Spiral waves in large aperture laser model

Krents A.A., Anchikov D.A., Molevich N.E., Pahomov A.V.
Department of Physics / Theoretical sector
Samara State Aerospace University / Samara branch of Physical Institute of RAS
Samara, Russia

***Abstract* — In this paper spontaneous formation of spiral waves in large aperture laser model has been reported. As a mathematical model the system of Maxwell-Bloch equations has been used. Linear stability and Floquet analyses were done. This analysis allows us to plot a bifurcation diagram and predict a characteristic size of exited spiral waves. Numerical simulations are in good according with theoretical analysis.**

Keywords* — *spiral waves, Floquet exponents, wide-aperture lasers

I. Introduction

It is well known that transversely extended laser systems may appear rich spatiotemporal dynamics of optical field. In this paper the formation of spiral waves in large aperture lasers has been reported. In optical systems spiral waves appear in the phase structure of the optical field. Spiral waves in lasers may evolve from two mechanisms: through hard [1] and spontaneous excitation when the system becomes unstable on varying control parameters. In this paper, we study the spiral wave formation through the spontaneous excitation in the large aperture laser model.

II. Teoretical model and analisys

The transversely extended two-level laser system used for investigation is described by the Maxwell–Bloch equations, given as [2]:

$$
\begin{aligned}
&dE/dt=\sigma(P-E)+ia\Delta E,\\
&dP/dt=-(1+i\delta)P+DE, \qquad (1)\\
&dD/dt=-\gamma[D-r+0.5(PE^*+EP^*)],
\end{aligned}
$$

where ΔE is transverse two-dimensional Laplace operator, E,P are dimensionless electric field and polarization amplitudes and D-dimensionless population inversion. σ, γ are dimensionless electric field decay rate and inversion decay rate respectively. System (1) has steady state solution. Linear stability analysis of this solution was done. This solution undergoes a Hopf or a Wave instability depending on value of the parameters r, δ, γ and σ. It is well known that the Hopf instability excites the homogeneous oscillations. The wave instability excites the travelling or the standing waves. More complicated structures can be formed as a result of the competition of two unstable modes. Linear stability analysis is no more effective in this case because Hopf and wave modes may interact now.

Floquet exponents is an appropriate mathematical tool which allow us to make the stability analysis of homogeneous oscillations excited by Hopf instability [3]. It shows how a limit cycle will behave when perturbation with a finite wave number is added. The stability criterion is then obtained from a linear eigenvalue problem of the Monodromy matrix. Homogeneous oscillations are stable if all exponents are negative for all wave numbers. No spatial patterns are exited. By varying control parameters the exponent for some wave number k becomes positive. Then the system suffers a bifurcation: the homogeneous state is destabilized by a standing wave with wave number k. This analysis allows us to plot a bifurcation diagram and predict a characteristic size of exited spatiotemporal structures.

III. Numerical simulation

2D Numerical simulation of model (1) with the initial small random noise near the steady state have been performed using a pseudo-spectral method and periodic boundary conditions. Parameters δ=-1, γ=0.1 and σ=2 are fixed, only parameter r may vary. This numerical solutions have been compared with the theoretical predictions of the Floquet analysis. Below the instability threshold, the homogeneous steady state solution remains stable. Near the Hopf instability (r=25), the homogeneous oscillations are stabilize. By increasing control parameter (r=30) the Floquet exponent with approximately k=8 becomes positive. Numerical simulation shows the spiral waves formation (Fig. 1a). Characteristic size of spiral wave is in good agreement with theoretical predictions (Fig. 1b).

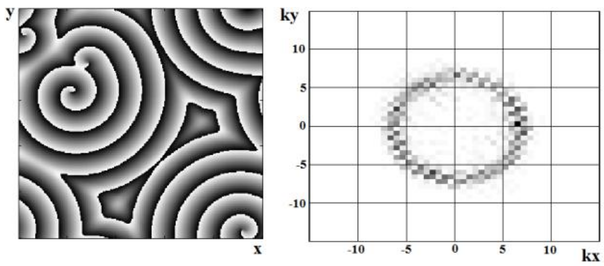

Fig. 1. Phase of the electric field and far field image, here x,y - are transverse coordinates and kx, ky are appropriate wavenumbers.

References

[1] *Dejin Yu, Weiping Lu, Harrison L.G.* // J. Opt. B: Quantum Semiclass. Opt. – 1999. – Vol. 1. – P.25-30.

[2] *Jacobsen P.K., Moloney J.V., Newell A.C., Indik R.* // Physical Review A. – 1992. – Vol. 45. - P. 2076-2086.

[3] *Pena B., Bestehorn M.* // Eur. Phys. J. Special Topics. – 2007. – Vol. 146. – P.301-311.

This work was financially supported by The Ministry of education and science of Russia, under project 608 and by RFBR under grants 13-01-97001, 14-02-97030, 14-02-31419.