Fault Detection and Diagnosis for GTM UAV with Dual Unscented Kalman Filter

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This paper presents an applicable procedure for Fault Detection and Diagnosis (FDD) in a realistic nonlinear six degree-of-freedom unmanned aerial vehicle (UAV) model. The work has been developed based on the Matlab/Simulink environment of the NASA Generic Transport Model (GTM) UAV under the NASA Aviation Safety Program (AvSP). By introducing the partial loss fault in aircraft actuators into the GTM model, the dual Unscented Kalman Filter (UKF) algorithm is implemented for online estimation of both flight states and fault parameters, and for making statistical decisions associated with fault detection and diagnosis.

Nomenclature

V _{EAS}	=	Mass of the vehicle	
α	=	Angle of attack (rad)	
q	=	Pitch angle rate (rad/sec)	
θ	=	Pitch angle (rad)	
т	=	Mass of GTM	
δ_Z	=	Position of surface Z (δ_{ele} : elevator, $\delta_{throttle}$: throttle)	
λ	=	Composite scaling parameter	
L	=	Dimension of the state	
R ^r	=	Process noise covariance	
Rª	=	Measurement noise covariance	

I. Introduction

In flight, loss of control has become one of the causes of airplane crashes and crash-related fatal accidents worldwide for many years. Thus, it is necessary to enable aircraft to increase the fault tolerance ability where unexpected faults occur in the aircraft. When a fault occurs in the aircraft, the first and main problem to be solved is to detect what and where the fault is and to diagnose it, and then to give a solution for it. This is the motivation for the Fault Detection and Diagnosis (FDD) and Fault Tolerant Control (FTC) in the aviation industry¹. A good FDD scheme should be able to report detailed information for the post-fault system as accurate as possible. On the purpose of investigating flight dynamic and studying the behavior of the aircraft in upset conditions, NASA built a test bed which is the Generic Transport Model (GTM). GTM is a 5.5% dynamically scaled, turbine powered fixed-wing Unmanned Aerial Vehicle (UAV)². In this paper, we will focus on developing application of a FDD scheme for Linear Parameter Varying (LPV) model^{3,4,5} of the GTM in the event of actuator faults or failures. The scheme utilizes a Dual Unscented Kalman Filter (DUKF)^{6,7} with real time fault parameters identification based on the measured outputs of the sensors and the control inputs to actuators.

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For a high performance FDD/FTC scheme, when a fault/failure occurs either in an actuator or sensor, the FDD scheme will detect and diagnose the source and the magnitude of the fault timely. The reconfiguration scheme will design the reconfigurable controller based on this information to balance and adapt to the faults/failures. Therefore the entire dynamic system can still achieve acceptable level of performance and keep stability of the airplane. Figure 1 depicts the general structure of FDD/FTC. In this work, both the system state variables and the actuator fault parameters are estimated by using DUKF in the FDD module.



Figure 1. General structure of a fault tolerant control system (FTCS)

Many researchers focus on developing methodologies to detect and diagnose actuator faults. An actuator fault may enable aircraft becoming unstable and may cause crash. In this paper, in order to simplify the presentation, we will focus on one actuator fault: elevator partial loss fault and we will investigate how elevator partial loss fault affects the performance of the GTM in the longitudinal motion. Although the proposed FDD scheme is tested based only on the longitudinal motion of the six degree of freedom (DOF) nonlinear GTM, the developed FDD scheme is suitable to both longitudinal and lateral motion of the UAV. Investigation and implementation of the proposed FDD scheme of the proposed FDD scheme on both longitudinal and lateral motion are one of our future works.

The known FDD approaches can be classified into two categories¹: 1) model-based and 2) data-based (model-free) schemes; these two schemes are also known as quantitative and qualitative approaches. In general, we can use a set of equations of motion to describe the dynamic motion of a flight vehicle. In this paper, faults occurring in the GTM are considered as additive random biases. Hence, we can approach fault detection and diagnosis as a model-based bias estimation problem^{8,9}.

For nonlinear aircraft system, the Extended Kalman Filter (EKF) has been applied widely. The EKF is the nonlinear version of the Kalman filter, and it only simply linearizes about the current equilibrium point with the characteristics of random state variables described by mean and covariance. Hence, the EKF can only preserve the first-order system statistics and may quickly diverge if the process is not modeled correctly, due to the linearization operation. The UKF is the improvement of the EKF to replace the EKF in nonlinear filtering problems. In the UKF, the probability density is approximated by the nonlinear transformation of a random variable, which returns much more accurate results than the first-order Taylor expansion of the nonlinear functions used in the EKF. The approximation utilizes a set of sample points, which guarantees accuracy with the posterior mean and covariance to

the second order for any nonlinearity⁶.

The paper is organized as follows. In Section II, the Linear Parameter Varying (LPV) model of the nonlinear GTM model is briefly introduced. In Section III, the DUKF is presented. In Section IV, fault detection and diagnosis schemes are described. In Section V, simulation results and analysis for the proposed FDD scheme implemented in the six DOF of nonlinear GTM are reported. A brief overall conclusion is given in Section VI.

II. Linear Parameter Varying Model of GTM

In this section, we will briefly introduce the concept of Linear Parameter Varying (LPV) models of nonlinear longitudinal motion of GTM⁵. The GTM is a dynamically scaled small unmanned aerial vehicle developed by NASA to investigate modeling and control of large transport vehicles in upset conditions².

The nonlinear equations for the longitudinal motion of GTM are given by:

$$V_{\overline{c}AS} = \frac{1}{m} (F_{\chi} \cos \alpha + F_{\overline{c}} \sin \alpha) \tag{1}$$

$$\dot{\alpha} = \frac{1}{mV_{g,qg}} \left(-F_{g} \sin \alpha + F_{g} \cos \alpha \right) + q \tag{2}$$

$$\vec{\vartheta} = q \tag{3}$$

$$\dot{q} = \frac{\omega_y}{\tau_{yy}} \tag{4}$$

These equations contain transcendental functions and aerodynamic data which are obtained through wind tunnel testing and flight tests. Since our FDD is model-based bias estimation, LPV model is chosen for FDD design to the nonlinear model of GTM due to real-time implementation consideration.

LPV modeling and control of nonlinear systems have been widely studied since the early 1990's^{3,4}. LPV model will simulate the actual nonlinear system by using time-varying real parameters like altitude and/or speed to obtain smooth semi-linear models. The state-space matrices A, B, C and D of a LPV model depend continuously on some vector of time-varying parameters $\rho(t)$. Parameters $\rho(t)$ are assumed to be measured at the current time and not known in advance although its value is constrained *a priori* to lie in some known, bounded set and is continuous. There are three techniques for obtaining LPV models from a nonlinear system⁵. The first method is to use Jacobian linearization at a number of selected equilibrium points. The second technique is based on exact state transformations at a number of selected equilibrium points. The last method corresponds to obtain a LPV model at a unique trim point by decomposing the nonlinear function.

The transcendental functions can be approximated by third-order Taylor series extension. The aerodynamic data which are obtained by using look-up table in the nonlinear model of GTM can be approximated by polynomial equations¹⁰. The LPV model of longitudinal motion of the GTM has state variables $x = [EAS \ \alpha \ q \ \theta]^T$, with equivalent airspeed (*EAS*), pitch angle rate (q), angle of attack (α), and pitch angle (θ) and input $u = [\delta_{ele} \ \delta_{thrattle}]^T$, with δ_{ele} representing elevator deflection and $\delta_{thrattle}$ representing throttle deflection.

In the original LPV model of the GTM, fault models were not included. Partial loss of control effectiveness in elevator has been implemented for FDD purpose in this work.

III. Dual Unscented Kalman Filter Algorithm

In this section, we will present an overview of the Dual UKF state-parameter estimation scheme implemented for estimation of the reduction of the actuator's control effectiveness.

UKF was originally developed by Eric A. Wan and Rudolph van der Merwe in 2000⁶, and it is mainly used to nonlinear system identification, training of neural networks and dual estimation problems. The model is highly non-linear, and the UKF picks a minimal set of sample points which are called *sigma points* around the mean by the unscented. These sigma points are then propagated through the non-linear functions and the covariance of the estimate is then recovered. It captures the posterior mean and covariance accurately to the 3rd order (Taylor series expansion) for any nonlinearity. Therefore, the UKF captures both the first-order and second-order statistics of the nonlinear system. It has been demonstrated that the UKF has better filter performance compared with EKF and is equivalent to the performance of second-order EKF^{6,7}. Figure 2 shows an example of the unscented transformation (UT) for mean

and covariance propagation.



Figure 2. Example of the UT for mean and covariance propagation

From Figure 2, we can find that the result of UKF is more accurate to capture the true mean and covariance than other Kalman filters do.

A. Unscented Transformation

The unscented transformation is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation⁶. It is built on the principle that it is easier to approximate a probability distribution than an arbitrary nonlinear function. The approach is illustrated in Figure 3.



Figure 3. The principle of the unscented transform

Consider a nonlinear model y = f(x), and the random variable x whose dimension is L and assume x has mean \overline{x} and covariance P_x . To calculate the statistics of y, one needs obtain:

$$\chi_{\mathbf{0}} = \overline{\mathbf{x}} \tag{5}$$

$$\chi_i = \overline{x} + \left(\sqrt{(L+\lambda)P_x}\right)_i \quad i = 1, \cdots, L \tag{6}$$

$$\chi_i = \overline{x} - \left(\sqrt{(L+\lambda)P_x}\right)_i \quad i = L = L + 1, \cdots, 2L \tag{7}$$

$$W_{\varrho}^{(m)} = \frac{\lambda}{L+\lambda}$$
(8)

$$W_{\varepsilon}^{(c)} = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta)$$
⁽⁹⁾

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{\{2(L+\lambda)\}} \quad i = 1, \cdots, 2L$$
(10)

where $\lambda = \alpha^2 (L + k) - L$ is a scaling parameter. In this paper, $\alpha = 1, \beta = 2, k = 3 - L$.

B. State Estimation

Consider a nonlinear transform of a random variable:

$$y = f(\mathbf{x})$$

Given: $\overline{\mathbf{x}} = E[\mathbf{x}] \quad \mathbf{P}_{\mathbf{x}} = E[(\mathbf{x} - \overline{\mathbf{x}})(\mathbf{x} - \overline{\mathbf{x}})^T]$
Find: $\overline{\mathbf{y}} = E[\mathbf{y}] \quad \mathbf{P}_{\mathbf{y}} = E[(\mathbf{y} - \mathbf{y})(\mathbf{y} - \overline{\mathbf{y}})^T]$

A set of 2L+1 sigma points are derived from the augmented state and covariance where L is the dimension of the augmented state.

$$\chi^{a}_{k-1|k-1} = \chi^{a}_{k-1|k-1} \tag{11}$$

$$\chi^{i}_{k-1|k-1} = \chi^{a}_{k-1|k-1} + \left(\sqrt{(L+\lambda)P^{a}_{k-1|k-1}}\right)_{i} \quad i = 1, \cdots, L$$
(12)

$$\chi^{i}_{k-1|k-1} = \chi^{a}_{k-1|k-1} - \left(\sqrt{(L+\lambda)P^{a}_{k-1|k-1}}\right)_{i} \quad i = L = L+1, \cdots, 2L$$
(13)

where $\left(\sqrt{(L+\lambda)P^{a}_{k-1|k-1}}\right)_{i}$ is the *i*th column of the matrix square root of $\sqrt{(L+\lambda)P^{a}_{k-1|k-1}}$.

Using the definition: square root A of matrix B satisfies

$B \equiv AA^T$

The complete state estimation of the UKF is given below:

 $\hat{\boldsymbol{x}}_{\boldsymbol{g}} = \boldsymbol{E}[\boldsymbol{x}_{\boldsymbol{g}}] \tag{14}$

$$\boldsymbol{P}_{\boldsymbol{\varrho}} = \boldsymbol{E}[(\boldsymbol{x}_{\boldsymbol{\varrho}} - \hat{\boldsymbol{x}}_{\boldsymbol{\varrho}})(\boldsymbol{x}_{\boldsymbol{\varrho}} - \hat{\boldsymbol{x}}_{\boldsymbol{\varrho}})^{\mathrm{T}}]$$
(15)

$$\widehat{\boldsymbol{x}}^{\boldsymbol{\alpha}}_{\boldsymbol{\theta}} = \boldsymbol{E}[\boldsymbol{x}^{\boldsymbol{\alpha}}] = [\widehat{\boldsymbol{x}}^{T}_{\boldsymbol{\theta}} \quad \boldsymbol{0} \quad \boldsymbol{0}]^{T}$$
(16)

$$P_{g}^{a} = E[(x_{g}^{a} - \hat{x}_{g}^{a})(x_{g}^{a} - \hat{x}_{0}^{a})^{T}] = \begin{bmatrix} P_{g} & 0 & 0\\ 0 & R^{v} & 0\\ 0 & 0 & R^{n} \end{bmatrix}$$
(17)

For $k \in \{1, \dots, \infty\}$,

Calculate the sigma points:

$$\chi_{k-1}^{a} = \begin{bmatrix} \hat{x}_{k-1}^{a} & \hat{x}_{k-1}^{a} + \gamma \sqrt{P_{k-1}^{a}} & \hat{x}_{k-1}^{a} - \gamma \sqrt{P_{k-1}^{a}} \end{bmatrix}$$
(18)

The time-update equations are:

$$\chi_{k|k-1}^{x} = F(\chi_{k-1}^{x}, u_{k-1}, \chi_{k-1}^{v})$$
(19)

$$\hat{x}_{k}^{-} = \sum_{i=0}^{2L} W_{i}^{(m)} \chi_{i,k|k-1}^{x}$$
(20)

$$P_{k}^{-} = \sum_{i=0}^{2L} W_{i}^{(c)} (\chi_{i,k|k-1}^{x} - \widehat{x}_{k}) (\chi_{i,k|k-1}^{x} - \widehat{x}_{k}^{-})^{T}$$
(21)

$$\mathcal{Y}_{k|k-1} = H(\chi^{x}_{k|k-1}, \chi^{n}_{k-1})$$
(22)

$$\hat{y}_{k}^{-} = \sum_{i=0}^{2L} W_{i}^{(m)} \mathcal{Y}_{i,k|k-1}$$
(23)

and the measurement-update equations are:

$$\boldsymbol{P}_{\hat{y}_{k}\bar{y}_{k}} = \sum_{i=0}^{2l} W_{i}^{(c)} (\boldsymbol{y}_{i,k|k-1} - \hat{y}_{k}^{-}) (\boldsymbol{y}_{i,k|k-1} - \hat{y}_{k}^{-})^{l}$$
(24)

$$\boldsymbol{P}_{x_{k}y_{k}} = \sum_{i=0}^{2L} W_{i}^{(c)} (\chi_{i,k|k-1} - \widehat{x}_{k}) (\chi_{i,k|k-1} - \widehat{x}_{k})^{T}$$
(25)

The complete parameter estimation of UKF is given below:

$$\hat{\boldsymbol{W}}_{\boldsymbol{\theta}} = \boldsymbol{E}[\boldsymbol{w}] \tag{26}$$

$$\boldsymbol{P}_{w_0} = E[(w - \hat{w}_0)(w - \hat{w}_0)^T]$$
(27)

For $k \in \{1, \dots, \infty\}$, the time update and sigma-point calculation are given by:

$$\hat{w}_k^- = \hat{w}_{k-1} \tag{28}$$

$$P_{W_k}^- = P_{W_{k-1}} + R_{k-1}^r \tag{29}$$

$$W_{k|k-1} = \begin{bmatrix} \hat{w}_{\bar{k}} & \hat{w}_{\bar{k}} + \gamma \sqrt{P_{\bar{w}_{\bar{k}}}} & \hat{w}_{\bar{k}} - \gamma \sqrt{P_{\bar{w}_{\bar{k}}}} \end{bmatrix}$$
(30)

$$\mathcal{D}_{k|k-1} = G(\boldsymbol{x}_k, \boldsymbol{W}_{k|k-1}) \tag{31}$$

Option 1: $\hat{d}_k = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{D}_{i,k|k-1}$

Option 2: $\hat{d}_k = G(x_k, \hat{w}_k)$

and the measurement-update equations are:

$$\boldsymbol{P}_{\tilde{d}_{k}\tilde{d}_{k}} = \sum_{i=0}^{2L} W_{i}^{(C)} \left(\boldsymbol{\mathcal{D}}_{i,k|k-1} - \hat{d}_{k} \right) \left(\boldsymbol{\mathcal{D}}_{i,k|k-1} - \hat{d}_{k} \right)^{T} + \boldsymbol{R}_{k}^{e}$$
(32)

$$\boldsymbol{P}_{w_k, d_k} = \sum_{i=0}^{2L} W_i^{(c)} \left(W_{i,k|k-1} - \hat{W}_k^{-} \right) \left(\mathcal{D}_{i,k|k-1} - \hat{d}_k \right)^T$$
(33)

$$\mathcal{K}_{k} = \boldsymbol{P}_{W_{k}d_{k}}\boldsymbol{P}_{\tilde{d}_{k}\tilde{d}_{k}}^{1} \tag{34}$$

$$\hat{w}_k = \hat{w}_k + \mathcal{K}_k (d_k - \hat{d}_k) \tag{35}$$

$$\boldsymbol{P}_{w_k} = \boldsymbol{P}_{w_k}^{-} - \mathcal{K}_k \boldsymbol{P}_{\boldsymbol{\partial}_k \boldsymbol{\partial}_k} \mathcal{K}_k^{\mathrm{T}}$$
(36)

where $\gamma = \sqrt{L + \lambda}$, λ is the composite scaling parameter, L is the dimension of the state, $\mathbf{R}^{\mathbf{r}}$ is the process noise covariance, $\mathbf{R}^{\mathbf{r}}$ is the measurement noise covariance.

C. UKF Dual Estimation

In the dual UKF estimation, both states of the dynamical system and its parameters are estimated simultaneously, given only noisy observations. At every time sample, a UKF state filter estimates the state using the current model estimate \hat{w}_k , while the UKF parameter filter estimates the parameters using the current state estimate \hat{x}_k . The estimation scheme is shown in Figure 4.



Figure 4. Sequential approach of DUKF designed to pass over the data one point at a time

D. Actuator Fault Model

Faults that develop in a linear time-varying system associated with the actuators can be represented by an equation as follows:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) - B\mathbf{y}\mathbf{u}(t) + \mathbf{u}\mathbf{r}(t)$$
(37)

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{v}(t) \tag{38}$$

with $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^l$ and $y(t) \in \mathbb{R}^l$ is the state, control input and output variables, respectively. u(t) and v(t) denote white noise. $\gamma = diag(\gamma_1, \dots, \gamma_m)$ where γ_i are scalars satisfying $0 \le \gamma_i \le 1$. If $\gamma_i = 0$, the *i*th actuator is working perfectly whereas if $\gamma_i > 0$, a fault is present. Hence, to avoid estimating **B** directly, we switch to estimate the control effectiveness γ which is formulated in above equations. The objective of FDD is to determine the extent of the loss in the control effectiveness, $\gamma_i^{8,9}$.

IV. Fault Detection and Diagnosis Schemes

The state and parameter estimation methods for FDD are based on the concept that faults typically affect the physical coefficients of the process. By estimating the parameters of a process model on-line, residuals are computed as the parameter estimation errors and will be passed to the reconfiguration controller scheme. To successfully diagnosis faults and failure, the mapping from the model coefficients to the process parameters is necessary. Part of the diagnosis task is to recognize the changes in a dynamic system. The detection is based on statistical hypothesis test which involves two phases.

The first phase is to get the statistical quantities of the normal operations after system becomes stable, like mean values and variances. Assume that the residuals from the output estimates and from the estimated fault parameters follow the Normal or Gaussian distribution, or very close. The second phase determines the same statistical quantities when the system is not normal by using smaller moving window. By defining an appropriate statistical detection variable to accentuate the deviation in the statistical quantities from their normal values, the detection and diagnosis of a loss of control effectiveness can be achieved. To carry out an on-line fault detection and isolation, the recursive calculation of the detection variables is highly desirable.

Phase I: Define $\hat{\gamma}(\mathbf{k}) \sim \mathbb{D}(\overline{\mu}_{\gamma^0}, \sigma^2_{\gamma^0})$, where $\hat{\gamma}(\mathbf{k}) \in \mathbb{R}^p$ denotes the chosen residuals vector from the estimated fault parameters and the measurement residuals of the filter. $\overline{\mu}_{\gamma^0}$ represents the mean value of $\hat{\gamma}(\mathbf{k}), \sigma^2_{\gamma^0}$ denotes the associated variance.

or
$$i = 1, \cdots, N_i$$
, using

$$\overline{\mu}_{\widehat{\gamma}_i} \circ (k) = \frac{1}{k} \sum_{j=1}^k \hat{\gamma}_i (j)$$
(39)

to obtain the mean, and covariance can be obtained by

$$\boldsymbol{\sigma}^{2}_{\hat{\boldsymbol{\gamma}}_{t}}\boldsymbol{\boldsymbol{\upsilon}}(\boldsymbol{k}) = \frac{1}{\boldsymbol{k}-1} \sum_{j=1}^{\boldsymbol{k}} \left[\hat{\boldsymbol{\gamma}}_{t}(\boldsymbol{j}) - \overline{\boldsymbol{\mu}}_{\hat{\boldsymbol{\gamma}}_{t}}\boldsymbol{\boldsymbol{\upsilon}}(\boldsymbol{k}) \right]^{2}$$
(40)

Or in recursive form:

$$\overline{\mu}_{\widehat{\gamma}_{i}} \circ (k) = \frac{k-1}{k} \overline{\mu}_{\widehat{\gamma}_{i}} \circ (k-1) + \widehat{\gamma}_{i}(k)$$

$$\tag{41}$$

$$\sigma^{2}_{\hat{\gamma}_{l}} \circ(k) = \frac{k-2}{k-1} \sigma^{2}_{\hat{\gamma}_{l}} \circ(k-1) + \frac{1}{k} \left[\hat{\gamma}_{l}(k) - \overline{\mu}_{\hat{\gamma}_{l}} \circ(k-1) \right]^{2}$$
(42)

 N_1 is sample size of a discrete random vector, and generally it is chosen to ensure a sufficient accuracy of getting the statistical quantities of the normal operations.

Phase II: to determine the statistical quantities of the abnormal operation.

Define the following moving data window based statistical quantities

$$\begin{split} \bar{\mu}_{\hat{\gamma}_{i}}(k) &= \frac{1}{N_{2}} \sum_{j=k-N_{2}+1}^{\kappa} \hat{\gamma}_{i}(j) \\ &= \bar{\mu}_{\hat{\gamma}_{i}}(k-1) - \frac{1}{N_{2}} [\hat{\gamma}_{i}(k-N_{2}) - \hat{\gamma}_{i}(k)] \\ &= \bar{\mu}_{\hat{\gamma}_{i}}(k-1) - \frac{1}{N_{2}} \delta_{i}(k) \end{split}$$
(43)

$$\sigma^{2}_{\hat{\gamma}_{l}l}(k) = \frac{1}{N_{2}-1} \sum_{j=k-N_{2}+1}^{k} \left[\hat{\gamma}_{i}(j) - \bar{\mu}_{\hat{\gamma}_{l}}^{0} \right]^{2}$$
(44)

$$\sigma^{2}_{\hat{\gamma}_{i}H}(k) = \frac{1}{N_{2}-1} \sum_{j=k-N_{2}+1}^{k} [\hat{\gamma}_{i}(j) - \bar{\mu}_{\hat{\gamma}_{i}}(k)]^{2}$$
(45)

Then, a fault in the system corresponding to the tth residual is declared at time k if the following detection variable

$$d_{i}(k) = \frac{\sigma^{2} \gamma_{i} \sigma^{(k)}}{\sigma^{2} \gamma_{i} \sigma^{(k)}} - \ln \frac{\sigma^{2} \gamma_{i} \sigma^{(k)}}{\sigma^{2} \gamma_{i} \sigma^{(k)}} - 1, i = 1, \cdots, p$$
(46)

exceeds a predetermined threshold $\boldsymbol{\varepsilon}_{i}$.

$$d_{\tilde{t}}(k) \underset{H_{0}}{\overset{H_{1}}{\geq}} \varepsilon_{\tilde{t}}$$

$$(47)$$

where $H_{\emptyset} = \{i$ th residual no fault indication}, $H_i = \{i$ th residual fault indication}. The selection of the window length, N_2 , and the threshold, ε_i , represents some trade-off between the probability of false alarm the probability of missed detection.

V. Simulation Results of GTM

The FDD method introduced in the previous section is implemented in the LPV model of GTM, and simulation results and analysis will be presented in this section. The application to GTM is based on LPV model. In this paper, we assume that the collective elevator actuator has the failure while others are remain healthy. The response to the airplane is captured through equivalent airspeed (EAS), pitch angle (q), angle of attack (α), and pitch attitude (θ). The throttle is kept constant at its trim setting through out the maneuver.

In the following, two fault scenarios are simulated: 1) a 50% of loss of control effectiveness fault in elevator at 5 sec. 2) a 20% of loss of control effectiveness fault in elevator at 6 sec. The measurement interval is T = 0.01 sec.

The UKF parameters are listed as follows:

 $\alpha = 1, \beta = 2, K = 3 - L$, where L is the dimension of the augmented state.

$$\begin{aligned} Rr_{ukfpe} &= diag\left(\left(\frac{1}{0.09995} - 1\right)\right) Pw, \ Re_{ukfpe} &= 0.009^2 eye(4.4), Rv = 0.01^2 eye(4.4), \\ Rn &= eye(4.4), Px = eye(4.4), Pw = 0.001 eye(2.2) \end{aligned}$$

As shown in Figure 5, it can be easily seen that the states outputs of TSKF and UKF equivalent airspeed (EAS), pitch angle (q), angle of attack (α) , and pitch attitude (θ) matched well with measured outputs.

1) A 50% of loss of control effectiveness fault in elevator at 5 sec.



Figure 5. Results of estimated states



Figure 6. Results of estimated parameters

2) A 20% of loss of control effectiveness fault in elevator at 6 sec.



Figure 7. Results of estimated states



Figure 8. Results of estimated parameters

Figure 6 and Figure 8 show that the better performance has been achieved by the Dual UKF. In this case, the actuator faults have been detected at 5.3 sec and 6.3 sec, with 0.3 sec delay. For comparison purpose, a TSKF (Two-Stage Kalman Filter) developed in [8,9] is also applied to GTM, it can be viewed that similar parameter estimation accuracy has been achieved by both DUKF and TSKF.

VI. Conclusion and Future Work

This paper presents results of an on-line Fault Detection and Diagnosis (FDD) design based on nonlinear recursive parameter estimation in the discrete-time stochastic system. Dual UKF has been introduced for aircraft on-line state and parameter dual estimation. Dual UKF can correctly estimate all the states and fault parameters within the given time limits in the LPV model of nonlinear aircraft model. The Dual UKF has the advantage that it separates state estimation and parameter estimation, which is more accurate, compared with other Kalman filters, and furthermore Dual UKF can use the nonlinear model of system directly, no need to linearize the system. However, the Dual UKF is also computationally more expensive. Through experience in this work, it can be seen that the Dual UKF is a powerful recursive state and parameter estimation algorithm and it improves the reliability of parameter estimates in the nonlinear systems.

In this paper, only actuator faults have been considered while others are remained proper. In reality, the faults can also occur in the sensors and system components such as wing damages. Furthermore, due to the limited time, only partial loss type faults have been considered. Our future work includes consideration of actuator stuck failures and wing damages, improvement of robustness and performance of Dual UKF based FDD algorithms, and integration to fault tolerant control to form a complete active fault tolerant control system.

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