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Single-qubit Rotation Gate Using Three-level Lambda Systems

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Abstract: In this paper we investigate the effect of time separation and delay between two f-STIRAP on single-qubit rotation gate based on Lacour et al (2006 Opt. Commun. 264 362). The f-STIRAP is a basic method used to adiabatically transfer population between lower states, where the two pulses terminate simultaneously while maintaining a constant ratio of amplitudes. Furthermore, we obtain numerically the optimal values for the time separation and delay for a perfect single-qubit rotation gate.

Key words: Quantum Rotation Gates, Adiabatic Process, STIRAP, f-STIRAP

INTRODUCTION

The physical implementation of quantum computer requires series of accurately controllable quantum operations. These quantum operations can be implemented using quantum geometric phases (Wilczek, 1984), where some parameters are controlled around a defined curved. This method is known as holonomic or geometric quantum computation, and has become one of the key approaches to achieve quantum computation that is resilient against errors (Erik Sj, 2008). In 1999 Zanardi and Rasetti (1999) laid the theoretical foundations of holonomic quantum computation by showing that any quantum circuit can be generated by using suitable Hamiltonians that depend on experimentally controllable parameters, such as those related to the bosonic mode in a quantum optical systems (Pachos, 2000). At the same time, Jones *et al* (2000) demonstrated experimentally a quantum gate based on geometric phase that was able to entangle a pair of nuclear spins in a nuclear magnetic resonance (NMR) setup. Deutsch presented in 1989 a three qubits quantum gate and showed that these gates together with arbitrary one-qubit rotations are sufficient to create any quantum network (Lacour, 2002). Such a set of gates are called universal gates for quantum computation. Since then many sets of gates were proved to be universal (Barenco, 1995). Recently Lacour *et al* (2002) have proposed experimental technique to implement single-qubit quantum gates based on Stimulation Raman Adiabatic Process (STIRAP) and static laser phases. This technique requires only the control of relative phase of the driving fields, which can be implemented precisely, and do not involve dynamic or geometric phases. It is well known that STIRAP technique is very efficient and robust theoretically as well as experimentally. An other important technique called fractional STIRAP (f-STIRAP). It is a generalization of STIRAP and it is based upon (partial) adiabatic population transfer between two states through an intermediate state. Geometric phases accumulated during a STIRAP process were previously investigated for tripod systems and used to implement single-qubit rotations gates. In this paper we extend the work in Ref. (Lacour *et al,* 2002) and investigate the effect of time separation and delay of pulses on the single-qubit rotation gates. We utilize a recent idea by Moller *et al* (2007) who applied STIRAP to implement quantum logic gates. We numerically obtain optimal values for the time separation and delay. The work is organized as follows. In sec. II we present the atomic system under consideration. In sec.III we review the generalized rotation gate proposed by Lacour *et al* (2002). In sec. IV we study the effect of time separation and time delay between two pulses used by the two f-STIRAP. A conclusion is given in sec. V.

Model and Equations of Motion:

We consider an atom in three-level lambda configuration depicted in Fig.1. The two lower levels |0> and |1> are long-lived atomic states. They are coupled to the upper state |*e*> by two non resonant coherent laser fields with Rabi frequencies $\Omega_{+(-)}$ In practice the lower states can be ground Zeeman or hyperfine sub-levels, and the upper state is an electronically excited state or excited state manifold. The Hamiltonian of the model can be expressed in the basis states {|0>, |1>, |e>} by

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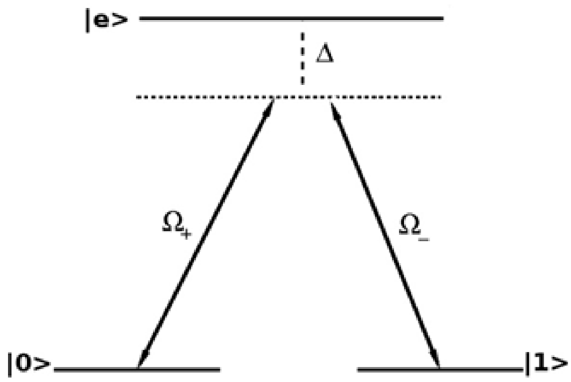

Fig. 1: Three-level lambda system with lower levels driven by two non resonant coherent fields. The detuning Δ, the common one-photon detuning of the two laser fields, is very large compared to the Rabi frequencies $\Omega_{\pm}$, so that the excited state |e> can be adiabatically eliminated.

$$H(t) = \frac{\hbar}{2}\begin{bmatrix} 0 & \Omega_-(t) & 0 \\ \Omega_-(t) & 2\Delta & \Omega_+(t) \\ 0 & \Omega_+(t) & 0 \end{bmatrix},$$

where Δ is the common one-photon detuning of the two laser fields which have time-dependent Gaussian profile given by

$$\Omega_j(t-t_j) = A_j e^{\frac{(t-t_j)^2}{\sigma^2}}$$

where σ is the width, Ω_j is the amplitude and t_j is the time delay. The detuning Δ is taken to be large enough so that the excited state |e> *is* therefore never populated during the coherent pumping process. Thus, our qubit consists of the two lower states {|0>, |1>}.
The evolution of the system is governed by the Schrödinger equation

$$i\frac{\partial}{\partial t}|\square(t)> = H|\square(t)>,$$

This Hamiltonian *H* has three eigenvalues

$$\omega_0 = 0,$$

$$\omega_{\pm} = \frac{1}{2}\left(\Delta \pm \sqrt{\Delta^2 + |\Omega_+|^2 + |\Omega_-|^2}\right).$$

They are the instantaneous adiabatic eigenvalues. Their correspondent eigenstates are

$$|\square_0> = \cos\theta\,|0> + e^{i\phi}\sin\theta\,|1>,$$

$$|\square_+> = \sin\theta\cos\psi|0>$$

$$+ e^{i\phi}\cos\theta\cos\psi\,|1> + \sin\psi|e>,$$

$$|\square_-> = \sin\theta\sin\psi|0>$$

$$+ e^{i\phi}\cos\theta\sin\psi\,|1> - \cos\psi|1>,$$

where φ is the relative phase between the two coherent lasers, $|\square_0 >$ is the dark state, $|\square_\pm >$ are the bright states, and

$$tan\,\theta = \frac{|\Omega_+|}{|\Omega_-|},$$

$$tan\,\psi = \frac{\Delta + \sqrt{\Delta^2 + |\Omega_+|^2 + |\Omega_-|^2}}{\sqrt{|\Omega_+|^2 + |\Omega_-|^2}},$$

In the next section, we review the implementation of the generalized rotation gate proposed in Ref. (Lacour, 2002).

Generalized Single-qubit Rotation Gate:

To construct the generalized rotation gate

$$R(\alpha,\phi) = \begin{bmatrix} cos\,\alpha & e^{i\phi}\,sin\,\alpha \\ -\,e^{i\phi} sin\,\alpha & cos\,\alpha \end{bmatrix}$$

we briefly review the technique given by Lacour *et al* in Ref. (Lacour, 2002). Two f-STIRAP processes separated by *T* in time are used to transfer population among the lower levels. Each f-STIRAP process has two pulses separated by τ in time. The first f-STIRAP process is a reversed f-STIRAP with elliptic and σ-pulses starting with a constant ratio $\Omega_+ / \Omega_- \to$ cot α and ending such that the σ- pulse vanishes first

$$\Omega_+ = \Omega_0(t + T - \tau) + \Omega_0(t + T + \tau)\,cos\,\alpha$$

$$\Omega_- = \Omega_0(t + T + \tau)\,sin\,\alpha$$

The second f-STIRAP process is a standard f-STIRAP where the pulses are switched on counter intuitively and switched off in a given constant ratio $\Omega_+ / \Omega_- \to$ tan α

$$\Omega_+ = \Omega_0(t - T - \tau)\,sin\,\alpha$$

$$\Omega_- = \Omega_0(t - T + \tau) + \Omega_0(t - T - \tau)\,cos\,\alpha$$

As stated before, for large detuning, Δ>>1, the excited state |e> can be adiabatically eliminated. The evolution of the system from the initial time t_i to the end of the first f-STIRAP $t`_f$ is given in the basis {|0>, |1>} by

$$U_1(t_i, t'_f) = \begin{bmatrix} e^{-iA_-} & 0 \\ 0 & 1 \end{bmatrix} R(\alpha,\phi)$$

where *A*- is the dynamical phase acquired by the eigenstate $|\square_- >$ during the first f-STIRAP. From the time $t`_f$ to the end of the second f-STIRAP the evolution of the system is given by

$$U_1(t'_f, t_f) = R(\alpha,\phi) \begin{bmatrix} 1 & 0 \\ 0 & e^{-iA'_-} \end{bmatrix},$$

where $A`_-$ is the dynamical phase acquired by the eigenstate $|\square_- >$ during the second f-STIRAP. Combining U_1 and U_2 the resulting operation from t_i to t_f leads to the propagator

$$U(t_i, t_f) = U_1 U_2 = R(\alpha,\phi) \begin{bmatrix} e^{-iA_-} & 0 \\ 0 & e^{-iA'_-} \end{bmatrix} R(\alpha,\phi).$$

If the two f-STIRAP have the same pulse shapes with the same delay, the dynamical phases acquired by the dark state $|\square_0 >$ in the two f-STIRAP are the same, $A_- = A'_-$. Thus, a compensation of the dynamic phase is achieved. When the pulse sequences is applied to our model, the process leads to the rotation gate *R*(2α, φ) up to a global phase. In Fig. 2 we plot the time-depend Rabi f requencies. In the next section we discuss the effect of the two parameters *T* and τ on the quantum rotation gate R(π/4φ).

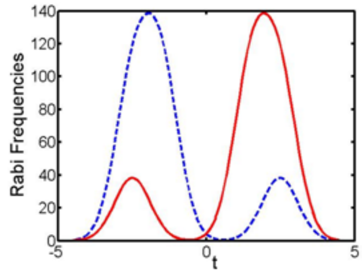

Fig. 2: Double f-STIRAP process with Gaussian pulses. Solid (dashed) line represents the time dependence of $\Omega_{+(-)}$. The parameters are as follows. Δ = 350, A_0 = 100, τ = 0.487, T = 2 and a = π/8.

The Effect of T and τ on the Rotation Gate:

To illustrate the effect of *T* on the single-qubit rotation gate we choose a special gate *R*(π/4,0), we numerically get the solutions of Schrödinger equation and plot the populations ρ_{11} and ρ_{11} as a function of *T*, assuming the atom is prepared initially in the state |0>, Δ = 350, A_0 = 100, τ = 0.487, *a* = π/8 and the final time t_f = 6. We can clearly see from Fig. 3 that there are four points, T =0.17, 0.36, 0.46, 2 at which $\rho_{11} = \rho_{22}$ = 0.5. One can see that at the first three points the slopes are high, which means that any small deviation from these points will result in imperfect rotation gate. At the last point which corresponds to *T*=2 the slope is small. This is the optimal value of *T* that can be used to get a perfect rotation gate for τ = 0.487.

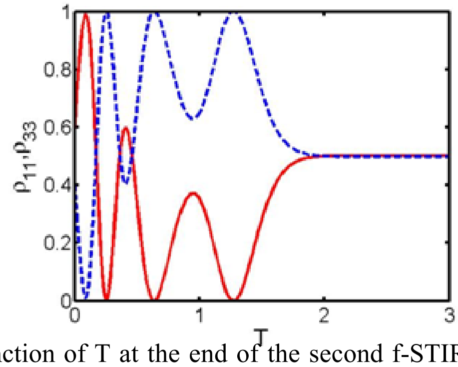

Fig. 3: The population as a function of T at the end of the second f-STIRAP. Solid line is for ρ_{11} and dashed line for ρ_{22}. The parameters are the same as Fig.2.

In Fig. 4 we plot the evolution of the populations as a function of time *t* for the optimal value *T* = 2. One can see from these figures that at the end of the second f-STIRAP we obtain a perfect single-qubit rotation gate R(π/4, 0).

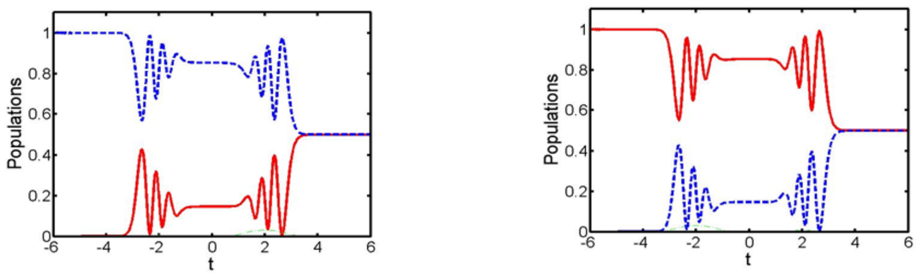

Fig. 4: Numerical solutions for the populations as a function of time. The parameters are: the same as Fig.2, the upper figure, the atom is initially prepared in the state |0>. The lower figure, the atom is initially prepared in the state |1>. Solid line for ρ_{11} and dashed line for ρ_{22}.

Now we turn our attention to the effect of time delay τ. In Fig. 5 we plot the populations of the lower levels as a function of τ. We can clearly see that there are three points to be selected τ = 0.40, 0.487, 0.56 where $\rho_{11} = \rho_{22}$ = 0.5. The point at τ = 0.487 is the optimal one because at this point the slope is small compared to the slope at other points.

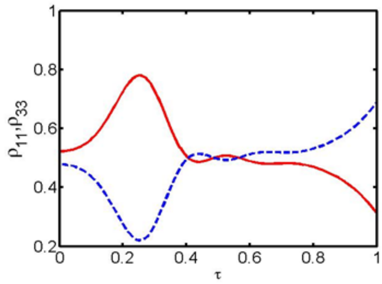

Fig. 5: The population as a function of τ at the end of the second f-STIRAP. Solid line is for ρ_{11} and dashed line for ρ_{22}. The parameters are the same as Fig. 2.

Conclusions:

In conclusion, we have investigated the effect of time separation and time delay of two f-STIRAP on the single-qubit rotation gate. One has to be careful when selecting the values of T and τ to obtain a perfect single-qubit rotation gate. We give optimal values τ = 0.487, T=2 for the special rotation gate R(π/4, 0).

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