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About Some Important Parameters in Dynamic Buckling Analysis of Plated Structures Subjected to Pulse Loading

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In the paper the influence of following factors: initial imperfections, shape and duration of pulse loading on the dynamic response of plate structures is presented. The effect of material properties in the plastic range and the estimation of structure capacity to sustain dynamic pulse loadings based on different dynamic stability criteria is discussed as well.

Keywords: Thin–walled plate structures, dynamic buckling, pulse load

1. Introduction

The problem of dynamic buckling of thin walled structures such as shells and plates subjected to in–plane pulse loading has been widely investigated starting from sixties of previous century [see e.g. works of Simitses [14, 15] and Gryboś [2]]. These pulse loads may be of various durations and shapes (rectangular, sinusoidal, triangular, trapezoidal, etc.) being approximations of real load courses. Depending on the so–called "pulse intensity" different phenomena may occur – impact for pulses of high amplitudes and durations in range of microseconds or quasi–static behavior if amplitude is low and duration is twice of period of fundamental natural vibrations. For pulses of intermediate intensity (amplitudes in range of static buckling load and durations close to period of fundamental natural vibrations) the phenomenon of dynamic buckling occurs. It is known that at pulse loads of short duration (in range of milliseconds) the dynamic structure carrying capacity is larger than static one. However it should be remembered that for plate structure, in contrary to the static behavior, the bifurcation dynamic load does not exist. The phenomenon of dynamic buckling takes place only for initially imperfect structures. Initial imperfections' magnitude in connection with pulse shape and its duration are crucial parameters in dynamic buckling load estimation.

Dynamic buckling load is usually determined on the basis of dynamic buckling criterion that oneself seems to be problematic. Commonly used Budiansky–Roth–Hutchinson criterion [3] was formulated for structures having limit point or unstable postbifurcation path. Its application to plate structure behavior, with stable post-buckling path, is based rather on intuition. Therefore in subject literature one can find a lot of stability criteria – most of them are based on state of displacements or state of stress. Some of them formulate conditions for dynamic buckling load estimation (e.g.Volmir [19], Budiansky–Hutchinson [3], Ari–Gur and Simonetta [1]), the others aim to determine the dynamic failure load (e.g. Petry and Fahlbusch [14], Ari–Gur and Simonetta, Weller et al. [18]).

Some degree of uncertainty of all mentioned criteria brought the researches of new dynamic stability criteria basing on Jacobian matrix eigenvalues analysis (see Kubiak [10]) or applying phase portraits criterion (see Teter [17]). It should be noted that both mentioned works deal with dynamic interactive buckling of plated structures with open cross-sections.

In most publications the considerations are conducted in the elastic range. Usually the unlimited elasticity is assumed. However it is well known that in static buckling of plate structures with low ratio width to thickness and larger imperfection amplitude some regions become plastic at loadings close to static buckling load (see Kołakowski, Kowal–Michalska [5]). Therefore, to determine static failure load, it is necessary to take into account the material characteristic. When a load is applied dynamically the material properties change (e.g. the value yield limit can be twice static one). In investigations concerning dynamic loading the material stress-strain curve obtained in static tests has been usually assumed [6], [7], [8], [9], [14]. Recently, in works of Mania [11], [12] the effect of strain rate on material characteristic has been accounted for in dynamic response of short columns of closed cross–sections.

The subject of this paper is the analysis of the influence of all mentioned earlier factors (shape and duration of pulse load, initial geometric imperfection, applied dynamic stability criteria, elasto–plastic material properties) on the dynamic response of rectangular plates and thin–walled columns of closed cross–sections.

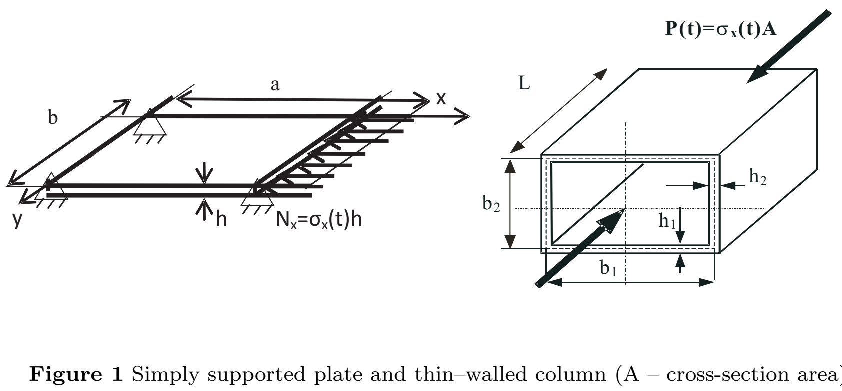

Figure 1 Simply supported plate and thin–walled column (A – cross-section area)

2. Considered structures and loading

The results of numerical calculations, being the subject under discussion, were received for isotropic plates, simply supported at all edges and for isotropic columns built of rectangular plates (Fig. 1). It was assumed that all plate edges and loaded edges of a column remain straight and mutually parallel during loading. Considered structures subject to pulse in–plane compressive load of a shape shown in Fig. 2 and described by the relation:

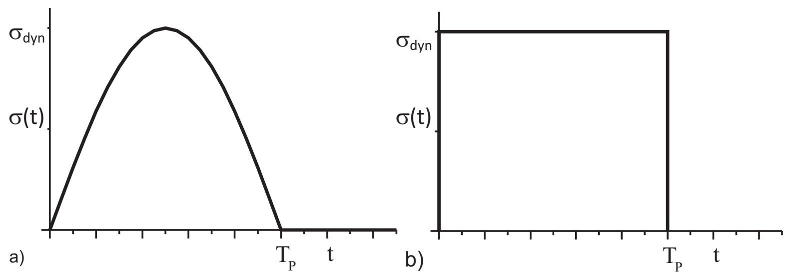

Figure 2 Exemplary shapes of pulse loading a) sinusoidal b) rectangular

$$\begin{aligned}
&\text{for} \quad 0 \le t \le T_p \quad P(t) = P_0 \sin\left(\frac{\pi t}{T_p}\right) \ \text{(Fig. 2a)} \qquad \sigma(t) = \sigma_{dyn} \ \text{(Fig. 2b)} \\
&\text{for} \quad t > T_p \quad P(t) = 0 \ \text{(Fig. 2a)} \qquad \sigma(t) = 0 \ \text{(Fig. 2b)}
\end{aligned} \tag{1}$$

where T_p - pulse duration, equal to 1 or 1/2 of T_0 (period of fundamental natural flexural vibrations).

The solutions to the problem of dynamic buckling were obtained numerically on the basis of ANSYS software, plate structures were meshed with four nodes isoparametric nonlinear shell elements. The detailed procedure was described in earlier publications (e.g. [6], [12]). In all considered cases the form of initial deflection was assumed as identical to the lowest static buckling mode.

The results of calculations have been presented as the relations between maximal dimensionless dynamic defection of a structure and ratio of a pulse amplitude versus static buckling load determined for perfect structure (i.e. bifurcation load). In literature, following Budiansky and Hutchinson, the quotient of pulse amplitude and static bucking load is termed as Dynamic Load Factor (DLF= σ_{dyn}/σ_{cr}).

It should be mentioned that for elastic isotropic materials these relations do not depend on the material type whereas the strong effect of geometric parameters, initial deflections, boundary conditions and pulse loading parameters can be observed.

3. Factors affecting dynamic response of plate structure

3.1. *Initial imperfections*

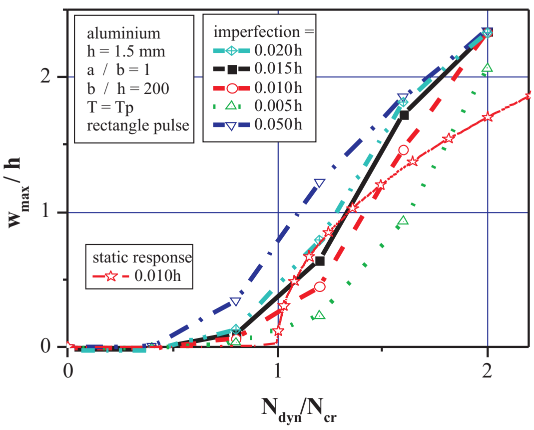

Figure 3 Influence of initial deflection amplitude on plate dynamic response

In Fig. 3 the courses of plate dimensionless maximal deflection (the quotient of maximal deflection to plate thickness) as a function of dimensionless dynamic load have been shown. The static response has been presented as well. The values of initial deflection amplitude varied in range of 0.005 h to 0.05h. It can be easily seen that for very small imperfections the dynamic buckling load determined on the basis of Budiansky–Hutchinson criterion is greater than static one (N_{cr}) and in some range of loads N_{dyn}/N_{cr} the dynamic deflections are smaller than static ones. For larger values of imperfections the character of curves changes (it becomes similar to static course) and dynamic buckling load is less than static one for perfect plate.

Therefore the question arises – should the buckling load of imperfect structure be determined relative to the static bifurcation load as it is commonly assumed? Perhaps the information of dynamic load carrying capacity would be more evident with regard to static buckling load determined in similar way for imperfect structure.

3.2. *Pulse shape and duration*

In Figs 4 and 5 the results obtained for pulses of rectangular and sinusoidal shape, of equal duration T_p, are shown. The pulses of equal area under the curve describing the time dependence of load – equal impulses (Fig. 4 and 5) and pulses of equal amplitude (Fig. 5) were considered.

For square plate simply supported along all edges of rather large imperfection (w_{0max}=0.1 h) the strong influence of pulse shape and duration T_p on character and values of dynamic deflections is visible.

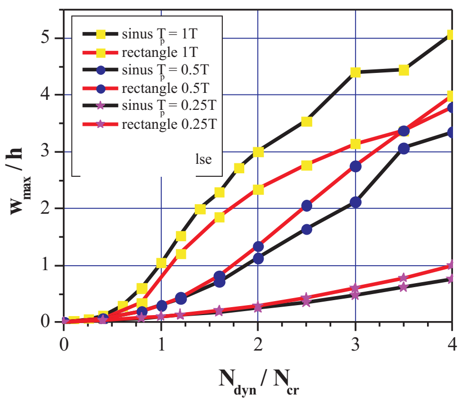

Figure 4 Influence of pulse shape and duration on plate dynamic response (geometric parameters the same as in Fig.3)

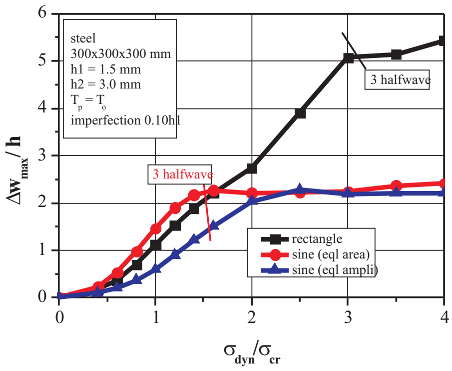

Figure 5 Influence of pulse shape on column dynamic response (cubic column of different wall thickness)

For pulses of duration equal to the T_0 (period of fundament flexural vibrations) the dynamic buckling load (determined on the basis of Budiansky–Hutchinson criterion) is smaller than static critical load and sinusoidal pulse involves larger deflections than rectangular one. For $T_p = 1/2T_0$ the situation changes – the dynamic buckling load grows rapidly and in the same time the deflections become larger for rectangular pulse. It can be noted that the character of the curves for $T_p = 1/2T_0$ becomes similar to the character of courses for a plate with small initial imperfections at $T_p = T_0$ (see Fig. 3). For very short pulses, in considered range of loading, the dynamic deflections are so small that it is impossible to apply any dynamic buckling criterion (the phenomenon of buckling does not occur).

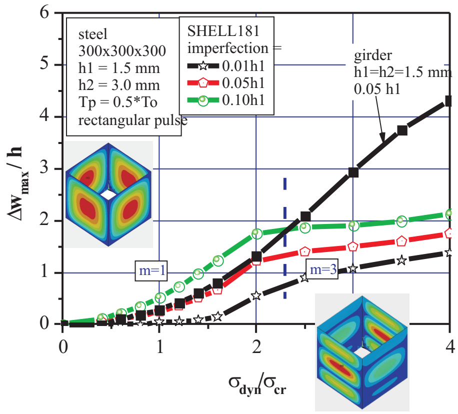

Figure 6 Influence of imperfections and different wall thickness on column dynamic response

For a column of cubic outline but of different wall thickness (Figs 5 and 6) the dynamic deflections versus dynamic load amplitude are presented. Three types of pulse load: rectangular and two sinusoidal (one of the same amplitude value as rectangular one and the second of the same area) were considered (Fig. 5). The duration was kept constant $T_p = T_0$ (Fig. 5) or $T_p = 1/2T_0$ (Fig.6) whereas different values of initial deflection amplitude were taken into account. In all cases for some value of dynamic load the rapid change of buckling mode (from one to three half–waves) occurs. For comparison in Fig. 6 the curve for cubic column of constant wall thickness has been drawn – in whole range of loading the buckling mode is the same and the behavior of a column is similar (but not the same – see ref. [8] for details) to the behavior of a square plate simply supported along all edges. Therefore the change of buckling modes occurs in case corresponding to the plate of unloaded edges clamped. It can be noticed that for longer rectangular pulses ($T_p = T_0$ – Fig. 5) this phenomenon appears at higher dynamic amplitude than for shorter ones ($T_p = 1/2T_0$ – Fig. 6). Comparing the curves of dynamic deflections for columns of the same imperfection amplitude (0.1h) loaded by rectangular pulse of different duration (see Figs 5 and 6) the same conclusions as mentioned earlier can be drawn – the pulse duration influences the character of courses $w_{max}/h = f(N_{dyn}/N_{cr})$ and the dynamic buckling load is greater for shorter pulses (estimated on the basis of Budiansky-Hutchinson criterion critical value of σ_{dyn} stays in range 0.8-1.0σ_{cr} – for $T_p = T_0$ and 1.2-1.6σ_{cr} for $T_p = 1/2T_0$).

During calculations it was observed that pulse duration affects also the time in which the maximal deflections appear (Fig. 7) [6], [8]. For shorter pulses almost independently on initial imperfections magnitude and on pulse amplitude value the maximum deflection appeared after the load was released. For pulses of T_p=T_0 the maximal deflection took place within pulse duration except limited range of dynamic loads in cases of small imperfection amplitude.

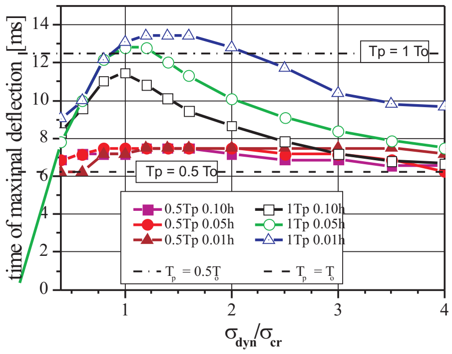

Figure 7 Influence of pulse duration on time of maximal deflection appearance (cubic column of equal wall thickness, rectangular pulse)

3.3. *Material characteristics*

In most publications concerning the problem of dynamic buckling of plates the unlimited material elasticity has been assumed. It is known that in order to determine satisfactorily static ultimate load of thin–walled plated structure it is necessary to take into account the postbuckling state in the elasto–plastic range together with initial imperfections. The strength reserve of the statically compressed plate element in the post–buckling state strongly depends on the width to thickness ratio and on the material properties (magnitude of yield limit, shape of characteristic in plastic range). This reserve may disappear when the width to thickness ratio is relatively small and then the unfavorable effect of initial imperfections can bring such a decrease of the load carrying capacity that it becomes significantly lower than static buckling load for perfect structure.

On the other hand it is known that material properties change at loadings applied dynamically (see: e.g. [4]). The fact that mild steel is strain rate sensitive is well known and widely documented in literature. It is reported that the yield limit value of a mild steel under dynamic loading increases and the hardening part of strain–stress curve lies over static characteristic.

Consideration of a column made of steel with rather low initial yield limit σ_y=100MPa allows to investigate the problem of dynamic buckling in the elastic–plastic range even for low pulse amplitudes. Exemplary results of theses analyses are presented in Fig. 8 (when static material characteristic was taken into account) and in Fig. 9 (the effect of strain rate was included). The effect of strain rate on the dynamic buckling of short columns of closed cross-sections has been widely described by Mania [11], [12].

The curves presented in Fig. 8 showing the relations between nondimensional dynamic deflection and effective stress ratio ($S_{ratio} = \sigma_{eff}/\sigma_Y$ –where σ_{eff} –effective stress calculated accordingly to Huber-Mises formula, σ_Y – yield stress) were obtained under assumption that static material characteristic is bilinear. It can be seen that for $\sigma_{dyn}/\sigma_{cr} > 2$ deflections grow almost linearly. For $\sigma_{dyn}/\sigma_{cr} > 3$ the plastic regions become pronounced the curves tend to infinity.

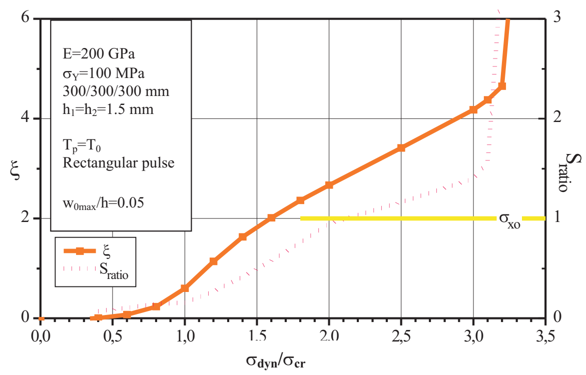

Figure 8 Dimensionless maximal dynamic deflection $\xi = \Delta w_{max}/h$ and effective stress ratio for cubic column (material characteristic)

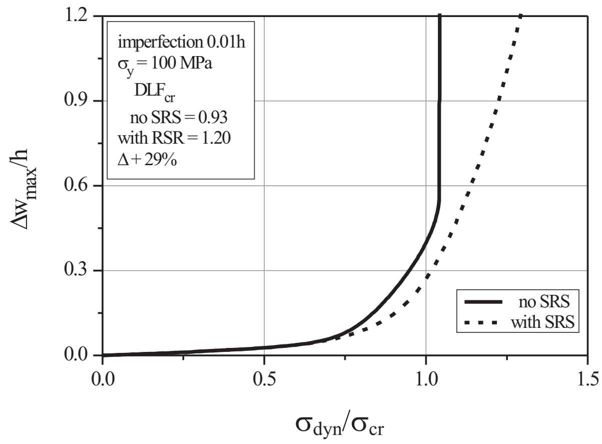

Figure 9 Maximal column deflections versus dynamic load (rectangular pulse) (strain rate effect included)

In Fig. 9 the courses of maximal deflections are shown for cubic column of equal wall thickness ($b/h = 71$) when the strain–rate effect was included into analysis. In this case - accordingly to Budiansky–Hutchinson criterion – the dynamic buckling load is 29% greater for rate sensitive material than for strain rate independent behavior. The analysis of a column made of material without strain rate effect was limited to dynamic amplitudes (DLF) not larger than 1.5. At greater pulse amplitudes large deformations appeared at the column corners and the plastic solution process was not convergent. For more results concerning viscoplastic materials see works of Mania e.g. [11], [12].

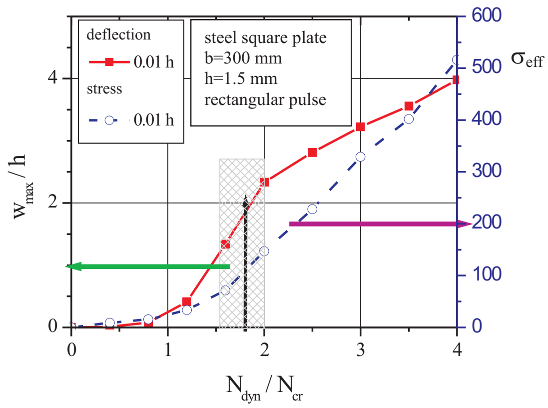

Figure 10 Comparison of dynamic buckling load estimated on the basis of three buckling criteria

3.4. *Dynamic buckling load versus assumed stability criteria*

The results of calculations presented in the paper allow to discuss the effect of application of the following criteria:

- the simplest criterion, proposed by Volmir [19] – the dynamic critical load corresponds to the amplitude of pulse load (of constant duration) at which the maximal plate deflection is equal to some constant value k (k=one half or one plate thickness),
- Budiansky&Hutchinson [3] stability criterion that states: dynamic stability loss occurs when the maximal plate deflection grows rapidly with the small variation of the load amplitude.
- Petry and Fahlbusch [14] presented a dynamic failure criterion for isotropic plates: a dynamic response caused by a pulse load is defined to be dynamic stable if the condition that the effective stress σ_{eff} (found by Huber–Mises formula) is not greater than limit stress σ_L, is fulfilled at every time everywhere in the structure. This criterion was presented for linearly elastic – perfectly plastic materials and the limit stress was assumed as equal to yield stress.
- Ari Gur and Simonetta [1] analyzed the behaviour of laminated columns and plates with all edges clamped subjected to sinusoidal pulse loading and formulated four buckling criteria. One of them connects the dynamic buckling load with the phenomenon of buckling mode change.

The simple and quick simple comparison of first three mentioned earlier criteria was made for square steel plate of yield limit value equal to 200 MPa (Fig. 10). It can be seen that Volmir's criterion is the most conservative although in many cases it gives the results staying very close to the results obtained on the basis of Budiansky–Hutchinson criterion. According to Petry–Fahlbusch the dynamic buckling load is much greater (see also Fig. 8 when S_{ratio}=1).

As it is shown in Figs. 5 and 6 in case of a column of different walls thickness the buckling mode changes from one to three halfwaves . More half–waves overlap causing lower deflection amplitude. In this situation one of the Ari–Gur and Simonetta criteria can be used. It defines the critical condition for dynamic load as such for which the shape change in dynamic response occurs. According to this condition the dynamic critical loads are ca 50% greater than the values determined with application of the Budiansky–Hutchinson criterion.

Therefore the relation between the determined value of dynamic buckling load and the applied dynamic stability criterion can be seen. This is one of basic differences between dynamic and static buckling analysis where for the last the bifurcation buckling load exists.

4. Conclusions

The calculations presented in this paper confirmed the facts well known from the subject literature – the geometric imperfections, shape and duration time of pulse loading are the factors that strongly affect the dynamic behaviour of plates. In most works the influence of pulse shape was investigated under the assumption of equal amplitude at constant duration for different pulses and then the rectangular pulse always causes the largest deflections. In this paper the pulses of equal area were also compared. Then it showed that for pulses of short duration the deflections caused by rectangular loading grow more rapidly but for the duration equal to the period of natural vibrations larger deflections correspond to the sinusoidal pulse.

It should be also noticed that usually the analysis of dynamic stability is performed under the assumption of unlimited elastic range. Taking into account the material properties obtained from static tests it can be easily seen that the limit state (determined by the moment when the effective stress reaches the yield stress) appears for rather low values of pulse amplitude. It was proved that accounting for the strain-rate dependence of material properties in dynamic buckling analysis results in higher values of dynamic buckling loads.

The dynamic stability criterion, applied in the dynamic buckling analysis, influences the drawn conclusions and the critical load value.

References

[1] **Ari–Gur, J. and Simonetta, S.R.**: Dynamic pulse buckling of rectangular composite plates, Composites, Part B, 28B, 301–308, **1997**.

[2] **Gryboś, R.**: Stability of structures under impact load, /in Polish/, *PWN*, Warsow–Poznan, **1980**.

[3] **Hutchinson, J.W. and Budiansky, B.**: Dynamic buckling estimates, *AIAA Journal*, 4–3, 525–530, **1966**.

[4] **Jones, N.**: Structural impact, *Cambridge University Press*, **2003**.

[5] **Kołakowski, Z., Kowal–Michalska, K., (eds)**: Selected problem of instabilities in composite structures, A Series of Monographs, Łódź, **1999**.

[6] **Kowal–Michalska, K. (ed.)**: Dynamic stability of composite plated structures, (in Polish), Warszawa – Łódź, *WNT*, **2007**.

[7] **Kowal–Michalska, K., Kołakowski, Z. and Mania, R.**: Estimation of dynamic load factor for orthotropic plates subjected to in–plane pulse loading, Proc. of Fourth Int. Conf. on *Thin–Walled Structures*, Loughborough, UK, 415–422, **2004**.

[8] **Kowal–Michalska, K. and Mania, R.**: Some aspects of dynamic buckling of plates under in–plane pulse compression, *Mechanics and Mechanical Engineering*, vol.12, no 8, 135–146, **2008**.

[9] **Kowal–Michalska, K. and Mania, R.**: Dynamic buckling load of thin–walled columns of closed cross–section subjected to pulse compression, in Shell Structures Theory and Applications, eds. Pietraszkiewicz W., Szymczak Cz., Taylor and Francis/Balkema, 353–358, **2005**.

[10] **Kubiak, T.**: Interactive dynamic buckling of thin–walled columns, /in Polish/, *Scientific Bulletin of Łódź Technical University*, Łódź, **2007**.

[11] **Mania, R.J.**: Dynamic buckling of thin–walled viscoplastic columns, /in Polish/, *Scientific Bulletin of Łódź Technical University*, Łódź, **2010**.

[12] **Mania, J.R. and Kowal–Michalska, K.**: Parametric analysis of thin–walled structures dynamic stability by FEM, in *Numerical analysis of chosen problems in mechanics,* Niezgoda T. (ed.) /in Polish/, WAT, Warsaw, 229–246, **2007**.

[13] **Mania, J.R. and Kowal–Michalska, K.**: Elasto–plastic dynamic response of thin–walled columns subjected to pulse compression, in Shell Structures Theory and Applications, Vol.2. Pietraszkiewicz W., Kreja I., (eds), Taylor and Francis/Balkema, 183–186, **2009**.

[14] **Petry, D. and Fahlbusch, G.**: Dynamic buckling of thin isotropic plates subjected to in–plane impact, *Thin–Walled Structures*, 38, 267–283, **2000**.

[15] **Simitses, G.**: Instability of dynamically loaded structures, *Applied Mech. Rev.*, 40, 10, 1403–1408, **1987**.

[16] **Simitses, G.J.**: Dynamic stability of suddenly loaded structures, *Springer Verlag*, New York, **1990**.

[17] **Teter, A.**: Multi–modal buckling of thin–walled stiffened columns loaded by compressive pulse, /in Polish/, *Scientific Bulletin ofŁódź Technical University*, Łódź, **2010**.

[18] **Weller, T., Abramovich, H. and Yaffe, R.**: Dynamic buckling of beams and plates subjected to axial impact, *Computers &Structures*, 37, 835–851, **1989**.