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STATISTICAL MODELING OF EXTREME RAINFALL PROCESSES IN THE CONTEXT OF CLIMATE CHANGE

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M-H. Yeo[1] and V-T-V. Nguyen[1]

[1] Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, Quebec, Canada

Abstract

This paper proposes a Statistical Downscaling (SD) approach to link climate predictors provided by General Circulation Models (GCMs) to extreme rainfalls at a local site. More specifically, the suggested approach is based on the combination of a *spatial downscaling* method for linking the large-scale predictors to extreme rainfall and a *temporal downscaling* method for describing the relationships between daily and sub-daily annual maximum precipitations (AMPs). Results of the numerical application to observed AMPs (Quebec, Canada) and climate change scenarios have indicated that it is feasible to link large-scale predictors given by GCM-simulation outputs with daily and sub-daily AMPs at a local site.

Keywords: Statistical downscaling, extreme rainfalls, scaling concept, IDF relations, climate change.

Résumé

Cette communication propose une méthode par réduction d'échelle spatiale pour relier les prédicteurs de climat fournis par les modèles climatiques globaux (MCG) aux pluies extrêmes sur un site local. Plus précisément, l'approche suggérée se fonde sur la combinaison d'une méthode de *réduction d'échelle spatiale* établissant le lien entre les prédicteurs à grande échelle et les pluies extrêmes au moyen de SDRain, et d'une méthode de *réduction d'échelle temporelle* pour décrire les relations entre les précipitations maximales annuelles (MA) journalières et plurijournalières à l'aide de la distribution des valeurs extrêmes généralisée avec changement d'échelle. Les résultats de l'application numérique aux précipitations MA (Québec, Canada) et aux scénarios de changements climatiques fournis par les simulations de scénarios CGCM3 et HadCM3 ont montré qu'il était faisable de relier les prédicteurs à grande échelle résultant de la simulation par modèle climatique global (MCG) aux précipitations MA journalières et plurijournalières sur un site local.

Mots clés Mise en échelle, pluies extrêmes, invariance d'échelle, relations IDF, changement climatique.

1. Introduction

The estimation of extreme storms for a given duration and for a selected return period is often necessary for the planning and design of various hydraulic structures such as dams, reservoir, storm sewers, and culverts. This estimation is usually based on frequency analyses of annual extreme rainfalls. Results of these statistical analyses can be summarized by the relationship between intensity, duration, and frequency (IDF) for a given site. Several probability distribution functions have been proposed to model the characteristics of annual maximum (AM) rainfalls at a single site [1,11,12]. However, the inferences by these functions are applicable only to the particular time scale associated with the data used. Therefore, it has necessitated the need for formulating models which could statistically describe properties of AM rainfall process at different levels of aggregations. The main practical implication of these models is that we could infer the statistical properties of AM rainfall process at a shorter time scale (e.g. hourly maximum rainfall) using those at the longer time scale (e.g. available daily maximum rainfall). Another major advantage of such models is the parsimonious parameterization because these models require a smaller number of parameters for the construction of the IDF curves for rainfall events at all durations [2]. In particular, Nguyen et al. [5] proposed the scaling invariance model based on Generalized Extreme Value (GEV) distribution (called scaling-GEV method) to estimate sub-daily AM rainfall for a given return period from statistical properties of observed daily AM rainfalls.

Climate change impacts on water resources have been considered as one of the most critical issues for water management around the world. General Circulation Models (GCMs) have been commonly used in these impacts studies since these models could describe reasonably well the main features of the distribution of basic climate parameters at global scale. However, the coarse-scale outputs from these GCMs are not suitable for hydrological impacts assessment at the regional or local scale. Therefore, several downscaling methods have been developed in order to link large-scale climate variables to local-scale hydrological variables such as precipitation. Once this linkage could be established, then the projected change of climate conditions based on the GCMs could be used to predict the change of local precipitation properties in the future.

In view of the above-mentioned issues, the present study therefore proposes a statistical downscaling approach to link the climate change scenarios by GCMs to extreme rainfall event at a local site. More specifically, the suggested approach is based on the combination of a *spatial downscaling* method for linking large-scale climatic variables provided by GCMs to extreme rainfall for a local site using the statistical downscaling method (SDRain) proposed by Nguyen and Yeo [8] and a *temporal downscaling* method for describing the relationships between daily AM precipitation and sub-daily AM using the scaling-GEV. The proposed spatial-temporal downscaling approach was tested using observed AM precipitation data at nine raingage stations in Quebec (Canada) and climate simulations under four different climate change scenarios provided by the Canadian GCM version 3 (CGCM3) and the UK Hadley Centre Coupled Model 3 (HadCM3) (i.e., A1 and A1B scenarios for CGCM3, and A2 and B2 scenarios for HadCM3) for the current 1961-2000, as well as for future 2020s, 2050s, and 2080s periods. Results of this numerical application have indicated that it is feasible to establish the relationship between daily climatic variables provided by GCM simulations and observed daily AM rainfalls at all local stations. Hence, the proposed spatial downscaling method could be used for projecting the variability of AM daily precipitations for future periods for different climate scenarios. Moreover, it was found that AM precipitations in concerning stations present 'simple' scaling properties. On the basis of these results, the IDF curves were constructed for

the current period (1961-1990) and the future periods (2020s, 2050s, and 2080s) using the proposed Statistical Downscaling (SD) method for the different climate scenarios considered.

2. A Statistical Downscaling Approach

As mentioned in the previous section, the presented statistical downscaling method consists of two steps: (1) *spatial downscaling* approach is used to link daily precipitation data to large-scale atmospheric variables for a local site using the SDRain [8], and (2) *temporal downscaling* approach is employed to derive sub-daily AM precipitation data from daily AM precipitation using scaling-GEV method in order to construct IDF curves [5].

2.1. A spatial downscaling method using SDRain

For assessing local climate change impacts, the SDRain is used to model the daily precipitation process in the basis of two components: the modeling of daily precipitation occurrences and the modeling of daily precipitation amounts. Daily time series of precipitation occurrence is defined by two values (O_i = 0 if dry, O_i = 1 if wet). The daily probability (π_i) of non-zero precipitation for a day *i* is formulated as follows:

$$\ln\left(\frac{\pi_i}{1-\pi_i}\right) = a_0 + a_1 X_1 + a_2 X_2 + \cdots + a_m X_m \tag{1}$$

in which X_j, *j* = 1, 2, …, m, are the significant large-scale climate predictors, and the *a*'s are the regression parameters. A uniformly distributed random number r_i ($0 \le r_i \le 1$) is used to determine whether it is a wet or dry day. In addition, similar to the Statistical Downscaling Model (SDSM) [10], the relationship between the local daily precipitation amount (R_i) and the large-scale climate predictors (X_i`s) is described by the following nonlinear expression:

$$R_i = \exp\left(b_0 + b_1 X_1 + b_2 X_2 + \cdots + b_m X_m + SE \times \delta_i\right) \tag{2}$$

in which *b*'s are the regression parameters, and SE is the standard error in non-linear regression model, and δ_i is a normal distributed random number with mean of 0 and standard deviation of Variance Inflation Factor (VIF). The VIF term plays the role of compensating for the variance with respect to empirical data.

Results of the numerical application of the SDRain downscaling method to different climatic regions have indicated that the method is adequately able to describe the local daily precipitation occurrences and amounts with large-scale climate predictors provided by GCMs for the assessment of climate change impacts [8]. Moreover, it can be used to generate "synthetic predictands" associated to the predictors simulated by GCMs' scenarios.

The daily AM precipitations are extracted from the daily precipitation series generated by SDRain with large-scale predictors from GCMs' scenarios. However, the extracted AM precipitations are not usually comparable to observed. In other words, the differences (e.g. residuals) exist between observed daily AM precipitations and simulated with GCMs'. Hence,

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Nguyen et al. [6] proposed the adjustment method in order to improve the accuracy of spatial SD for at-site daily AM precipitations. The proposed procedure is described as the follows:

Let

$$y_\tau = \hat{y}_\tau + e_\tau \qquad (3)$$

in which y_τ is the adjusted daily AM precipitation at a probability level τ, $\hat{y}_\tau$ is the corresponding GCM-SDRain estimated daily AM precipitation, and e_τ is the residual associated with $\hat{y}_\tau$. The residual (e_τ) is modelled by the second order polynomial regression as the follows:

$$e_\tau = m_0 + m_1\hat{y}_\tau + m_2\hat{y}_\tau^2 + \varepsilon \qquad (4)$$

where m_0, m_1, and m_2 are regression parameters in the function, $\hat{y}_\tau$ is the estimated AM precipitation, and ε is the modelling error term.

2.2. A temporal downscaling method using the scaling-GEV distribution

The application of GEV distribution has been dealt with to model properties of AM precipitation series and to construct IDF curves. The cumulative distribution function, F(x), for the GEV distribution is

$$F(x) = \exp\left[-\left(1 - \frac{\kappa(x - \xi)}{\alpha}\right)^{1/\kappa}\right] \qquad (\kappa \neq 0) \qquad (5)$$

in which ξ, α, and κ are the location, scale, and shape parameter, respectively. For linking GEV parameters and quantiles with scaling property, the k-th order of non-central moment (NCM), μ_k, of the GEV distribution can be expressed as

$$\mu_k = \left(\xi + \frac{\alpha}{\kappa}\right)^k + (-1)^\kappa \left(\frac{\alpha}{\kappa}\right)^\kappa \Gamma(1 + k\kappa) + k\sum_{i=1}^{k-1}(-1)^i\left(\frac{\alpha}{\kappa}\right)^i\left(\xi + \frac{\alpha}{\kappa}\right)^{k-1}\Gamma(1 + ik) \qquad (6)$$

in which $\Gamma(\cdot)$ is the gamma function. Therefore, it is possible to estimate parameters (ξ, α, and κ) of GEV distribution using first three NCMs. The quantiles (X_τ) corresponding to a return period can be calculated by the inverse distribution function as follows

$$X_\tau = \xi + \frac{\alpha}{\kappa}\left\{1 - \left[1 - \ln(p)\right]^\kappa\right\} \tag{7}$$

where p is the exceedance probability of interest.

The proposed temporal downscaling method is based on the concept of scale-invariance (or scaling). According to the definition, if a function *f(x)* has a proportional relationship with respect to scaled *λx*, the *f(x)* follows scaling behaviour for all positive values of the scale factor λ. With scaling property, the relationship between *C(λ)* and *f(x)* can be expressed such that

$$f(x) = C(\lambda)f(\lambda x) \tag{8}$$

C(λ) can be readily derived as

$$C(\lambda) = \lambda^{-\beta} \tag{9}$$

in which *β*, called a scaling exponent, is a constant for a local site. Therefore, Equation [8] can be transformed such that

$$f(x) = x^\beta f(1) \tag{10}$$

Consequently, since *k*-th order NCMs (μ_k) are proportionally associated with each other, the μ_k can be expressed in the same form of Equation [10] as

$$\mu_k(x) = E\left\{f^k(x)\right\} = \alpha(k)x^{\beta(k)} \tag{11}$$

where *x* is duration of precipitation, $\beta(k) = k\beta$ under simple scaling condition. When *x* is the reference value (e.g. 1minute), it is found that $\alpha(k) = \left\{f^k(1)\right\}$ is satisfied.

Further, for a simple scaling process, it can be shown that the statistical properties of the GEV distribution for two different time scales *t* and *λt* are related as follows:

$$\kappa(\lambda t) = \kappa(t) \tag{12}$$

$$\alpha(\lambda t) = \lambda^\beta \alpha(t) \tag{13}$$

$$\xi(\lambda t) = \lambda^\beta \xi(t) \tag{14}$$

$$X_T(\lambda t) = \lambda^\beta X_T(t) \tag{15}$$

Hence, based on these relationships it is possible to derive the statistical properties of sub-daily AM precipitations using the properties of daily AM precipitations. Then, the derived NCMs (μ_k) are used to estimate three parameters of GEV distribution for sub-daily extreme rainfalls. Therefore, the proposed scaling GEV method can be used to construct IDF curves taking account of climate-related impacts on extreme rainfalls.

3. Numerical Application

The application of the proposed spatial-temporal downscaling is illustrated using AM precipitation data observed (1961-2000) at nine raingage stations located in Quebec (Canada) and using climate simulations under four different climate change scenarios provided by CGCM3 and HadCM3 (i.e. CGCM3 A1B & A2, and HadCM3 A2 & B2). At site AM precipitation data for several durations ranging from 5 minutes to 24 hours used in this study are available for the period of 1961-2000. The observed data for 1961-1990 were used for calibration and verification of downscaling models, and the remaining observed data (1991-2000) were employed for validation purposes. The computational procedure for the suggested SD method in this study can be summarized as follows:

  a) Calibrate the spatial statistical downscaling model with local site daily precipitation series and large-scale variables in four GCMs' scenarios by SDRain. Generate 100 daily precipitation series for a local site based on the calibrated SDRain, and extract daily AM precipitations.

  b) Apply the adjustment function to extracted AM precipitation corresponding to the calibration period (1961-1990) via Equation [3] and Equation [4]. For validating the bias-correction function, employ the function to daily AM precipitations for the validate period (1991-2000).

  c) Generate daily AM precipitations via the calibrated SDRain and four GCMs' scenario for the 2020s, 2050s, and 2080s. Apply the bias-correction function derived in (b) to generated AM series.

  d) Investigate scaling properties among the first three NCMs for all durations. Under simple scaling regime, apply the temporal downscaling method to observed daily AM precipitation. Compare the quantiles for sub-daily AM estimated by traditional and the proposed (i.e. scaling GEV) (Nguyen et al., 2007) method.

  e) Apply scaling GEV method to the adjusted daily AM precipitations for the period of 2020s, 2050s, and 2080s. Construct IDF curves for each period with respect to GCMs' scenarios.

For purposes of illustration, Figure 1 shows the Quantile-Quantile plots (Q-Q plots) of the observed daily AM precipitations against the simulated values given by CGCM3-A2 and HadCM3-A2 for Dorval station. It was found that the GCM-based estimated daily AM precipitations do not agree well with the observed values. In order to improve this agreement, the second–order error-adjustment functions were used as shown in Figure 2. Hence, after making the bias-correction adjustment using the fitted second-order functions (see Equation 3), a very good agreement can be achieved between the adjusted mean of GCM-downscaled amounts and the observed at-site values as shown in Figure 3. The bias-correction functions established based on data for the 1961-1990 calibration period were then applied to the

downscaled AM precipitation for the 1991-2000 validation period to assess their validity. Table 1 shows the improved agreement (smaller relative root mean square error) between the adjusted downscaled AM precipitations and the observed values as compared to the unadjusted downscaled AM amounts for all stations, except for Drummondville due to the presence of the extremely high rainfall amount in the validation period. Hence, it is feasible to use the bias-correction function derived from data for the 1961-1990 calibration period for other time periods in the future.
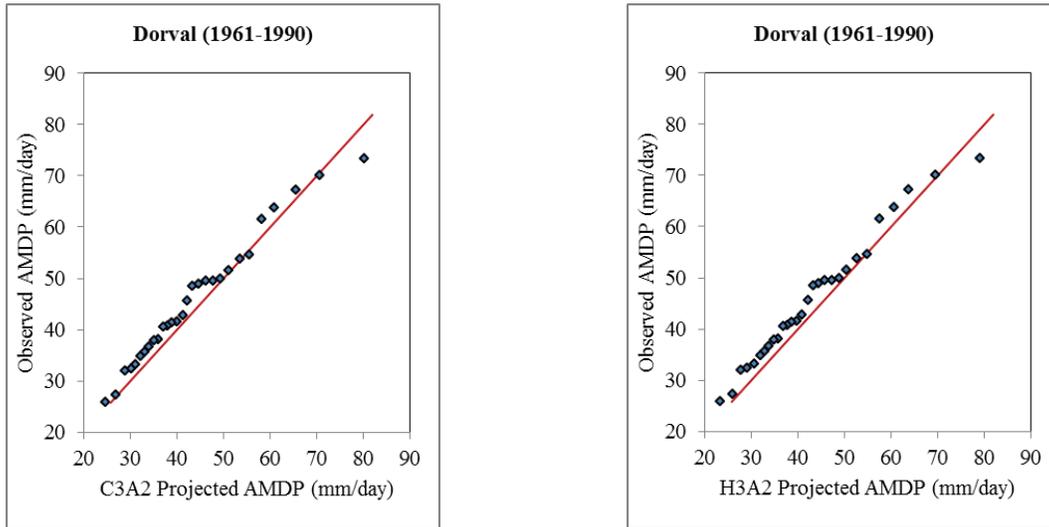
Figure 1. Quantile-Quantile plot of observed daily AM precipitation and simulated by CGCM3-A2 and HadCM3-A2 for the calibration period (1961 – 1990) before error-adjustment for Dorval station.
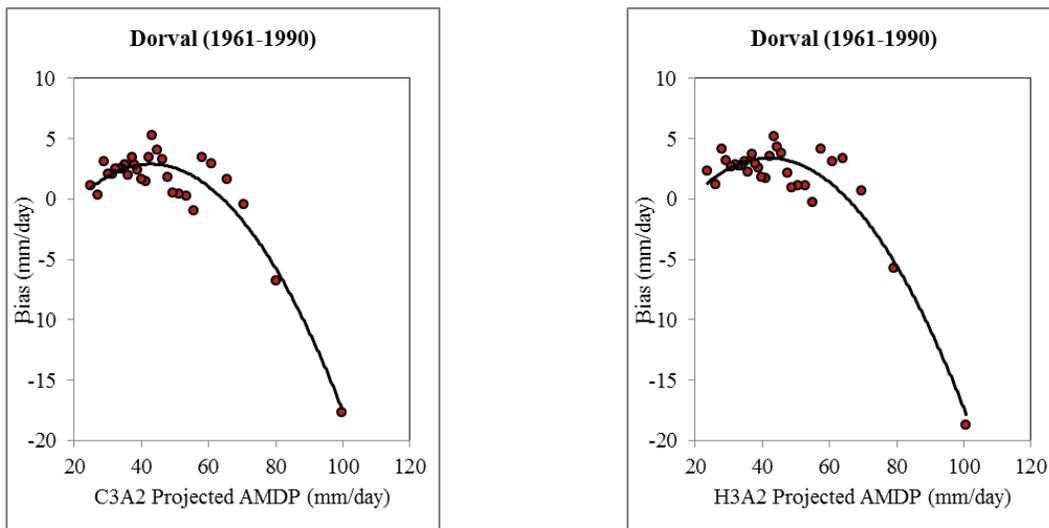
Figure 2. Application of error-adjustment functions to biases of daily AM precipitations from downscaled CGCM3-A2 and HadCM3-A2 the calibration period (1961 – 1990) for Dorval station.
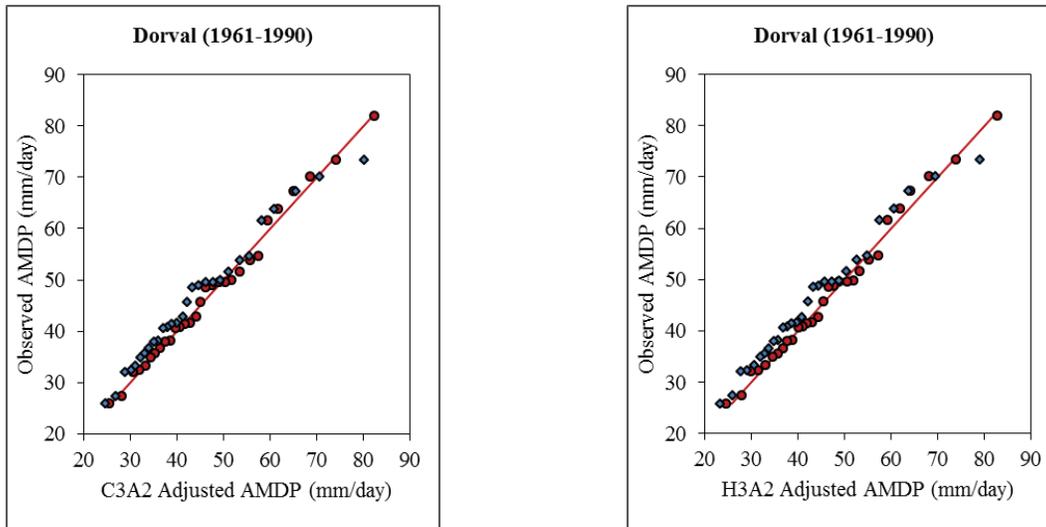
Figure 3. Quantile-Quantile plot of observed daily AM precipitation and simulated by CGCM3-A2 and HadCM3-A2 for the calibration period (1961 – 1990) before and after error-adjustment for Dorval station. Blue diamond denotes the quantiles before error-correction, and red circle does those after error-correction.

Table 1. Values of relative root mean square errors (RRMSE) for the daily AM precipitations without and with bias correction for the validation period (1991 – 2000).

Site	without bias correction				with bias correction			
	H3A2	H3B2	C3A1B	C3A2	H3A2	H3B2	C3A1B	C3A2
Dorval	0.126	0.122	0.135	0.135	0.113	0.104	0.124	0.124
Bagotville	0.222	0.198	0.181	0.181	0.136	0.135	0.134	0.134
Drummondville	0.107	0.122	0.181	0.181	0.203	0.233	0.189	0.189
Gaspe	0.109	0.122	0.129	0.129	0.058	0.054	0.058	0.058
Goosebay	0.213	0.223	0.228	0.228	0.167	0.173	0.214	0.214
Kuujjuaq	0.281	0.292	0.166	0.166	0.177	0.176	0.153	0.153
Kuujjuarapik	0.421	0.415	0.406	0.406	0.306	0.299	0.294	0.294
Mont Joli	0.217	0.178	0.193	0.193	0.179	0.161	0.152	0.152
Natashquan	0.129	0.141	0.186	0.186	0.086	0.092	0.135	0.135

To examine the scaling properties of the AM precipitations for all durations, graphical analyses were carried out for all stations using the first three NCMs of AM precipitations. According to Equation 11, the relationship between scaling property and NCMs can be plotted within log-log regime, i.e., the log-linearity in this regime indicates the power law dependency of NCMs with durations. For the purpose of illustration, Figure 4 shows the scaling relationships with respect to all duration for Dorval station. The log-linearity of NCMs exhibits for two time intervals: from 5 minutes to 30 minutes and from 30 minutes to daily. In addition, the linearity of the scaling exponents $\beta(k)$ against the order of NCMs of AM precipitation for Dorval as shown in Figure 5 has indicated that the AM data in the interest station can be described by a simple scaling model. Hence, it is possible to estimate the NCMs and parameters of the distribution of AM

precipitation for shorter durations using available AM data for longer time scale within the same scaling regime.
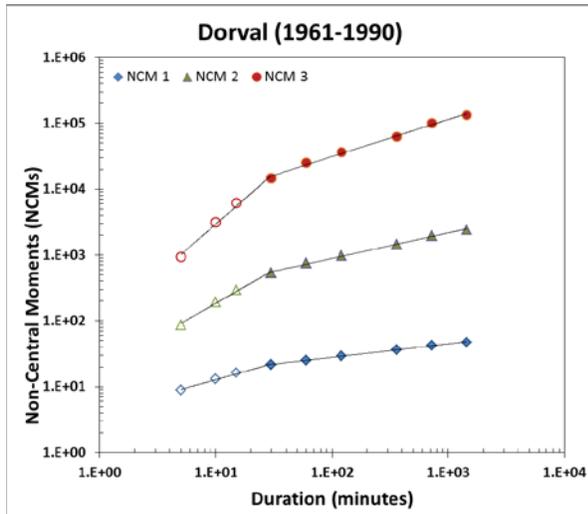
Figure 4. Log-log plots of Non-central Moments (NCMs) of the first three orders against several durations for Dorval station.
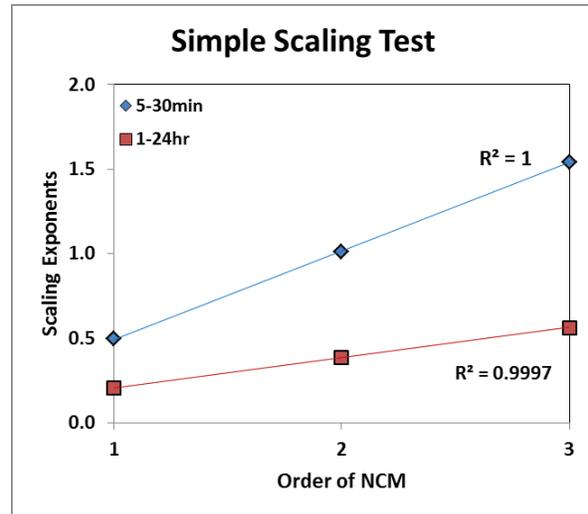
Figure 5. Plot of the scaling exponents $\beta(k)$ against the order of NCMs of AM precipitation for Dorval
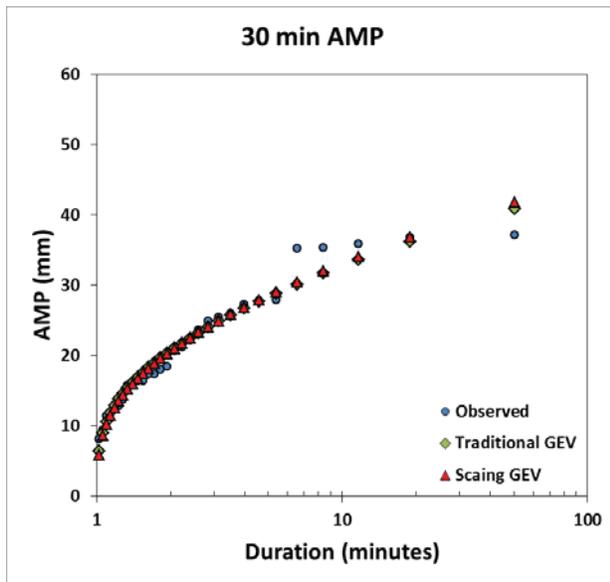
(a)
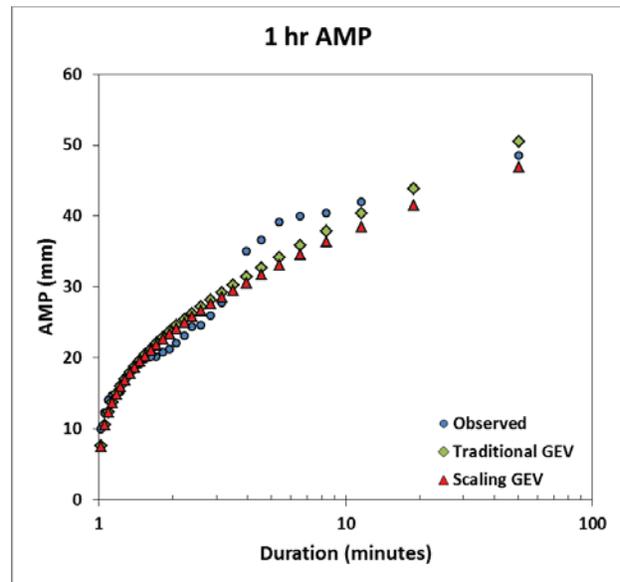
(b)

Figure 6. Probability plots of 30-minutes (a) and 1-hour (b) Observed AM precipitations, and estimated using traditional and scaling GEV distributions for the 1961-1990 for Dorval station.

On the basis of the simple scaling relationship, Figure 6 shows the comparison between the observed and estimated AMP by traditional and scaling GEV distributions for 30-minutes and 1 hour as duration periods at Dorval station. It can be seen that the quantiles derived from the

daily AMP via the established scaling relationships agree very well with those values given by the traditional fitted GEV distribution as well as with the observed values. Similar results were found for other duration and stations.
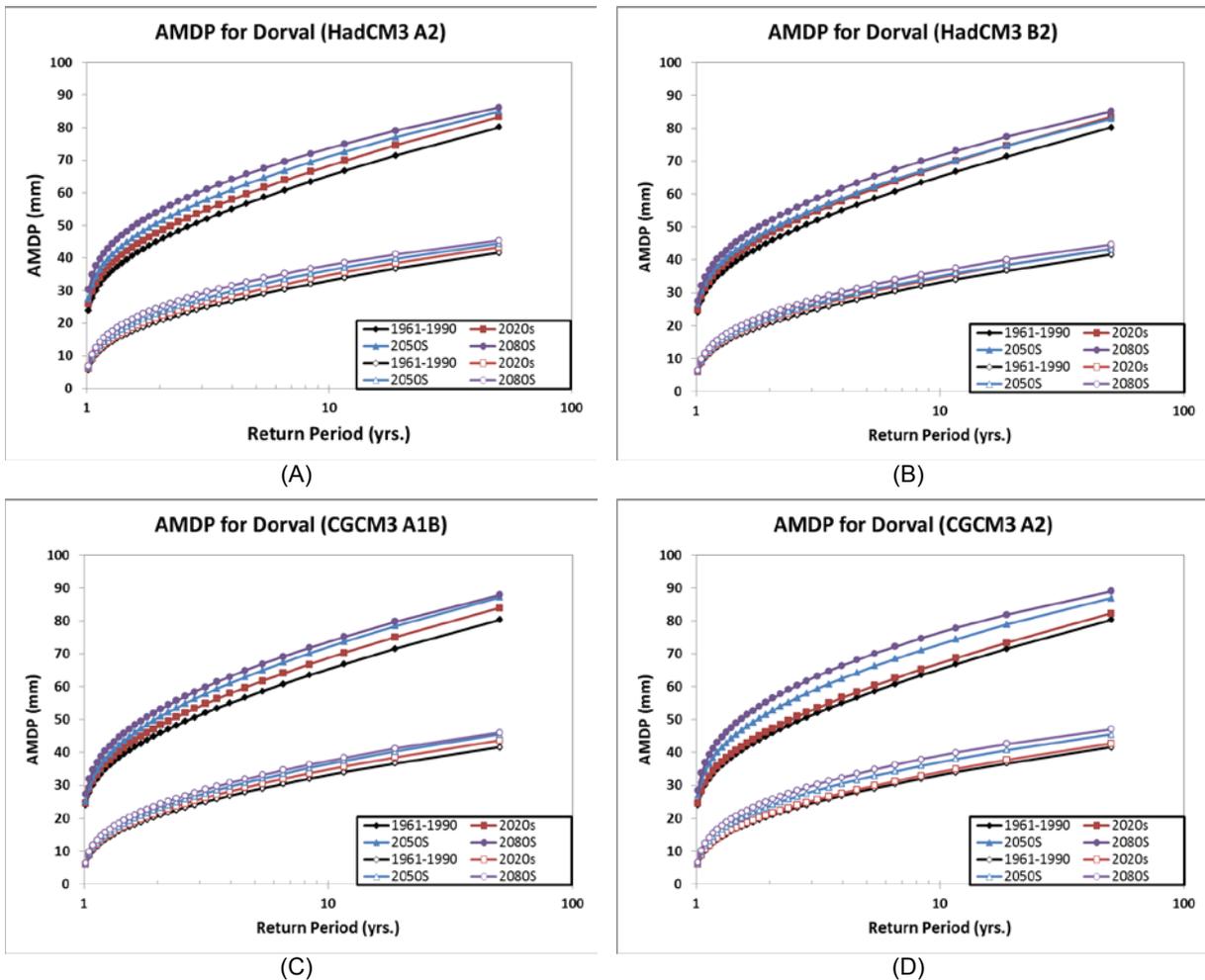
Figure 7. Probability plots of daily and 30-minute AM precipitations projected (a) from CGCM3A1B, (b) from CGCM3A2, (c) from HadCM3A2 and (d) from HadCM3B2 scenarios for the 1961-1990 period and for future periods (2020s, 2050s, and 2080s) for Dorval station. Solid markers denote daily AM precipitations and blank markers denote 30-minute AM precipitations.

Figure 7 shows the probability plots of daily and 30 minutes AM precipitations for the current (1961-1990) and future periods (2020s, 2050s, and 2080s) under different climate change scenarios (HadCM3 A2 & B2 and CGCM3 A1B & CGCM3 A2) using the proposed spatial-temporal downscaling for the Dorval station. It was found that increasing trends of AM precipitations for future periods were suggested by these four scenarios for Dorval station. In addition, changes of AM precipitations in the future given by the CGCM3 are more pronounced than changes given by the HadCM3.

4. Conclusions

In this study, a spatial-temporal SD method was proposed to describe the relationship between large-scale atmospheric variables and at-site daily and sub-daily AM precipitations for high-quality climate change impact assessment studies. More specifically, this study consists of two components: (1) to test the feasibility of SDRain for representing AM series; (2) to derive sub-daily precipitations from estimated AM daily precipitations by SDRain on the basis of four climate simulations from HadCM3 and CGCM3 and available observed daily and sub-daily AM precipitations at nine raingage stations located in Quebec, Canada. Results of the numerical application have indicated that it is feasible to describe the relationship between daily climatic variables given by GCMs and daily AM rainfalls at all local stations considered. Moreover, it was found that AM precipitations at these selected stations present 'simple' scaling properties. The statistical relationship of the scaling properties can be used to link AM precipitations of different durations. Results of the estimated sub-daily AM on the basis of the simple scaling behavior are very comparable to the observed values. Therefore, it can be concluded that the proposed spatial-temporal downscaling method could be used for describing the linkage between large-scale climate change variables provided by GCMs to daily and sub-daily AM precipitations at a given location of interest. This relationship would be useful for various climate-related impact assessment studies for a given region.

Finally, the proposed SD method was successfully used to construct the IDF curves for a given location for the current (1961-1990) and for future periods (2020s, 2050s, and 2080s) under the four selected climate change scenarios given by both HadCM3 and CGCMs. It was found that AM precipitations at a local site downscaled from the HadCM3 displayed a smaller change in the future, while those values estimated from the CGCM3 indicated a large increasing trend for future periods. This result has demonstrated the presence of high uncertainty in climate simulations provided by different GCMs. Further studies are planned to assess the feasibility and reliability of the suggested downscaling approach using other GCMs and data from regions with different climatic conditions.

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6. Biographies

Myeong-Ho is a Research Assistant in Civil Engineering Department at McGill University. His research interests are in Hydrology and climate change impacts on water resources.

Van Nguyen is Professor and Chair of Department of Civil Engineering and Applied Mechanics as well as Director of the Brace Centre for Water Resources Management at McGill. His scientific and professional contributions over more than 30 years have been mostly in the areas of Hydrology and Water Resources Management. He is author or co-author of over 200 articles in refereed journals, specialized monographs and conference proceedings.