(s+\lambda)^2 + s\mu_1} + D_3 p^*(2,0,s) \tag{27}$$

Where $D_3 = \left[\Upsilon_2 D_2 + \Upsilon_6 D_1 R^2 - \frac{(\mu_2+\mu_3)}{K(\Upsilon_1)} \Upsilon_6 \Upsilon_1^2 + \Upsilon_3\Upsilon_1 \right]$

$$p^*(0,n,s) = \left[D_1 R^n - \frac{(\mu_2+\mu_3)}{K(\Upsilon_1)} \Upsilon_1^n \right] p^*(2,0,s), \qquad 2 \le n \le c-1 \tag{28}$$

$$p^*(0,c,s) = D_4 p^*(2,0,s) \tag{29}$$

Where $D_4 = \Upsilon_2 D_1 R^{c-1} + \left\{ \Upsilon_3 - \frac{(\mu_2+\mu_3)\Upsilon_2\Upsilon_1^{-1}}{K(\Upsilon_1)} \right\} \Upsilon_1^c$

$$p^{*}(1,c+1,s) = D_5 p^{*}(2,0,s) \tag{30}$$

Where $D_5 = \Upsilon_4 \Upsilon_2 D_1 R^{c-1} + \left\{ \Upsilon_3 - \frac{(\mu_2+\mu_3)\Upsilon_2 \Upsilon_1^{-1}}{K(\Upsilon_1)} \right\} \Upsilon_1^{c} \Upsilon_4 + \Upsilon_5 \Upsilon_1^{c+1}$

$$p^{*}(1,n,s) = \begin{bmatrix} \Upsilon_4^2 \Upsilon_2 D_1 R^{n-3} + \left\{ \Upsilon_3 - \frac{(\mu_2+\mu_3)\Upsilon_2 \Upsilon_1^{-1}}{K(\Upsilon_1)} \right\} \Upsilon_1^{n-2} \Upsilon_4^2 \\ +\Upsilon_5 \Upsilon_1^{n} \left(1+\Upsilon_4 \Upsilon_1^{-1}\right) \end{bmatrix} p^{*}(2,0,s)$$

where, $c+2 \le n \le d-1$ (31)

and

$$p^{*}(1,n,s) = \Upsilon_1^{n} p^{*}(2,0,s)\,,\ n>0 \tag{32}$$

Using the normalizing condition

$$\sum_{n=0}^{c} p^{*}(0,n,s) + \sum_{n=c+1}^{d-1} p^{*}(1,n,s) + \sum_{n\ge d} p^{*}(2,n,s) = \frac{1}{s}$$

$$P^{*}(2,0,s) = \left[\frac{\lambda^2}{s[(s+\lambda)^2 + s\mu_1]} \right] \begin{bmatrix} D_2 + D_3 + D_4 + D_5 + D_1 R^{n} - \frac{(\mu_2+\mu_3)\Upsilon_1^{n}}{K(\Upsilon_1)} + \\ \Upsilon_4^2 \Upsilon_2 D_1 R^{n-3} + \left\{ \Upsilon_3 - \frac{(\mu_2+\mu_3)\Upsilon_2 \Upsilon_1^{-1}}{K(\Upsilon_1)} \right\} \Upsilon_1^{n-2} \Upsilon_4^2 \\ +\Upsilon_5 \Upsilon_1^{n} \left(1+\Upsilon_1^{-1}\right) + \Upsilon_1^{n} \end{bmatrix}^{-1} \tag{33}$$

3. STEADY STATE PROBABILITIES

Let $\lim_{s\to 0} \Upsilon_1 = \theta_1, \lim_{s\to 0} \Upsilon_2 = \theta_2, \lim_{s\to 0} \Upsilon_3 = \theta_3, \lim_{s\to 0} \Upsilon_4 = \theta_4, \lim_{s\to 0} \Upsilon_5 = \theta_5, \lim_{s\to 0} \Upsilon_6 = \theta_6$, then,

$\theta_1 = \frac{\lambda}{\lambda+\mu_2+\mu_3}, \theta_2 = \frac{\lambda}{\lambda+\mu_1}, \theta_3 = \frac{\mu_2+\mu_3}{\lambda+\mu_1}, \theta_4 = \frac{\lambda}{\lambda+\mu_2}, \theta_5 = \frac{\mu_2+\mu_3}{\lambda+\mu_2}\quad \theta_6 = \frac{\mu_1}{\lambda+\mu_1}$ and $\theta = \lim_{s\to 0} R$

Hence from (26) to (32), we get the steady state probabilities as

$$P(0,0) = P^{*}(2,0)\xi_2 \tag{34}$$

$$P(0,1) = P^{*}(2,0)\xi_3 \tag{35}$$

$$P(0,n) = \left[\xi_1 \theta^{n} - \frac{(\mu_2+\mu_3)\theta_1^{n}}{h(\theta_1)} \right] P^{*}(2,0) \text{ where, } 2 \le n \le c-1 \tag{36}$$

$$P(0,c) = \xi_4 P^{*}(2,0) \tag{37}$$

$$P(1,c+1) = \xi_5 P^{*}(2,0) \tag{38}$$

$$p(1,n) = \left[\theta_4^2 \theta_2 \xi_1 \theta^{n-3} + \left\{ \theta_3 - \frac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)} \right\} \theta_1^{n-2}\theta_4^2 + \theta_5\theta_1^n \left(1+\theta_4\theta_1^{-1}\right) \right] p^*(2,0),$$

Where, $c+2 \le n \le d-1$ (39)

$$P(2,n) = P^*(2,0)\theta_1^n, \ n > 0 \qquad (40)$$

where

$$\xi_1 = \frac{1}{\theta_2\theta_4^2\theta^{d-4}} \left[\frac{1}{\theta_1} - \frac{(\mu_2+\mu_3)\theta_1^d}{\lambda(1-\theta_1)} - \left\{ \theta_3 - \frac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)} \right\} \theta_1^{d-3}\theta_4^2 + \left(1+\theta_4\theta_1^{-1}\right)\theta_5\theta_1^{d-1} \right]$$

$$\xi_2 = \left(\frac{\lambda+\mu_1}{\lambda^2} \right) \left[\begin{array}{l} \mu_1 \left\{ \theta_6\xi_1\theta^2 - \dfrac{(\mu_2+\mu_3)\theta_6\theta_1^2}{h(\theta_1)} + \theta_3\theta_1 \right\} + \\ \mu_2 \left[\displaystyle\sum_{n=c+2}^{n=d-1} \left[\mu_2\theta_4^2\theta_2\xi_1\theta^{n-3} + \left\{ \theta_3 - \dfrac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)} \right\} \theta_1^{n-2}\theta_4^2 \right. \right. \\ \left. \left. + \left(1+\theta_4\theta_1^{-1}\right)\theta_5\theta_1^n \right] \right] + (\mu_2+\mu_3) \end{array} \right]$$

$$\xi_3 = \left[\theta_2\xi_2 + \theta_6\xi_1\theta^2 - \frac{(\mu_2+\mu_3)}{h(\theta_1)}\theta_6\theta_1^2 + \theta_3\theta_1 \right]$$

$$\xi_4 = \theta_2\xi_1\theta^{c-1} + \left\{ \theta_3 - \frac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)} \right\} \theta_1^c$$

$$\xi_5 = \theta_4\theta_2\xi_1\theta^{c-1} + \left\{ \theta_3 - \frac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)} \right\} \theta_1^c\theta_4 + \theta_5\theta_1^{c+1}$$

Using the Final value theorem on Laplace transforms, we get

$$P^*(i,j) = \lim_{t\to\infty} P^*(i,j,t) = \lim_{s\to 0} sP^*(i,j,s)$$

$$P^*(2,0) = \lim_{s\to 0} sP^*(2,0,s)$$

$$= \left[\begin{array}{l} \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_1\theta^n - \dfrac{(\mu_2+\mu_3)\theta_1^n}{h(\theta_1)} + \theta_4^2\theta_2\xi_1\theta^{n-3} \\ + \left\{ \theta_3 - \dfrac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)} \right\} \theta_1^{n-2}\theta_4^2 + \theta_5\theta_1^n \left(1+\theta_4\theta_1^{-1}\right) + \theta_1^n \end{array} \right]^{-1}$$

$$= \begin{bmatrix} \xi_2+\xi_3+\xi_4+\xi_5+\xi_1\left\{(1-\theta)^{-1}(\theta^2-\theta^c)\right\}-\dfrac{(\mu_2+\mu_3)\left\{(1-\theta_1)^{-1}(\theta_1^2-\theta_1^c)\right\}}{h(\theta_1)} \\ +\theta_4^2\theta_2\xi_1\left\{(1-\theta)^{-1}(\theta^{c-1}-\theta^{d-3})\right\}+\left\{\theta_3-\dfrac{(\mu_2+\mu_3)\theta_2\theta_1^{-1}}{h(\theta_1)}\right\}\theta_4^2\left\{(1-\theta_1)^{-1}(\theta_1^c-\theta_1^{d-2})\right\} \\ +\theta_5\left(1+\theta_4\theta_1^{-1}\right)\left\{(1-\theta_1)^{-1}(\theta_1^{c+2}-\theta_1^d)\right\}+\theta_1(1-\theta_1)^{-1} \end{bmatrix}^{-1}$$

$$= B \tag{41}$$

4. EXPECTED QUEUE LENGTH

If the number of customers in the system is n, ($1\le n\le c$) then there will be a queue of size n-1 with probability P(0,n). If the system size is greater than c and less than d ($c+1\le n\le d-1$) then there will be no queue. when the system size is greater than d ($n\ge d$) then both the servers are busy, so arrival customers will have to wait for service until one of the server become idle. Hence, in this case a queue will be formed with probability P(2,n), $n\ge 0$.

$$L_q=\sum_{n=2}^{c}(n-1)P(0,n)+\sum_{n\ge 1}nP(2,n)$$

Using (36), (37),(40) and (41), we get

$$L_q = \begin{bmatrix} \xi_1\theta^2\left\{(1-\theta^{c-1})(1-\theta)^{-2}-(c-1)\theta^{c-2}(1-\theta)^{-1}\right\} \\ -\dfrac{(\mu_2+\mu_3)}{h(\theta_1)}\theta_1^2\left[(1-\theta_1)^{-2}(1-\theta_1^{c-1})-(c-1)\theta_1^{c-2}(1-\theta_1)^{-1}\right] \\ +c\xi_4+\theta_1(1-\theta_1)^{-2} \end{bmatrix} B \tag{42}$$

5. NUMERICAL ILLUSTRATION

For example, let us consider λ = 10, μ_1=6, μ_2=3, μ_3=2,c=5 and d=10. The steady state probabilities are computed by using the equations (34) to (40) and the numerical results are presented in table 1

Table 1

n	P(0,n)	n	P(1,n)	n	P(2,n)
0	0.1369	6	0.0596	1	0.0157
1	0.1480	7	0.0463	2	0.0105
2	0.1533	8	0.0402	3	0.0070
3	0.1368	9	0.0347	4	0.0046
4	0.1207			5	0.0031

5	0.0764			6	0.0021
				7	0.0014
				8	0.0009
				9	0.0006
				10	0.0004
				11	0.0003
				12	0.0002
				13	0.0001
				14	0.0001
				15	0.0001
				≥ 16	0.0000
Sum of the probabilities =1					

In table 2, 3 and 4 a few numerical results of queue length for λ = 10, μ_2=3 μ_3=2 and different values of μ_1 (μ_1=6,7 and 8) are given. In table 2 and 5 a few numerical results of queue length for λ = 10, μ_1=6, μ_3=2 and different values of μ_2 (μ_2=3 and 2) are given. In table 2 and 6 a few numerical results of queue length for λ = 10, μ_1=6, μ_2=3 and different values of μ_3 (μ_3=2 and 1) are given. In table 2, 7, 8 and 9 a few numerical results of queue length for μ_1=6, μ_2=3 μ_3=2 and different values (λ = 10, 11, 12 and 15) are given

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	1.009	0.7951	0.6500	0.5456	0.4692	0.3692
3	-	1.1306	0.9483	0.8178	0.7225	0.5981
4	-	-	1.3816	1.2101	1.0852	0.9225
5	-	-	-	1.6874	1.5245	1.3122
6	-	-	-	-	2.0125	1.7426

Table 2

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	0.9795	0.7611	0.6110	0.5077	0.4365	0.3521
3	-	1.0695	0.8890	0.7663	0.6821	0.5830
4	-	-	1.2644	1.1116	1.0076	0.8859
5	-	-	-	1.4999	1.3703	1.2193
6	-	-	-	-	1.7392	1.5545

Table 3

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	0.9542	0.7271	0.5743	0.4741	0.4090	0.3394
3	-	1.0092	0.8306	0.7146	0.6399	0.5603
4	-	-	1.1549	1.0160	0.9273	0.8334
5	-	-	-	1.3307	1.2240	1.1118
6	-	-	-	-	1.5017	1.3686

Table 4

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	1.4470	1.1358	0.9022	0.7302	0.6050	0.4486
3	-	1.4384	1.1785	0.9889	0.8518	0.6813
4	-	-	1.5600	1.3416	1.1851	0.9918
5	-	-	-	1.7515	1.5680	1.3430
6	-	-	-	-	1.9713	1.7073

Table 5

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	1.4076	1.0819	0.8504	0.6868	0.5712	0.4300
3	-	1.4017	1.1388	0.9544	0.8247	0.6669
4	-	-	1.5336	1.3168	1.1651	0.9817
5	-	-	-	1.7346	1.5539	1.3364
6	-	-	-	-	1.9608	1.7027

Table 6

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	1.1734	0.9344	0.7626	0.6369	0.5435	0.4186
3	-	1.2675	1.0591	0.9071	0.7942	0.6438
4	-	-	1.4987	1.3043	1.1604	0.9687
5	-	-	-	1.7976	1.6128	1.3670
6	-	-	-	-	2.1259	1.8159

Table 7

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	1.3588	1.0874	0.8887	0.7410	0.6297	0.4783
3	-	1.4135	1.1798	1.0063	0.8756	0.6982
4	-	-	1.6197	1.4036	1.2411	1.0207
5	-	-	-	1.9057	1.7007	1.4229
6	-	-	-	-	2.2309	1.8841

Table 8

c↓ / d→	**4**	**5**	**6**	**7**	**8**	**10**
2	1.9779	1.6185	1.3409	1.1255	0.9571	0.7181
3	-	1.9087	1.6036	1.3667	1.1813	0.9180
4	-	-	2.0213	1.7453	1.5295	1.2231
5	-	-	-	2.2446	1.9862	1.6195
6	-	-	-	-	2.5342	2.0920

Table 9

6. CONCLUSION

In this paper, we have carried out an analysis of a single and batch service queueing system with additional service station. We have developed a method to find the expected queue length. In this model, for any value of arrival rate λ choosing d sufficiently large and c sufficiently small then the queue length will be very small. The technique used in this paper can be applied to analyze waiting time of a customer and busy period of servers.

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