High-Performance Turbo-MIMO System Design with Iterative Soft-Detection and Decoding

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***Abstract*—In turbo-multiple-input multiple-output (Turbo-MIMO) systems, the soft-output MIMO detector can provide the priori information to the turbo decoder. Unfortunately, if Rayleigh fading channels are applied, the induced unreliable priori information would cause the system performance degradation. In this paper, we proposed an iterative method to acquire the high reliability priori information from MIMO soft-detector in Turbo-MIMO systems. Similar to the conventional updating rules in the turbo decoding algorithm, we utilize the extrinsic information from the turbo decoder to update the log-likelihood ratios (LLRs) based on log-MAP algorithm in the list sphere decoding (LSD) algorithm. To reduce the overall computational complexity, different iteration profiles are also discussed. Simulation results show that the proposed Turbo-MIMO system can significantly improve the system performance compared to that of the conventional Turbo-MIMO system.**

I. INTRODUCTION

Multiple-input multiple-output technique has been widely used in recent wireless communication standards. Under the limited bandwidth, each transmitted antennas can transmit different signals to increase the transmission rate, which are called as spatial multiplexing mode. To recover transmitted signals in receiver, various MIMO detection algorithms have been presented in [1]-[4]. One is the linear detection methods which use zero-forcing (ZF) or minimum mean square error (MMSE) criterion to generate the channel matrix inverse and then multiply its matrix by received signal vector. Although linear detection methods have low computational complexity, the bit error rate (BER) performance is not sufficient, especially in low signal to noise ratio (SNR) environments. Another method is the maximum likelihood (ML) scheme [1]. An optimal solution is obtained by minimizing the Euclidean distance between the received signals and the possible signal candidates. Since the ML scheme is equivalently to the exhaustive search method, a large computational complexity is required in MIMO systems with high modulation constellations. To simplify the exponentially complex search problem in ML detector, the sphere detectors are presented in [2]-[4] to achieve near-ML performance with reasonable complexity. Since these detectors only find the most possible transmitted signals, these methods are called as hard-output MIMO detector. For enhancing system performance, the soft-output MIMO detectors are presented in [5]-[7] to approximate the maximum a posteriori probability (MAP) detection and provide soft outputs, which are called as list sphere detector (LSD) [5]. Compared to the hard-output detectors, the soft-output MIMO detectors not only obtain the most possible candidates, but also remain some transmitted signal candidates.

In recent years, a Turbo-MIMO system which combines soft-output detector and Turbo decoder is considered to improve system performance. The soft-output MIMO detector is like as inner decoding to provide soft-input decisions to the outer turbo decoder. Furthermore, the turbo-principle can be applied between the soft-output MIMO detector and the outer decoder performing iterative detection and decoding. Since the off-the-self turbo decoder only generates the soft information of symmetric bits, the amount of the updated soft information between soft-output MIMO detector and the turbo decoder are different. Hence, the soft-input decisions of the turbo decoder cannot be fully updated by iterative detection and decoding. In this paper, we proposed an iterative method to acquire the high reliability priori information from MIMO soft-output detector by the modified updating algorithm. To reduce the overall computational complexity, the different iteration profiles are also discussed. Compared to the conventional Turbo-MIMO system, our system performance can be significantly improved when the modified updating algorithm and selected iteration profile are applied. This rest of paper is organized as follows. In Section II, we present the system model of turbo-MIMO systems and some related soft information computations. Section III introduces the proposed soft information updating algorithm and discusses different iteration profiles. Section IV shows the simulation results. Finally, conclusions are shown in Section V.

II. TURBO-MIMO SYSTEM MODEL

We consider the turbo-MIMO system model which consists M transmit antennas and N receive antennas in Fig. 1, where $N \geq M$. At the transmitter, the K_u information bits are encoded by using convolutional or turbo encoder with rate R_c and then the codeword $\mathbf{c}=[c_1, c_2, \ldots, c_K]^T$ are generated, where $K=K_u \cdot R_c$. To combat error bursts, the interleaver operation, which is represented by Π, is applied to permute the coded bits. After interleaved operations, the permuting coded bits will pass to modulation to form symbol sequences. These symbol sequences are transmitted in parallel from the M transmit antennas and represented as $\mathbf{s}=[s_1,s_2, \ldots, s_M]^T$, where s_m is modulation symbol of $\mathbf{x}_k$ and $\mathbf{x}_k$ is a $M_c\times 1$ vector of data bits on kth symbols. In Fig.1, the channel impulse response from the ith receive antennas and jth transmit antennas are denoted as $h_{i,j}$ and the channel matrix $\mathbf{H}$ with dimension N-by-M are shown as:

$$\mathbf{H}=\begin{bmatrix} h_{1,1} & \cdots & h_{1,M} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,M} \end{bmatrix}. \quad (1)$$

Let the transmitting signal vector $\mathbf{s}$ pass to the MIMO channel, the received signal vector $\mathbf{y}=[y_1\ y_2 \ldots y_N]^T$ can be represented as follows.

$$\mathbf{y}=\mathbf{Hs}+\mathbf{n}, \quad (2)$$

where $\mathbf{n}=[n_1,\ n_2,\ \ldots,\ n_N]^T$ is an independently identical distributed additive white Gaussian noise (AWGN) with zero-mean. At the receiver, the soft-output MIMO detector is applied to find the candidate nodes which include some possible transmitted symbols. In addition, we select LSD as soft-output MIMO detector and take received signal vector and a priori knowledge $\mathbf{L}_{A1}$ on the inner coded bits to compute new intrinsic soft information $\mathbf{L}_{E1}$. Through the deinterleaver operation, which is denoted as Π^{-1}, the $\mathbf{L}_{E1}$ is changed to the *a priori* information $\mathbf{L}_{A2}$ into the turbo decoder. In the turbo decoder, we can use maximum *a posteriori* (MAP) [8] or Bahl-Cocke-Jelinek-Raviv (BCJR) [9] algorithm to produce extrinsic information $\mathbf{L}_{E2}$ on the outer code bits. Then, the $\mathbf{L}_{E2}$ pass through re-interleave operation and fed back as *a priori* knowledge $\mathbf{L}_{A1}$ to the soft-output MIMO detector. It is worth noting that the subscript "1" and "2" denote processing blocks that are connected to MIMO detector and turbo decoder, respectively. After completing information exchange between the subscript "1" and "2", it means that one iterative computation is performed. In the MIMO detector, the intrinsic soft information for the lth bit on the kth symbol are shown as

$$L_{D1}^{(k,l)} = L_{A1}^{(k,l)} + L_{E1}^{(k,l)}, \quad (3)$$

where the superscript (k,l) denotes this soft information for lth bit on kth symbol. The $L_{E1}^{(k,l)}$ can expressed by

$$L_{E1}^{(k,l)} = \ln \frac{\sum_{x\in X_{k,l,+1}} \left\{ \exp\left[-\frac{1}{2\sigma^2}\|\mathbf{y}-\mathbf{Hs}\|^2 + \frac{1}{2}\mathbf{x}_{[k,l]}^T \cdot \mathbf{L}_{A1,[k,l]} \right] \right\}}{\sum_{x\in X_{k,l,-1}} \left\{ \exp\left[-\frac{1}{2\sigma^2}\|\mathbf{y}-\mathbf{Hs}\|^2 + \frac{1}{2}\mathbf{x}_{[k,l]}^T \cdot \mathbf{L}_{A1,[k,l]} \right] \right\}}. \quad (4)$$

,where $\mathbf{x}_{[k,l]}$ denotes the information bit vector with omitting $x_{k,l}$ and $\mathbf{L}_{A1,[k,l]}$ is equal to $\mathbf{L}_{A1}$ with omitting the value of $L_{A1}^{(k,l)}$. $X_{k,l,+1}$ and $X_{k,l,-1}$ are the set of $2^{M\cdot M_c-1}$ bit vectors $\mathbf{x}$ which include the $x_{k,l}$=+1 and $x_{k,l}$=-1, repressively. The detailed derivations are shown in [4]. Through the Max-log

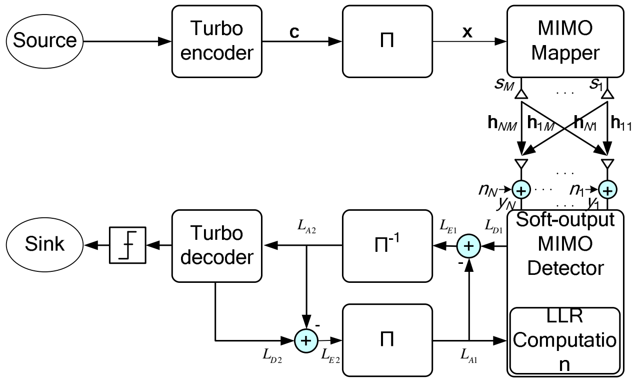

Fig. 1 Turbo-MIMO block diagram

approximation which proposed in [9], (4) can be represented as

$$\begin{aligned} L_{E1}^{(k,l)} \approx & \max_{x\in X_{k,l,+1}} \left\{ -\frac{1}{2\sigma^2}\|\mathbf{y}-\mathbf{Hs}\|^2 + \frac{1}{2}\mathbf{x}_{[k,l]}^T \cdot \mathbf{L}_{A1,[k,l]} \right\} \\ & - \max_{x\in X_{k,l,-1}} \left\{ -\frac{1}{2\sigma^2}\|\mathbf{y}-\mathbf{Hs}\|^2 + \frac{1}{2}\mathbf{x}_{[k,l]}^T \cdot \mathbf{L}_{A1,[k,l]} \right\}. \end{aligned} \quad (5)$$

Similar to turbo decoder, (4) and (5) are called as Max-log MAP and Log-MAP solutions, respectively.

III. PROPOSED ITERATIVE TURBO-MIMO SYSTEM

In this section, we consider an iterative system with the combination of soft-output MIMO detector and off-the-self turbo decoder. For the soft-MIMO detector, LSD is applied to find the candidate list. To avoid system degradation due to Max-log approximation, we use the log-MAP algorithm to compute the extrinsic information. In addition, we utilize the extrinsic information from turbo decoder to update the log-likelihood ratios (LLRs) based on log-MAP algorithm in LSD algorithm. Unfortunately, the conventional turbo decoder only calculates the extrinsic information of systematic bits. Since the amount of updated soft information between $\mathbf{L}_{D1}$ and $\mathbf{L}_{D2}$ are inconsistent, the unreliable priori information for parity-check bits would be occurred, especially for considering Rayleigh fading channels. Hence, the following subsection will describe how to introduce the extrinsic information of parity check bits and a modified updating rule in the proposed iterative turbo-MIMO system. Furthermore, the different iteration profiles are also discussed to reduce the computational complexity.

A. Introduce the log-MAP algorithm to compute parity-check information

In this section, we consider turbo decoding with serial concatenation scheme [10] which achieves the integration with two elementary decoders. Firstly, let $\mathbf{S}_k$ denote the set state transitions on time k, which are shown as $\mathbf{S}_k=\{(s,s')\}$, where s are s' denote the index of current state and next state, respectively. According to the value of systematic bit d_{sym}, the state transitions can be divided into $\mathbf{S}_k^{d_{sym}=0}$ and $\mathbf{S}_k^{d_{sym}=1}$. In addition, $\mathbf{S}_k^{d_{parity}=0}$ and $\mathbf{S}_k^{d_{parity}=1}$ are denoted as the two set of state transitions which correspond to the value of parity-check bit d_{parity}. Fig. 2(a) and 2(b) show the trellis diagram with generator polynomial $G(D)=1/(1+D^2)$ for systematic bit and parity-check bit, respectively. Hence, we can find $\mathbf{S}_k^{d_{sym}=0}$ ={(0,0), (1,2), (2,1), (3,3)}, $\mathbf{S}_k^{d_{sym}=1}$ ={(0,2), (1,0), (2,3), (3,1)}, $\mathbf{S}_k^{d_{parity}=0}$ ={(0,0), (1,0), (2,1), (3,1)}, and $\mathbf{S}_k^{d_{parity}=1}$ ={(0,2), (1,2), (2,3), (3,3)}. Similarly to the original MAP algorithm for symmetric bits, the intrinsic soft information L_{D2}^{parity} for parity-check bits based on $\mathbf{S}_k^{d_{parity}=0}$ and $\mathbf{S}_k^{d_{parity}=1}$ are shown as

$$LLR_p(d_k) = \ln \frac{\sum_{s'}\sum_{s}\gamma_1(x_k,s,s')\cdot\alpha_{k-1}(s)\cdot\beta_k(s')}{\sum_{s}\sum_{s'}\gamma_0(x_k,s,s')\cdot\alpha_{k-1}(s)\cdot\beta_k(s')}, \quad (6)$$

where the encoded bit d_k is belonged to parity-check bit d_{parity} and the forward recursion can be expressed as:

$$\alpha_k(s') = \frac{\sum_{s'}\sum_{i=0}^{1}\gamma_i(x_k,s,s')\cdot\alpha_{k-1}(s)}{\sum_{s'}\sum_{s}\sum_{i=0}^{1}\gamma_i(x_k,s,s')\cdot\alpha_{k-1}(s)}. \quad (7)$$

If q=0 then $\alpha_0(s')$=1. Otherwise, $\alpha_0(s')$=0. The backward recursion can be defined as:

$$\beta_k(s) = \frac{\sum_{s}\sum_{i=0}^{1}\gamma_i(x_{k+1},s,s')\cdot\beta_{k+1}(s')}{\sum_{s'}\sum_{s}\sum_{i=0}^{1}\gamma_i(x_{k+1},s,s')\cdot\alpha_k(s')}. \quad (8)$$

If p=0 then $\beta_N(s)$=1. Otherwise, $\beta_N(s)$=0. The branch transition probabilities are given by

$$\gamma_i\left(\left[x_k^{sym}, x_k^{Parity}\right], s, s'\right) = p\left(x_k^{Parity} \mid d_{parity}=i\right)\cdot p\left(x_k^{sym} \mid d_{parity}=i, s, s'\right)\cdot q\left(d_{parity}=i \mid s, s'\right)\cdot P_r\left(s' \mid s\right);\ i=0,1$$

where $q(d_{parity}=i\ |s,s')$=1 depending d_{parity} is associated with the state transition and equals 0 if it is not. $P_r(s'|s)$ is a fixed value dependent on *a priori* probabilities of d_{parity}. Table I shows the modified soft-information updating rule in LLR form. Firstly, we take the received signal vector and the corresponding *a priori* information $\mathbf{L}_{A1}$ which are initialized to zero to compute new intrinsic soft information $\mathbf{L}_{E1}$. Then, $\mathbf{L}_{E1}$ execute deinerleaver operation Π^{-1} and produce the *a priori* information $\mathbf{L}_{A2}$. In the turbo decoder, the MAP algorithm is applied to generate the intrinsic soft information $\mathbf{L}_{D2}^{sym}$ for symmetric bits by iterative operations. The notation $iter_{\text{turbo}}$ is defined as the maximum iteration number for turbo decoding, which is actually set to 8. Furthermore, according to (6), the intrinsic soft information $\mathbf{L}_{D2}^{parity}$ for each parity-check bits can be computed. Hence, we can use these intrinsic soft information and *a priori* information $\mathbf{L}_{A2}$ to compute the extrinsic information $\mathbf{L}_{E2}$. Then, the $\mathbf{L}_{E2}$ pass through re-interleave operation and fed back as *a priori* knowledge $\mathbf{L}_{A1}$. The above procedures are executed in an outer iteration and the maximum outer iteration is denoted as $iter_{\text{outer}}$.

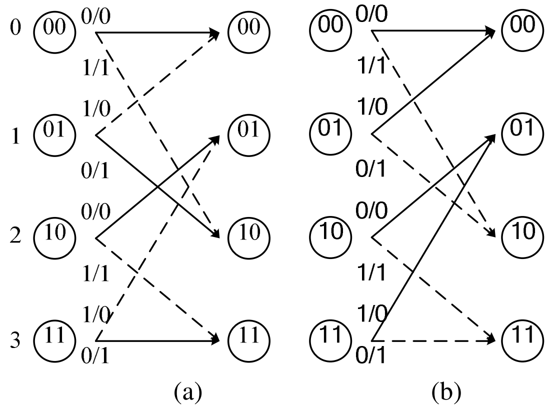

Fig. 2(a) Trellis diagram for considering symmetric bits (b) Trellis diagram for considering parity-check bits

TABLE I. MODIFIED SOFT-INFORMATION UPDATING RULE

Algorithm: Modified soft-information updating rule, LLR form
Input: The channel matrix **H** and the received signal vector **y**=[$y_1\ y_2 \ldots y_N$]^T
Outputs: the *a priori* information $\mathbf{L}_{A1}$
Initialize: $\mathbf{L}_{A1}$=0
Begin
for j=0 to $iter_{\text{outer}}$
$\mathbf{L}_{E1} \leftarrow \mathbf{L}_{D1}$-$\mathbf{L}_{A1}$, the computation is realized by (4)
$\mathbf{L}_{A2} \leftarrow \Pi^{-1}(\mathbf{L}_{E1})$
for i=0 to $iter_{\text{turbo}}$
$\mathbf{L}_{D2}^{sym} \leftarrow \mathbf{L}_{D2}^{sym}$
end (for i loop)
$\mathbf{L}_{D2}^{parity} \leftarrow LLRp$ for all parity-check bits by using (6)
$\mathbf{L}_{D2} \leftarrow [\ \mathbf{L}_{D2}^{sym}\ \ \mathbf{L}_{D2}^{parity}\ ]$
$\mathbf{L}_{E2} \leftarrow \mathbf{L}_{D2}$-$\mathbf{L}_{A2}$
$\mathbf{L}_{A1} \leftarrow \Pi^{-1}(\mathbf{L}_{E2})$
end (for j loop)
End

B. Discussion on different iteration profiles

In this subsection, we consider different the value of $iter_{\text{turbo}}$ and $iter_{\text{outer}}$. In [5], the Turbo-MIMO systems consider $iter_{\text{turbo}}$=8 and $iter_{\text{outer}}$=4 without computing LLR for parity-check bits, which are denoted as LLR_p. Hence, the number of turbo decoder computation is 45. Since (6) is similar to original LLR computation, we assume an LLR_p computation is equal to execute one turbo decoder computation. To increase the reliability of value of LLR_p, the value of $iter_{\text{outer}}$ would set to a high value to update LLR_p several times. In Table II, different iteration profiles are considered in the

TABLE II COMPUTATIONAL COMPLEXITY

Iteration numbers	W/ or W/O LLRp	Number of LLR computation at MIMO detector	Number of turbo decoder computation
$iter_{turbor}$=8, $iter_{outer}$=4	w/	5	45
	w/o	5	50
$iter_{turbor}$=6, $iter_{outer}$=5	w/	6	48
$iter_{turbor}$=4, $iter_{outer}$=6	w/	7	42
$iter_{turbor}$=5, $iter_{outer}$=6	w/	7	49
$iter_{turbor}$=4, $iter_{outer}$=7	w/	8	48

acceptable system performance. Since the turbo decoder computation is critical computing for soft information updating, we prefer choosing the lowest number of turbo decoder computation. Hence, the iteration profile is selected as $iter_{turbor}$=4 and $iter_{outer}$=6 under considering LLR_p computation in Turbo-MIMO systems.

IV. SIMULATION RESULTS

We consider a R_c=1/2 convolutional encoder with feedback generator polynomial $G_r(D)=1+D+D^2$ and feedforward generator polynomial $G_f(D)= 1+D^2$. The interleaver size of the turbo code is 9216 information bits. For 4×4 MIMO systems (M=4 and N=4), each transmitted symbol with carrying 4 information bits (M_c=4) is generated by 16QAM modulation. At the receiver, we applied the LSD algorithm with 512 candidate nodes to implement the soft-output MIMO detector. When the average signal energy per receive antenna is E_s, the signal energy per transmitted information bit at the receiver is $E_b=(N/R_cMM_c)\ E_s$. Hence, the modified SNR for Turbo-MIMO systems is given as

$$\left.\frac{E_b}{N_0}\right|_{dB} = \left.\frac{E_s}{N_0}\right|_{dB} + 10\log_{10}\left(1/2\right).$$

The simulation results shown in Fig. 3 are given on Rayleigh channel which generated by complex Gaussian distribution. Since max-log MAP algorithm uses approximated LLRs, its BER performance is worse than that of using log MAP algorithm. Whether the LLRs are computed by max-log MAP algorithm or log MAP algorithm, the performance gain with respect to the proposed soft information updating rule is 0.2~0.3 dB at BER $=10^{-3}$. According to Table II, we also consider different iteration profiles in Fig. 3. Compared to conventional iteration profile which is proposed in [5], the iteration profile, which is selected as $iter_{turbo}$=4 and $iter_{outer}$=6, has better performance. Since the LLRs for parity-check bits can be updated several times as increasing the number of $iter_{outer}$, the intrinsic information is more reliable.

V. CONCLUSIONS

In this paper, an iterative method to acquire the high reliability priori information from MIMO soft-detector by the modified updating algorithm is presented. Through applying the LLRs of the parity check bits and the new combination of

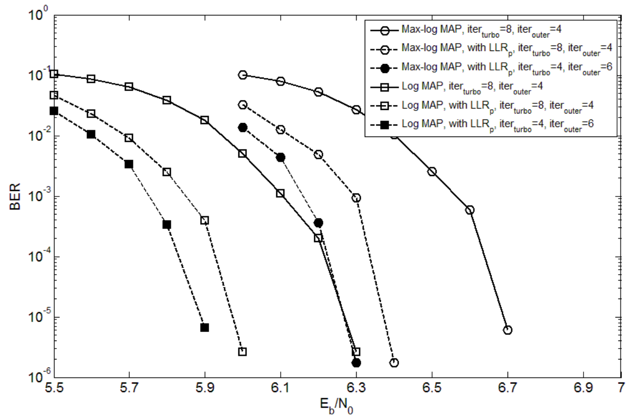

Fig. 3 Simulation results for Turbo-MIMO systems with LLR of parity check information under different iteration numbers.

the iteration numbers into the Turbo-MIMO systems, we can obtain about 0.4 dB gain than that of the original Turbo-MIMO systems. Additionally, with the LLRs of the parity check bits, the performance improvement obtained by increasing the number of outer iteration processes is more pronounced than that by increasing the number of iteration processes of the turbo decoder.

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