The Bivariable Fractal Interpolation Algorithm of Simulating the Mountains in the Distributed Navigation Simulation System

Dan Liu, Xiangjian He, and Yicheng Jin

Abstract--**3D scene system is an important part of computer imitation system, and its fidelity determines if an imitation system stands or falls. At present, such algorithms have been perfected, but a good algorithm is always so complicated that can not achieve the demands of calculation during the time required. So it is necessary to find a fast algorithm applied in the real time system. Fractal Geometry is a powerful tool to describe the complicated and anomalistic geometrical objects. A method of bivariable fractal interpolation combined with objects polyhedral technique to construct polyhedrons and vertex data of mountains in distributed simulation system is proposed. The implementing method and steps are given as well.**

Index Terms—**Bivariable fractal function, Bivariable fractal interpolation, Distributed navigation simulation system, Mountains simulating.**

I. Introduction

Computer simulation technique is a multi-subject synthesis technique based on system techniques. information techniques and related speciality techniques in the application fields. It uses computers and various physical domino affecting facilities and dynamically studies actual systems or systems in design by system models. Moreover, distributed navigation simulation system is a large distributed interactive computer simulation system combining virtual reality techniques, internet techniques, multimedia techniques and navigation techniques, which has been applied broadly in the fields such as system analysis, scientific research and engineering design. As an important part of the system, 3D scene system with accompanying sound system and interactive equipment makes a virtual reality environment with an experiential feeling for the users [1].

Since ships are always navigating, the mountains on the seacoast comprise the main contents of the navigation simulation system. Natural objects such as mountains are very anomalistic. These objects can only be approximated by polyhedrons in computer graphics, which is an effective figuration method of the geometrical objects with certain lubricous degree and leads to more errors in describing natural objects. Mountains within a certain range can be seen as a compact set of R^3 Space and fractal interpolation method is an effective way to construct Iterated Function System (IFS) using a group of existing data, we will discuss how to collect original data and use these data to construct IFS in detail in this paper: Fractal geometry obtains the attractor through infinite alternation, which can not be used by a real time system. Consequently some useful results of bivariable functions have been obtained by studying the bivariable fractal space and how to use bivariable fractal method to produce the basic data in the polyhedron presentation method. This is followed by a discussion of how to take full advantage of the validity of the fractal method and efficiency of the polyhedron presentation method are also be discussed.

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II. The Collection and Pretreatment of Original Data

A. The Collection of Original Data

The original data here is defined as a discrete point set obtained from data source. At present, there are three approaches to obtain the original data: using digital instruments to collect needed information from the chart papers; using a mouse to pick up information from raster charts; or using special software to pick up data automatically under operator's commands. From the contour lines of a mountain we can figure the detailed shape of the mountain. It is reasonable and feasible to get the characteristic information of reconstructing the mountain's shape. The method is to collect the 3D coordinates of a series points on the contour lines dispersedly. In fact, the planar coordinates can be obtained easily by tracking the location of digital collector or cursor. In order to gain the third coordinate, which is the height information, it is required to input the altitude value of current contour line as well when collecting the data. In general, to ensure the obtained information can express the mountain information effectively, it is requested that the sampling frequency is higher when the curvature of the contour line is bigger, and is lower when the curvature is smaller in order to reduce the number of data.

B. The Preprocessing of the Messy Data

The collected data can be used as interpolating data only after a series of pretreatments. The pretreatments include the coordinate transformation between different coordinates and the sort of the coordinate values. It can be seen as follows that both treatments are in the interest of satisfying the fractal interpolation methods with the interpolating data. In the following, we will take the example of collection by digital instrument to introduce the special process of data preprocessing.

First of all, we will introduce three coordinate systems used here; they are facility coordinate system, geographical coordinate system and world coordinate system. Facility coordinate system is the coordinate system of digital instrument itself. The origin is the lower-left corner of the digital instrument; right is the positive direction of X-Axis and up is the positive direction of Y-Axis, and the coordinate unit is pixel. Secondly, in order to simplify the problem, we introduce a third coordinate, measured in meters to express the height information of the point; Geographical coordinate system uses latitude and longitude to express the coordinate value of a point. We also introduce a third coordinate; World coordinate system is the left hand coordinate system used in the 3D scene system, the origin is the low-left corner of the chart, and the coordinate unit is meter. Since there is no need to change the height information in the course of transformation, we do not consider transforming the height value. In this case, we simplify the transformation between three-dimensional coordinate systems to the transformation between two-dimensional coordinate systems. The three plane coordinate system are X-Y plane of facility coordinate system, geographical coordinate plane of geographical coordinate system and X-Z plane of world coordinate system, and the height value (Z value) is transformed to the Y value in the world coordinate system directly.

After the coordinate of each point is transformed into world coordinate system, it will be sorted in X-Z plane. The sort is going along two directions; one is sorting every point ascendingly according to its X value along X direction. The points with same value can only be used once. In this way, there is a partition on the X-Axis composed of X value of each point: $\{[x_0, x_1],[x_1,x_2],\ldots,[x_{M-1},x_M]\}$, M+1 is the number of points with different coordinate value, $X_{min}= x_0$, $X_{max}= x_M$. It can be imagined that this partition is nonsymmetrical. In the same way, we can gain a partition along Z-Axis: $\{[z_0, z_1],[z_1,z_2],\ldots,[z_{N-1},z_N]\}$ by sorting interval $[Z_{min},Z_{max}]$, where, N has the same meaning with M.

C. Computing Output Data

When IFS is gained, we can iterate an arbitrary initial set to obtain the attractor of the IFS, which is the final output result. In order to obtain the graph with smaller error, it usually needs additional iteration. Such iteration process sometimes takes time, especially when there are more mappings in the IFS. Moreover, in a real time simulation application, in order to gain better visual effect, the updating rate should be greater than 20 frames per second. Thereby, it is infeasible to gain a frame image using several seconds or more by iteration. Current graphics softwares can support the protraction of polyhedrons better and the efficiency is higher. That is the reason that most applications use polyhedrons to denote geometrical object. So in order to combine fractal interpolation method with practical application, it is necessary to combine the method with traditional polyhedron protraction method. This method can ensure not only higher precision of the graphics obtained but also abundant details. We do it as follows: firstly, use IFS to compute offline, after a few iterations, abandon the middle result, and let the last data be the output result. Secondly, construct polyhedrons grid using this output discrete data by messy data interpolation method. In the course of the iteration, in order to make the iteration convergent to a given precision, usually use the point in the attractor as the original iteration point. In this case, we can get a satisfying result after 3 to 5 iterations. The method of constructing polyhedrons grid can select Delaunay triangular grid automatic generation algorithm in common use.

III. THE BIVARIABLE FRACTAL INTERPOLATION FUNCTION

Let X be the complete metric space and let $W_i : X \to X$, $i=1,2,\ldots,n$ be continuous mappings. Then the collection $\{X; W_i : i=1,2,\ldots,n\}$ is called an iterated function system, abbreviated IFS. A set $A \subseteq X$ is said to be an attractor of the IFS if A is nonempty, compact and with $A=\bigcup_{i=1}^{n} W_i(A)$. We now introduce some notations which we will put to use throughout the rest of this paper[2].

Let $\lambda \in (-1,1)$ be a constant, $m,n \geq 12$ be integers. Denote $M=\{1,\ldots,m\}$ and $N=\{1,\ldots,n\}$. Let $-\infty \leq x_0 < x_1 < \cdots < x_m < \infty$ and $-\infty < y_0 < y_1 < \cdots < y_n < \infty$. Denote $D=[x_0,x_m] \times [y_0,y_n]$ and $D_{i,j}=[x_{i-1},x_i]\times[y_{j-1},y_j]$ for all $i\in M$ and $j\in N$. Let C(D) denote the collection of real-valued continuous functions defined on D. The partition of D given by $D=\bigcup_{i,j} D_{i,j}$ will be denoted by Δ. For $i \in M$ and $j\in N$ define maps $A_i : [x_0,x_m] \to [x_{i-1},x_i]$ and $B_j : [y_0,y_n] \to [y_{j-1},y_j]$ by

$$A_i(x)=\frac{x_i - x_{i-1}}{x_m - x_0}(x-x_0)+x_{i-1}, x\in[x_0,x_m]$$

and

$$B_j(y)=\frac{y_j - y_{j-1}}{y_n - y_0}(y-y_0)+y_{j-1}, y\in[y_0,y_n]$$

respectively.

Definition 1. Let $W_{i,j} : D\times R \to D\times R, (i,j)\in M\times N$, be continuous mappings. The IFS $\{ D\times R; W_{i,j} : (i,j)\in M\times N \}$ is said to be iterated generating system, called IGS[3] for short, if there is a function $f \in C(D)$ such that $\{(x, y, f(x, y)):(x, y)\in D\}$, the graph of f, is the unique attractor of the IFS, in which case we also write.

IV. THE SPACE OF BIVARIABLE FRACTAL INTERPOLATION FUNCTIONS

Theorem 1 [4] .Let $\phi_{i,j} \in C(D),(i,j) \in M \times N$ be Lipschitz. Define $W_{i,j}: D \times R \to D \times R$ by

$$W_{i,j}(x,y,z) = (A_i(x), B_j(y), \lambda z + \phi_{i,j}(x,y))$$

For (x, y, z) $\in D \times R$ and (i, j) $\in M \times N$. then $\{D \times R; W_{i,j}: (i,j) \in M \times N\}$ is an IGS if and only if there exist $u \in C[x_0, x_m]$ and $v \in C[y_0, y_n]$ such that the following equations hold:

$$\lambda u(x) = \phi_{i,j}(x, y_n) - \phi_{i,j+1}(x, y_0)$$

$$x \in [x_0, x_m], i \in M, j = 1, \cdots, n-1$$

$$\lambda v(x) = \phi_{i,j}(x_m, y) - \phi_{i+1,j}(x_0, y)$$

$$y \in [y_0, y_n], j \in N, j = 1, \cdots, m-1$$

$$u(A_i(x)) = \lambda u(x) + \phi_{i,1}(x, y_0) - \phi_{i,n}(x, y_n)$$

$$x \in [x_0, x_m], i \in M$$

$$v(B_j(y)) = \lambda v(y) + \phi_{1,j}(x_0, y) - \phi_{m,j}(x_m, y)$$

$$y \in [y_0, y_n], j \in N$$

Definition 2 Let H be a linear subspace of C (D) then the set $F(\Delta, \lambda, H) =$

$\{ f \in C(D)$: there exist $\phi_{i,j} \in H, (i,j) \in M \times N$, such that

$\{ D \times R; W_{i,j}: (i,j) \in M \times N\} \Leftrightarrow f$, where

$W_{i,j}: D \times R \to D \times R$ is defined by

$W_{i,j}$(x, y, z)=($A_i(x), B_j(y), \lambda z + \phi_{i,j}(x,y,z))\}$

is called the set of fractal functions with respect to the partition Δ ,the contractive factor λ and the bottom space *H*.

Theorem 2.Denote $K=\{\phi \in C(D)$: there exist a, b, c and d in R such that ϕ (x, y)= axy+ bx + cy + d for all (x, y) $\in D\}$

Then $F(\Delta, \lambda, K)$ is a linear subspace of C (D). Furthermore, we have

$$\dim F(\Delta, \lambda, K) = \begin{cases} mn+3 & if \lambda \neq 0 \\ (m+1)(n+1) & if \lambda = 0 \end{cases}$$

Let us denote

$$I(\Delta) = \{(i,j): i = 1, \cdots, m-1; j = 1, \cdots, n-1\}$$

Called the interior indices, and

$$B(\Delta) = \{(i,j): i = 0, m; j = 0, 1, \cdots, n\} \cup \{(i,j):$$
$$i = 0, 1, \cdots, m; j = 0, n\}$$

Called the boundary indices.

V. CONCLUSION

Theorem 3. Suppose $\lambda \neq 0$, Let J be a subset of {(i,j): $i = 0, 1, \cdots, m$; $j = 0, 1, \cdots, n$ }.Denote $J_I = J \cup I(\Delta)$ and $J_B = J \cup B(\Delta)$ .Let $\{ Z_{i,j}: (i,j) \in J \}$ be a set of real numbers. Then we have (1). There exists an $f \in F(\Delta, \lambda, K)$ such that

$$f(x_i, y_i) = z_{i,j}, (i,j) \in J \quad (1)$$

If and only if the following equations

$$(x_m - x_{i-1})(z_{0,0} - z_{0,n}) - (x_m - x_0)(z_{i-1,0} - z_{i-1,n}) + (x_{i-1} - x_0)(z_{m,0} - z_{m,n}) = 0, i = 2, \cdots, m \quad (2)$$

$$(y_n - y_{j-1})(z_{0,0} - z_{m,0}) - (y_n - y_0)(z_{0,j-1} - z_{m,j-1}) + (y_{j-1} - y_0)(z_{0,n} - z_{m,n}) = 0 \quad (3)$$

$$j = 2, \cdots, n$$

In the unknowns $z_{i,j}(i,j) \in B(\Delta) \setminus J_B$,are linearly solvable.

There exists one and only $f \in F(\Delta, \lambda, K)$ such that (1) holds if and only if $J_I = I(\Delta)$ and the equation (2) and (3) in the unknowns $z_{i,j}(i,j) \in B(\Delta) \setminus J_B$ (see [2]), are linearly solvable.

Corollary. Suppose $\lambda \neq 0$ and let

$$J = \{(i,j): i = 0, 1, \cdots, m-1;$$
$$j = 0, 1, \cdots, n-1\} \cup \{(0,n)(m,0),(m,n)\}$$

Then for any set of mn+3 real numbers $\{ z_{i,j}: (i,j) \in J \}$, there exists a unique $f \in F(\Delta, \lambda, K)$ such that (1)holds. Furthermore, for each (p, q) $\in J$, let $g_{p,q} \in F(\Delta, \lambda, K)$ be the unique function with $g_{p,q}(x_i, y_j) = \begin{cases} 1 & if (i,j) = (p,q) \\ 0 & if (i,j) \in J \setminus \{(p,q)\} \end{cases}$

Then $\{ g_{i,j}: (i,j) \in J \}$ forms a basis of the linear space $F(\Delta, \lambda, K)$.

According to Theorem 3 and the Corollary, we can outline mountain in the screen plane of computer. Figure 1 is the experimental result of this method.

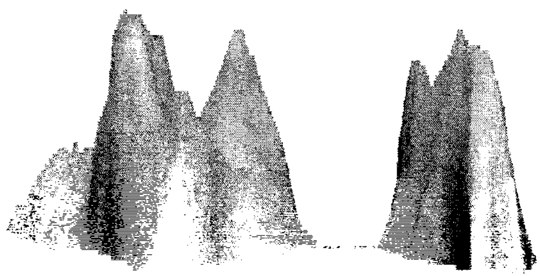

Fig. 1. The 3D model of three hill island , Dalian, China

In fact, common ship often navigates along the recommended sea-route when it navigates along the seashore. The outline of a mountain can be seen to be composed of the points of intersection between the normal of the center of the sea-route and the contour line with the most height and the points with the nearest distance to the sea-route center when the measurer observes the mountain in the positive horizontal direction. Therefore, the means to pick up the outline points of the mountain is: firstly, separate the center line of the sea-route with equal intervals; secondly, fine the intervals. The unequal interval partition can be used according to such factors as the ridge lines, the changes of height value and the curvature of the sea-route center line. Rearrange the sampling points in a certain order, and then put them into the computer. The following figure is the mountain outline of Huangbaizui, Dalian.

Fig. 2. The mountain outline of Huangbaizui, Dalian, China

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