EXPERIMENTS IN THE CONTROL OF UNBALANCE RESPONSE USING MAGNETIC BEARINGS

CARL R. KNOSPE, R. WINSTON HOPE, STEPHEN J. FEDIGAN AND RONALD D. WILLIAMS

Center for Magnetic Bearings, Department of Mechanical and Aerospace Engineering,
University of Virginia, Charlottesville, VA 22903, USA

Abstract -- Unbalance response is a common vibration problem associated with rotating machinery. For several years, researchers have demonstrated that this vibration could be greatly alleviated for machines using active magnetic bearings through active control. Many of the control strategies employed fall into a class which the authors have termed *adaptive open loop control*. In this paper, three algorithms in this class are presented and their performances are examined experimentally. These algorithms are (1) a non-recursive control law with simultaneous estimation, (2) a recursive control law with simultaneous estimation, and (3) a recursive control law with gain scheduling according to operating speed. Each algorithms was coded in *C* and executed on a high-speed, multi-tasking digital controller. The advantages and disadvantages of each algorithm are illustrated by examining experimental results from a laboratory magnetic bearing rotor rig. These results clearly demonstrate the high degree of synchronous vibration attenuation (over 30 dB) which can be achieved with adaptive open loop methods. The response of these algorithms to a sudden change in 'simulated imbalance' is used to evaluate their relative transient performances. These results indicate the benefits of recursive control laws in adapting the synchronous open loop control currents to cancel the vibration. The ability of each of the algorithms to adapt the open loop control during changes in rotor speed is also examined. On this test, the recursive gain scheduled algorithm shows superior performance: rotor midspan vibration is almost completely eliminated over the operating speed range. However, surprisingly, the non-recursive control law shows better performance than the recursive law with simultaneous estimation. This result is explained in terms of the stability of the adaptation process.

1. INTRODUCTION

Active magnetic bearings provide a number of advantages over conventional bearings for a variety of practical industrial applications. These include elimination of the lubrication system, friction free operation, decreased power consumption, operation at temperature extremes, and vibration control.

Recently there has been a great deal of interest in digital control of magnetic bearing systems. Digital control offers several major benefits over analog control systems for magnetic bearing supported rotors:

- quick tuning of a magnetic bearing system during installation

- implementation of some simple but powerful control strategies, such as gain scheduling
- application of fault tolerant controller architectures
- built-in monitoring and diagnostic capabilities

As the results presented here indicate, digital control provides capabilities for adaptive control which can be used to greatly alleviate the unbalance vibration of rotating machinery. This is often the worst vibration problem encountered during operation. The source of this vibration is the discrepancy between the geometric axis of the rotor and its inertial axis. When the rotor is spinning, this imbalance results in a centrifugal force which causes synchronous vibration throughout the machine. This problem is managed on conventional machinery through mechanical balancing -the addition or removal of a small amount of mass from the shaft to reduce the residual imbalance. Rotor balancing in the field, unfortunately, is usually time consuming and costly. The down-time incurred can also be very expensive in terms of lost production. Also for some machines where the imbalance changes often during operation, such as centrifuges, mechanical balancing will have a limited benefit.

Magnetic bearings, being active devices, offer the capability to establish new and beneficial relationships between rotor and casing vibration and applied bearing force. This capability has been employed by a number of researchers [1-10] investigating the control of unbalance response. One method to achieve unbalance response attenuation is through design of the feedback compensation. This has been achieved via the addition of filters to stabilizing controllers [1,8] or through the addition of pseudo-states in observer-based controllers [2,3]. Other researchers [4-10] have employed methods which the authors refer to as *adaptive open loop control*. These methods, as pointed out by Larsonneur [7,8] and Shafai et. al. [9], have the advantage that they may be added to feedback controllers that have been designed for optimum transient response without altering system stability or performance. These methods were first employed on a magnetic bearing supported rotor by Burrows and Sahinkaya [4] who solved a least-squares-balancing problem for the proper forces to apply using an off-line theoretical model. They later extended this work to obtain an estimate of an influence coefficient matrix through trial forces and the use of a recursive control law [5]. Higuchi et. al. [6] applied an adaptive open loop method (*periodic learning control*) that employed an estimate of the inverse transfer function in a recursive update equation. This method as presented can only be employed on systems with square influence coefficient matrices (number of actuators equals number of vibration sensors). Both References 5 and 6 are very similar to the convergent control algorithm presented by Knospe et. al [10] which uses a look-up table of influence coefficients obtained through off-line testing. Shafai et. al. [9] employ a very different method of adapting the open loop forces to cancel a synchronous signal. Here, only one Fourier coefficient of the open loop signal is changed per adaptation cycle in such a fashion as to decrease the residual error. This method, originally developed for SISO systems, was extended to square MIMO systems. Stability and performance robustness of this method (convergence to optimal open loop control) is guaranteed. This is in contrast to most of the model-based methods [4-7] where stability and performance robustness has generally not been examined. Also, the transient performance of the proposed adaptive open loop algorithms [4-9] to changes in imbalance or rotor speed has not been considered.

In this paper, three adaptive vibration control algorithms are evaluated experimentally. These algorithms were coded in *C* and executed on digital controller designed and built at the University of Virginia [11]. Feedback and adaptive unbalance control algorithms are coded and executed as separate tasks under a multi-tasking, real-time operating system written at the University [12].

2. ADAPTIVE OPEN LOOP CONTROL

The paradigm used for design of the control algorithms presented is that of *adaptive open loop control*. Ideally, an open loop controller pre-schedules the bearing forces to be applied to minimize the vibration. An advantage of this method is that if the scheduled forces are correct, nearly perfect performance can be realized since stability considerations impose no constraint upon achievable performance for an open loop controller. However, this control strategy relies on the accuracy of the pre-computed schedule; any error in calculating the scheduled forces cannot be corrected. Also, to calculate the proper schedule of forces, future disturbances to the rotor system must be known in advance. This imposes serious limitations on the type of rotor vibration problems which can be tackled with open loop control.

The first problem of open loop control methods, the inability to fix an incorrect schedule, can be overcome by re-computing the schedule periodically. If the length of the schedule is much longer than the largest time constant of the system, this *adaptation* of the open loop control does not appreciably affect the rotor's transient response and the system's dynamics may be considered to be unaltered. The adaptive open loop controller may also be viewed as a very slow, nonlinear feedback controller. However, this paradigm is not very useful in designing such controllers or in analyzing their behavior.

The second problem of open loop control methods, the requirement of knowing future disturbances so as to compute a schedule, restricts the successful application of adaptive open loop control to only a few disturbance sources, such as rotor imbalance, where future disturbances are known to be synchronous sinusoids.

3. ADAPTIVE OPEN LOOP CONTROL ALGORITHMS

The forced response of a rotor supported in magnetic bearings at any particular operating speed ω can be described by the following equation

$$X = TU + w^2 Q \tag{1}$$

where X is a $2n$ x 1 real vector of the synchronous Fourier coefficients of vibration (or signals to be attenuated) at n points of interest (perhaps, along the rotor and in the housing), U is a 2m x 1 real vector of the synchronous Fourier coefficients of the open loop control currents applied to the m magnetic actuators, T is a $2n$ x $2m$ real matrix of influence coefficients relating the open loop control forces to the synchronous vibrations, and Q is a $2n$ x 1 real vector of the synchronous Fourier coefficients of uncontrolled vibration at the n points of interest normalized by the square of the operating speed w. Here, each point of interest consists of only one direction of vibration (e.g., vertical). Also, two magnetic actuators (e.g., horizontal and vertical) constitute one magnetic bearing. Note that the vector of synchronous Fourier coefficients U determines the amplitude and phase for each of the synchronous open loop control currents and thus determines a schedule of applied open loop control forces. The vector X is determined during operation through a discrete time convolution of the vibration signals with 1x sine and cosine waves. The influence coefficient matrix T is a function of the operating speed and is determined by the dynamics of the rotor, bearings, and housing. All the algorithms explored in this paper are designed to minimize a weighted quadratic performance function:

$$J \equiv X^T W X \tag{2}$$

where W is a diagonal weighting matrix. This form of performance function was chosen since it has an analytic solution for the optimal control and generally yields good performance. The optimal control vector that minimizes this performance index is

$$U^* = -w^2 \left(T^T W T\right)^{-1} T^T W Q \tag{3}$$

The model given in Eqn. (1) is rewritten here with subscripts to reflect that a sequence of open loop control schedules is applied to the rotor:

$$X_i = TU_i + w_i^2 Q \tag{4}$$

Here the subscript i has been added to indicate that the values of the quantities are those during the i'th open loop schedule applied. This *quasi-steady* model of the machine's synchronous response assumes that the rotor speed is changing slowly enough that (1) the unbalance response can be considered to have reached its steady state value and (2) the Fourier coefficients of the vibration can be properly calculated. This model also assumes that any transients from the last update of the control vector (from schedule i-1 to schedule i) have decayed before the Fourier coefficients of the next (schedule i) vibration vector are calculated. This imposes a constraint on the update rate of the control force schedule.

The three adaptive open loop control algorithms examined in this paper fall into two categories based on whether continuous on-line estimation is used:

Simultaneous Estimation and Control Algorithms (SEC)
(1) a non-recursive control law that employs on-line estimation (NR-SEC)
(2) a recursive control law that employs on-line estimation (R-SEC)

Gain Scheduled Algorithm (GS or *Convergent Control*)
(3) a recursive control law that employs a gain schedule (R-GS).

The SEC algorithms repeatedly estimates the model's parameters (typically, T and Q) while applying control based on these estimates. In contrast, the R-GS algorithm does not update its estimate of the model's parameters but instead relies on the recursion of the control law to adapt the open loop control schedule. These three algorithms are now examined in detail.

3.1. Non-Recursive Simultaneous Estimation and Control with Least-Square Estimation

The non-recursive optimal control law is derived by minimizing the 2-norm of the vibration vector and substituting the estimates ($\hat{T}_i$ and $\hat{Q}_i$) into the control law for actual parameters (T and Q). It should be noted that this substitution, known as *certainty-equivalence*, yields a different result than if the uncertainty about the parameter estimates was used in deriving the control law.

Using the model given by Eqn. (4), the optimal control law that minimizes the weighted quadratic norm of the vibration vector is

$$U_{i+1} = -w_{i+1}{}^2\left(\hat{T}_i{}^T W \hat{T}_i\right)^{-1}\hat{T}_i{}^T W \hat{Q}_i \qquad (5)$$

(see Appendix). Note that the next control schedule applied is only dependent on the previous schedule in an indirect manner (through the estimation process). Therefore, this control law is referred to as *non-recursive*. The estimates $\hat{T}_i$ and $\hat{Q}_i$ are computed using a moving batch least squares estimator:

$$\begin{bmatrix}\hat{Q}_i & \hat{T}_i\end{bmatrix} = [X]_i [\underline{U}]_i{}^T \left([\underline{U}]_i [\underline{U}]_i{}^T\right)^{-1} \qquad (6)$$

where $[\underline{U}]_i$ and $[X]_i$ are matrices composed of the p most recent augmented controls and responses

$$[\underline{U}]_i \equiv \begin{bmatrix} w_i{}^2 & w_{i-1}{}^2 & \cdots & w_{i-p+1}{}^2 \\ U_i & U_{i-1} & \cdots & U_{i-p+1}\end{bmatrix} \qquad [X]_i \equiv \begin{bmatrix} X_i & X_{i-1} & \cdots & X_{i-p+1}\end{bmatrix} \qquad (7)$$

This identification algorithm may have numerical problems when the rotor is operating at a constant speed. In this case, the control vector is nearly constant and the columns of $\left[\underline{U}\right]_i$ will be nearly identical. Therefore, this matrix will be rank deficient and the inverse in Eqn. (6) will be poorly conditioned. To remedy this, the identification algorithm has been slightly modified. If the difference between the most recently applied control vector U_i and the most recent vector in the moving batch U_k as measured by

$$e \equiv (U_i - U_k)^T (U_i - U_k)$$

is greater than a predetermined constant $\bar{e}$ then the oldest vector in the batch is removed and the vector U_i is added to the batch. If $e < \bar{e}$, then the most recent vector in the batch U_k is removed and the new vector U_i is added to the batch. The same operations are also performed on the corresponding response vectors in the batch $\left[X\right]_i$. This modification prevents the control batch $\left[\underline{U}\right]_i$ from having rank less than its column dimension $2m$+1.

3.2. Recursive Simultaneous Estimation and Control with Least-Squares Estimation

To minimize the performance index $X_{i+1}{}^T W X_{i+1}$, the recursive algorithm employed calculates the new control vector using

$$\underline{U_{i+1} = U_i - \left(\hat{T}_i^T W \hat{T}_i\right)^{-1} \hat{T}_i^T W X_i} \tag{8}$$

(see Appendix). Note that since X_i is measured, only T needs to be estimated for this control law. The least-squares estimate of T is given by

$$\hat{T}_i = [\Delta X]_i [\Delta U]_i^T \left([\Delta U]_i [\Delta U]_i^T\right)^{-1} \tag{9}$$

where the batches $\left[\Delta U\right]_i$ and $\left[\Delta X\right]_i$ are defined by

$$\begin{aligned} \left[\Delta U\right]_i &\equiv \left[\Delta U_i \quad \Delta U_{i-1} \quad \cdots \quad \Delta U_{i-p+1}\right] \\ \left[\Delta X\right]_i &\equiv \left[\Delta X_i \quad \Delta X_{i-1} \quad \cdots \quad \Delta X_{i-p+1}\right] \end{aligned} \tag{10}$$

and

$$\Delta X_i \equiv X_i - X_{i-1} \qquad\qquad \Delta U_i \equiv U_i - U_{i-1} \tag{11}$$

Note that substitution of Eqn. (4) into Eqn. (8) yields

$$\underline{U_{i+1} = \left[I - \left(\hat{T}_i^T W \hat{T}_i\right)^{-1} \hat{T}_i^T W T_i\right] U_i - w^2 \left(\hat{T}_i^T W \hat{T}_i\right)^{-1} \hat{T}_i^T W Q} \tag{12}$$

This clearly shows the direct recursive relationship between one control schedule and the next. The stability of this algorithm, while difficult to analyze because of the updating of the estimates, is obviously related the eigenvalues of the matrix:

$$\left[I - \left(\hat{T}_i^T W \hat{T}_i\right)^{-1} \hat{T}_i^T W T_i \right]$$

For both the SEC controllers, a batch of *p* previously applied control vectors and responses must be available to begin estimation. For the estimation of the NR-SEC algorithm, the batch size *p* must be greater than or equal to 2*m*+1. For the estimation of the R-SEC algorithm, the batch size must be at least 2*m*. Therefore, in both these cases, the adaptive open loop control algorithm must be started with a series of at least 2*m*+1 test schedules.

3.3. Recursive Gain Scheduled Algorithm (Convergent Control)

The recursive gain scheduled control algorithm is very similar to the R-SEC controller, Eqn. (8), but with the R-GS control algorithm *T* is not estimated during adaptation of the control vector. The R-GS algorithm consists of iteratively applying the control law

$$U_{i+1} = U_i - A_w X_i \tag{13}$$

with

$$A_w \equiv \left(\hat{T}_w{}^T \hat{T}_w\right)^{-1} \hat{T}_w{}^T$$

Here, $\hat{T}_w$ is the estimate of *T* for the rotor operating speed ω. The A_w matrix is determined prior to the iterative application of Eqn. (13). If the influence coefficient matrix *T* is square (i.e., the number of actuators is equal to the number of vibration sensors), the control vector will converge to U^* if $\hat{T}_w$ is sufficiently close to *T*:

3.4. Performance Robustness for Square T

If the error in the estimate of a square *T* matrix is expressed as an additive error matrix E_a

$$\hat{T}_w = T + E_a \tag{14}$$

then the control law of Eqn. (13) will converge to U^* and *X* will converge to zero provided that

$$\bar{s}(E_a) < \underline{s}\left(\hat{T}_w\right) \tag{15}$$

(a sufficient condition) where $\bar{s}(\bullet)$ and $\underline{s}(\bullet)$ indicate the maximum and minimum singular values respectively (see Appendix B).

With an error in the estimate of a non-square T matrix , it is easy to demonstrate that the control vector generally will not converge to the optimal vector U^*. A bound on the degradation in the performance due to a certain size error $\bar{s}(E_a)$ is difficult to obtain (a bound will be presented by the authors in a future paper). However, it is easy to determine the stability robustness properties of the adaptation:

3.5. Stability Robustness of Adaptation for Non-Square T

If the error in the estimate of a non-square T matrix is expressed by an additive error matrix as in Eqn. (14), the R-GS adaptation algorithm will be stable (i.e., U will converge to some finite U_{ss}) if

$$\bar{s}(E_a) < \frac{1}{\bar{s}(A)} \tag{16}$$

(see Appendix B). Our experience indicates that the R-GS algorithm has good performance robustness for non-square T matrices as well as the indicated stability robustness.

For the results presented in this paper, the matrix $\hat{T}_w$ is estimated using the least squares estimator of Eqns. (9)-(11) for a set of speeds $\Omega \equiv \{w_1, w_2, \cdots, w_q\}$covering the operating speed range. Gain matrices A_k are then computed for each speed w_k in the set Ω and these are stored in a look-up table for use during operation. The matrix A_w to be used at any particular speed ω during operation is determined by element-wise linear interpolation between the matrices for the two nearest operating speeds in the table, w_k and w_{k+1}:

$$A_w = \left(\frac{w - w_k}{w_{k+1} - w_k}\right) A_{k+1} + \left(\frac{w_{k+1} - w}{w_{k+1} - w_k}\right) A_k \tag{17}$$

4. EXPERIMENTAL RESULTS

A laboratory test rig with two radial magnetic bearings, shown in Figure 1, was used to examine the efficacy of the adaptive balancing algorithms using a multi-tasking digital controller. The rotor of this rig has a 12.7 mm (0.5 inch) diameter and a 508 mm (20 inches) bearing span. Eddy current position sensors are located vertically and horizontally 32 mm (1.25 inch) outboard from the center of each bearing and 51 mm (2 inches) inboard from the center of the mid span disk. Bearing housing asymmetry resulted in unequal horizontal and vertical support stiffness. The rotor is supported using decentralized proportional-derivative control. The first critical speed of this rotor is at approximately 2700 rpm. In the experiments, both bearings (m=4) were used to reduce the vibration at the inboard, outboard, and midspan locations (n=6).

The control algorithms described in the previous section were implemented on a digital controller designed and built at the University of Virginia's Center for Magnetic Bearings. The digital controller is a 32 bit floating point machine using a Texas Instruments TMS320C30 digital signal processor [11]. Both the feedback control and the adaptive open loop control algorithms were coded in *C* and executed under a multi-tasking real-time operating system written at the University of Virginia [12]. The feedback algorithm executes as the highest priority task. The adaptive open loop algorithm executes during spare time between feedback updates.

For all the results presented herein, unless otherwise noted the following test conditions were used:

- the midspan vibration was weighted a factor of 6.3 greater than the inboard and outboard vibration.
- the batch size used in least square estimation, p, was 14.
- the Fourier coefficients of the vibration were computed over 15 revolutions.

4.1. Constant Speed Vibration Reduction-2700 RPM

The root-mean-square synchronous midspan, inboard, and outboard vibration amplitudes without adaptive open loop control and with the NR-SEC, R-SEC, and R-GS algorithms are shown in Figure 2. Note that all three algorithms achieve excellent attenuation of the midspan vibration, over 36 dB. The inboard vibration has increased slightly with the use of the adaptive controllers while the outboard vibration has decreased. The authors do not view the slight differences between the steady state performance of the three algorithms as significant. These differences fall within the variation that is normally seen on experiments with this test rig.

4.2. Sudden Change in Imbalance

A sudden change in the imbalance condition of the rotor was experimentally simulated by the addition of synchronous perturbation currents to the bearings. These currents act to produce an unknown force which rotates with the shaft much like a physical imbalance. For these tests, the rotor was operated at 2200 rpm. Figure 3 shows the midspan synchronous response of the rotor when adaptive open loop control is not used. For the first 10 updates, the rotor vibrates due to its physical imbalance. Then, the 'balance condition' of the rotor changes suddenly due to the applied synchronous perturbation currents. Figures 4, 5, and 6 show the transient response of the NR-SEC, R-SEC, and R-GS algorithms respectively during this test.

The NR-SEC responds rather poorly to the sudden change in imbalance. It updates its values of T and Q from the input and output data that has been collected. But, much of this information describes the system as it existed before the change. The estimator finds the parameters with the "best fit" to the available data, but performs poorly until all the old information (pre-dating the change) has moved out of the batch. Then, excellent vibration attenuation is once again recovered.

The R-SEC algorithm recovers much more quickly after the sudden change in imbalance than the NR-SEC algorithm. This quick recovery is due to:

(1) The data collected before the sudden change is still useful in determining the T matrix since this has not changed. Only a single set of measurements (ΔX and ΔU) in the batch straddles the sudden change and is therefore misleading in estimation.

(2) The convergent nature of the recursive algorithm used. Since the R-SEC is essentially a R-GS algorithm with the T matrix updated continuously, it has essentially the convergence property indicated in Eqns. (15) and (16). Therefore, the algorithm can produce good performance even when the estimate of T is in error due to the misleading measurement set.

As shown in Figure 6, the R-GS algorithm has a very quick recovery to the sudden change in imbalance. This high performance is due to:

(1) The algorithm does not estimate the T matrix. Since A_w is calculated from a look-up table, the algorithm assumes it changes only as a function of operating speed. A misleading data set (ΔX and ΔU) does not corrupt the estimate of T and hence the gain matrix A_w.

(2) The convergent nature of the recursive algorithm.

Note that the convergence property of this algorithm permits good vibration attenuation *even* if T changes although the R-GS algorithm does not track these changes through on-line estimation as do the SEC algorithms.

In summary, the recursive algorithms respond best to a change in the vector Q. The performance of the three algorithms during a change in the T matrix is examined next.

4.3. Changing Rotor Speed

As the rotor speed changes, the influence coefficient matrix T changes. Here, the performance of the three algorithms is examined during a 30 second run-up from 1200 to 3000 rpm. Figure 7 shows the amplitude of the synchronous midspan vibration without adaptive open loop control. The first critical speed is clearly visible at 2700 rpm. The performance of the NR-SEC, R-SEC, and R-GS algorithms during the run-up is shown in Figures 8, 9, and 10.

The NR-SEC is effective in reducing the midspan vibration from 1200 to 2500 rpm. Over most of this speed range, the influence coefficient matrix is changing slowly and the batch least-squares estimator can track these changes. Near the critical speed, the *T* matrix changes quickly and the estimator has difficulty in following this variation. Note that for the estimation performed at 2700 rpm in this test, the oldest data in the batch is from 2050 rpm.

The R-SEC algorithm also performs well from 1200 to 2500 rpm. But as with the NR-SEC, the estimates of the system parameters are increasingly in error near the critical speed. For the R-SEC algorithm, however, this results in adaptation instability as the poor estimate of *T* results in an eigenvalue of the matrix in Eqn. (12) being outside the unit circle. This is the cause of the loss of rotor support just below 2600 rpm.

The R-GS algorithm performs very well during changes in rotor speed as Figure 10 illustrates. This algorithm quickly adapts to the changes in rotor speed using the look-up table. Since the R-GS algorithm is computationally much simpler than either SEC algorithm, it executes faster and more often. Thus, the R-GS algorithm updates its control vector 50 times in 30 seconds while the NR-SEC updates only 36 times. Considering that a significant proportion of the time between updates is spent performing a discrete time convolution to obtain the Fourier coefficients (which is the same for all three algorithms), this is a significant increase in update rate. An increased update rate yields better tracking and vibration rejection.

5. DISCUSSION

The best algorithm for an industrial application may be a hybrid of the R-GS and SEC algorithms. This hybrid might use performance measures to determine when the estimates need to be updated. Under most conditions, it would function as recursive gain scheduled algorithm and would have a quick response during changes in unbalance response or rotor operating speed. But when its performance differed significantly from that expected (e.g., divergence of control action), the controller would begin to update its estimates. When the performance returned to acceptable, the estimation procedure would be halted. It would be important for this hybrid algorithm to distinguish between sudden changes in the imbalance condition resulting in momentary poor performance and controller divergence due to error in the estimate of *T*. In the first case, the proper action is to continue to employ the R-GS algorithm. In the second case, a new estimate of *T* needs to be obtained from recent on-line measurements. Thus, the criterion used to determine which mode of operation to adopt cannot be based solely on vibration level. A measure of the accuracy of the *T* matrix employed should also be used. This measure could be a norm of a propagated covariance matrix as used in discrete-time Kalman filters [13].

6. CONCLUSIONS

All three adaptive open loop control algorithms presented in this paper have been shown to provide excellent synchronous vibration attenuation in steady state operation. The performance of the three algorithms, however, differed greatly during the transient tests. The non-recursive SEC algorithm had sluggish vibration rejection after the sudden change in imbalance. Good performance was not recovered until all the measurements previous to the change had moved out of the batch. The recursive SEC algorithm performed much better at this since the corruption of its estimates was not severe enough to cause adaptation instability. The recursive gain scheduled algorithm had the best performance during this test. The midspan vibration was canceled after one adaptation.

During the rotor run-up test, the NR-SEC algorithm had poor performance near the critical speed since the estimator could not track the changes in the influence coefficient matrix. This performance was better, however, than that of the R-SEC algorithm. Poor tracking with this algorithm resulted in the adaptation process instability. The performance of the R-GS algorithm during this test was excellent; the midspan vibration was canceled over the entire operating speed range.

REFERENCES

[1] H. Habermann and M. Brunet, "The Active Magnetic Bearing Enables Optimum Damping of Flexible Rotors", ASME Paper 84-GT-117, 1984.

[2] T. Higuchi, T. Mizuno, and M. Tsukamoto, "Digital Control System for Magnetic Bearings with Automatic Balancing", *Proceedings of the 2nd International Symposium on Magnetic Bearings*, July 12-14, 1990, Tokyo, Japan.

[3] F. Matsumura, M. Fujita, and K. Okawa, "Modeling and Control of Magnetic Bearing Systems Achieving a Rotation Around the Axis of Inertia", *Proceedings of the 2nd International Symposium on Magnetic Bearings*, July 12-14, 1990, Tokyo, Japan.

[4] C.R. Burrows and M.N. Sahinkaya, "Vibration Control of Multi-Mode Rotor-Bearing Systems", *Proceedings of the Royal Society of London*, Vol. 386, pp. 77-94, 1983.

[5] C.R. Burrows, M.N. Sahinkaya, and S. Clements, "Active Vibration Control of Flexible Rotors: an Experimental and Theoretical Study", *Proceedings of the Royal Society of London*, Vol. 422, pp. 123-146, 1989.

[6] T. Higuchi, M. Otsuka, T. Mizuno, and T. Ide, "Application of Periodic Learning Control with Inverse Transfer Function Compensation in Totally Active Magnetic Bearings", *Proceedings of the 2nd International Symposium on Magnetic Bearings*, July 12-14, 1990, Tokyo, Japan.

[7] R. Larsonneur, *Design and Control of Active Magnetic Bearing Systems for High Speed Rotation*, PhD Thesis, Swiss Federal Institute of Technology, ETH Zurich, Switzerland, June 1988.

[8] R. Larsonneur and R. Herzog, "Feedforward compensation of Unbalance: New Results", IUTAM Symposium, Bath, UK, September 1994.

[9] B. Shafai, S. Beale, P. LaRocca, and E. Cusson, "Magnetic Bearing Control Systems and Adaptive Forced Balancing", *IEEE Control Systems*, Volume 14, No. 2, pp. 4-13, April 1994.

[10] C.R. Knospe, R.W. Hope, S.J. Fedigan, and R.D. Williams, "Adaptive On-Line Rotor Balancing Using Digital Control", *Proceedings of MAG '93 Magnetic Bearings, Magnetic Drives, and Dry Gas Seals Conference*, Technomic Publishing, Lancaster, PA, July 1993.

[11] J.K. Daddio, *An Integrated Magnetic Bearing Controller*, M.S. Thesis, University of Virginia, May 1992.

[12] S.J. Fedigan, *A Real-Time Operating System for a Magnetic Bearing Digital Controller*. M.S. Thesis, University of Virginia, May 1993.
[12] J. Mendel, *Lessons in Digital Estimation Theory*, Prentice Hall, Englewood Cliffs, NJ, 1987.

APPENDIX

A.1. Control Laws

A.1.1. Non-Recursive SEC

The NR-SEC control law minimizes the performance index

$$J = E\left\{X_{i+1}^{T} W X_{i+1}\right\} \tag{A1}$$

where $E\{\bullet\}$ is the expected value operator. Substitution of Eqn. (4) into this and changing the index to i+1 yields

$$J = E\left\{\left(TU_{i+1} + w_{i+1}^{2} Q_{i+1}\right)^{T} \left(TU_{i+1} + w_{i+1}^{2} Q_{i+1}\right)\right\} \tag{A2}$$

Assuming T is known (certainty equivalence assumption), this can be simplified to

$$J = U_{i+1}^{T} \hat{T}^{T} W \hat{T} U_{i+1} + w_{i+1}^{2} U_{i+1}^{T} \hat{T}^{T} W E\{Q_{i+1}\} + \left\{\text{terms not including } U_{i+1}^{T}\right\} \tag{A3}$$

Since there is no *a priori* information available about Q_{i+1}, the expected value of this vector is assumed to be equal to that experienced in cycle i (*a posteriori* information), that is

$$E\{Q_{i+1}\} = E\{Q_i\} = \hat{Q}_i \tag{A4}$$

Here, it is assumed that the estimate produced by the least-squares estimator is unbiased.
Substituting Eqn. (A4) into Eqn. (A3) and setting the first variation of J with respect to U_{i+1}^{T} equal to zero yields

$$0 = \hat{T}_i^{T} W \hat{T}_i U_{i+1} + w_{i+1}^{2} \hat{T}_i^{T} W \hat{Q}_i \tag{A5}$$

Now, solving for U_{i+1} yields the optimal control law

$$U_{i+1} = -w_{i+1}^{2} \left[\hat{T}_i^{T} W \hat{T}_i\right]^{-1} \hat{T}_i^{T} W \hat{Q}_i \tag{A6}$$

A.1.2.. Recursive SEC

Solving Eqn. (4) for Q_i yields

$$Q_i = \frac{1}{w_i^2}\left(X_i - TU_i\right) \tag{A7}$$

Substituting (A7) into (A4) yields

$$E\left\{\frac{X_i - TU_i}{w_i^2}\right\} = \hat{Q}_i \tag{A8}$$

or

$$\frac{1}{w_i^2}\left(X_i - \hat{T}_i U_i\right) = \hat{Q}_i \tag{A9}$$

(once again, using a certainty equivalence assumption). Substituting this expression into Eqn. (A6) yields

$$U_{i+1} = \left(\frac{w_{i+1}}{w_i}\right)^2 \left\{U_i - \left[\hat{T}_i^T W \hat{T}_i\right]^{-1} \hat{T}_i^T W X_i\right\} \tag{A10}$$

This control law was simplified for implementation by assuming that $w_{i+1} = w_i$ yielding Eqn. (8).

A.2. Stability and Performance Robustness of R-GS Algorithm

A.2.1 Stability and Performance Robustness for Square T

Assuming the rotor is at constant speed ω and that the uncontrolled vibration is constant, the vibration during schedule *i*+1 can be related to that during schedule *i* via

$$X_{i+1} = X_i + T\left(U_{i+1} - U_i\right) \tag{A11}$$

Substituting Eqns. (13) and (14) into this expression yields

$$X_{i+1} = \left\{I - \left[\hat{T}_w^T W \hat{T}_w\right]^{-1} \hat{T}_w^T W\left(\hat{T}_w - E_a\right)\right\} X_i \tag{A12}$$

For square T, this simplifies to

$$X_{i+1} = \left[\hat{T}_w^{-1} E_a\right] X_i \tag{A13}$$

If

$$\bar{s}\left(\hat{T}_w^{-1}E_a\right) < 1 \tag{A14}$$

then the 2-norm of the vibration vector will decrease with each adaptation, and therefore converges to zero indicating *both* stability and optimal performance. Condition (A14) will be satisfied if

$$\bar{s}\left(\hat{T}_w^{-1}\right)\bar{s}\left(E_a\right) < 1 \tag{A15}$$

Using the identity

$$\bar{s}\left(M^{-1}\right) = 1 / \underline{s}\left(M\right) \tag{A16}$$

Eqn. (A15) can be written as

$$\bar{s}\left(E_a\right) < \underline{s}\left(\hat{T}_w\right) \tag{A17}$$

A.2.2. Stability Robustness for Non-Square T

Substituting Eqns. (4) and (14) into Eqn. (13) yields

$$U_{i+1} = U_i - A_w\left\{\left(\hat{T}_w - E_a\right)U_i + w_i^2 Q_i\right\} \tag{A18}$$

The equilibrium solution U_{eq} to this difference equation must satisfy

$$U_{eq} = U_{eq} - A_w\left\{\left(\hat{T}_w - E_a\right)U_{eq} + w_i^2 Q_i\right\} \tag{A19}$$

Subtracting Eqn. (A19) from (A18) yields

$$dU_{i+1} = \left\{I - A_w\hat{T}_w + A_w E_a\right\}dU_i \tag{A20}$$

where dU_i is the difference between U_i and the equilibrium solution. Therefore, this difference becomes smaller with each iteration if

$$\bar{s}\left(I - A_w\hat{T}_w + A_w E_a\right) < 1 \tag{A21}$$

Since $A_w\hat{T}_w = I$, this equation simplifies to

$$\bar{s}\left(A_w E_a\right) < 1 \tag{A22}$$

Condition (A22) will be satisfied if

$$\bar{s}\left(A_w\right)\bar{s}\left(E_a\right) < 1 \tag{A23}$$

Therefore, a sufficient condition for stability of the adaptation process is

$$\bar{s}(E_a) < \frac{1}{\bar{s}(A_w)} \tag{A24}$$

Under this condition the control vector will converge to the equilibrium solution of Eqn. (A19)

$$U_{eq} = -w^2 \left[I - A_w E_a \right]^{-1} A_w Q_i \tag{A25}$$

and the adaptation process will be stable.

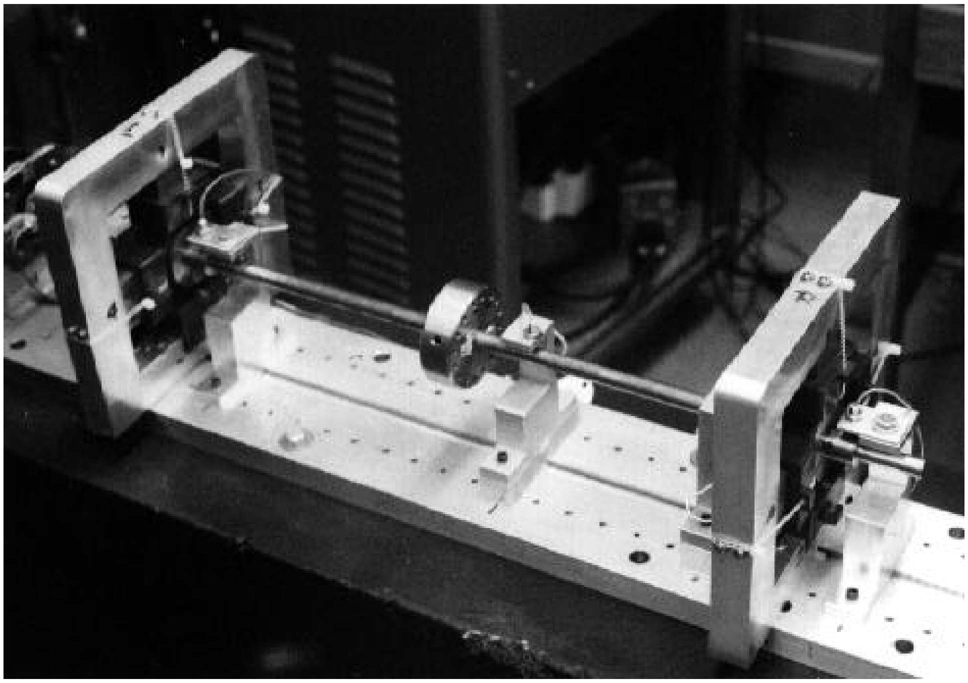

Figure 1: Magnetic bearing test rig

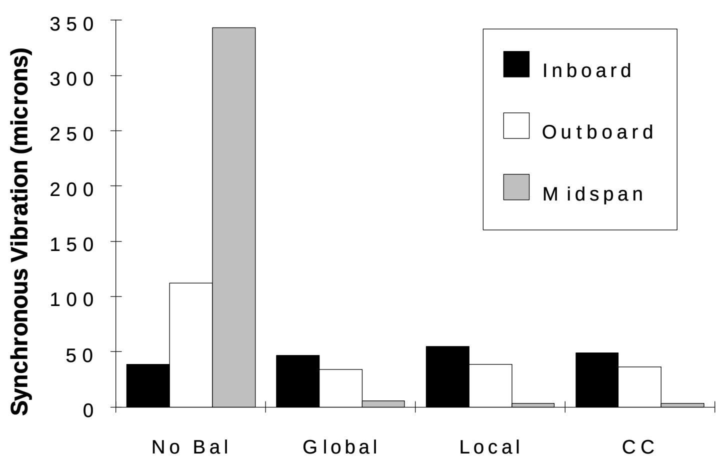

Figure 2: RMS Synchronous vibration amplitudes without and with the NR-SEC, R-SEC, and R-GS adaptive open loop control algorithms, 2700 rpm.

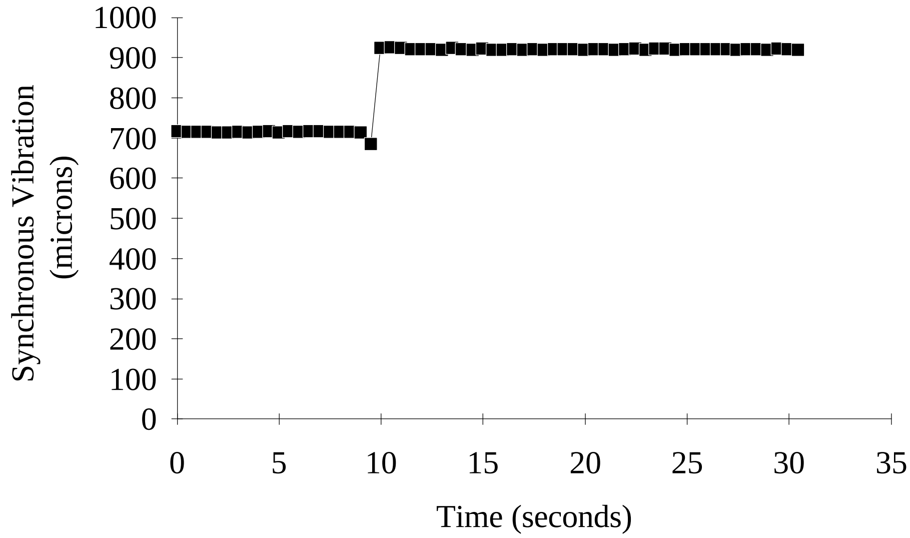

Figure 3: Synchronous midspan vibration amplitude without adaptive open loop control during the simulated change in balance, 2200 rpm.

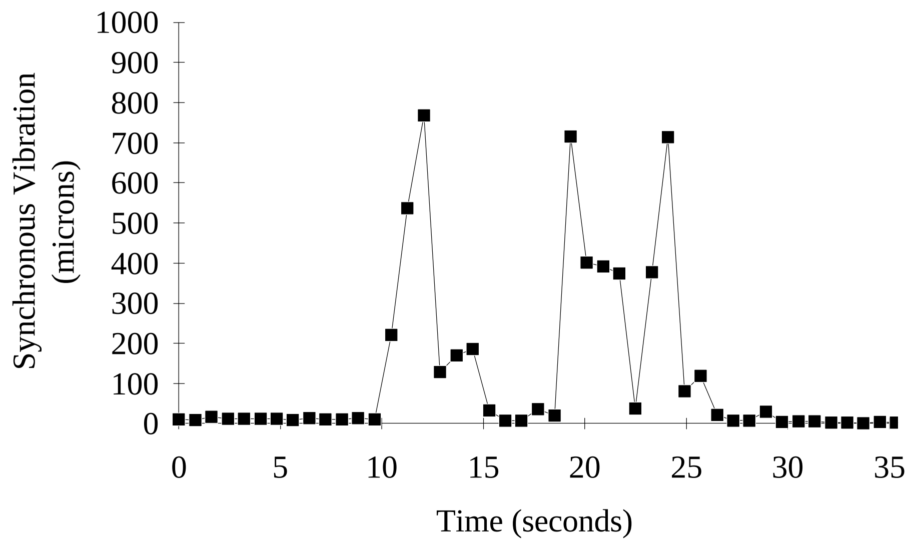

Figure 4: Synchronous midspan vibration amplitude with the NR-SEC algorithm during the simulated change in balance, 2200 rpm.

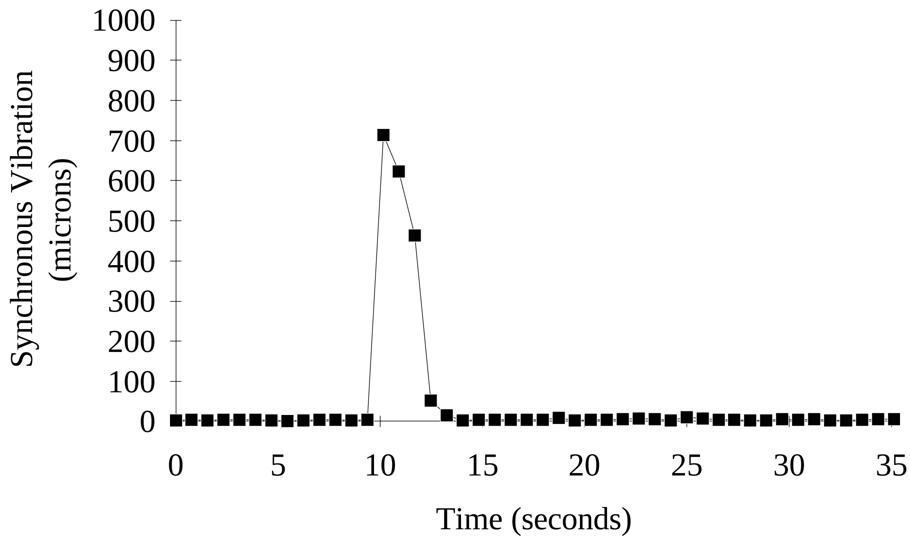

Figure 5: Synchronous midspan vibration amplitude with the R-SEC algorithm during the simulated change in balance, 2200 rpm.

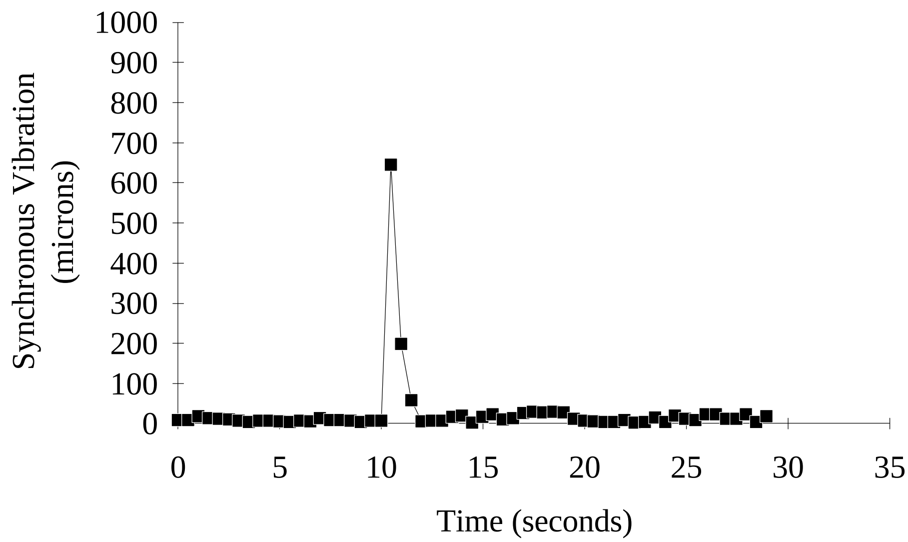

Figure 6: Synchronous midspan vibration amplitude with the R-GS algorithm during the simulated change in balance, 2200 rpm.

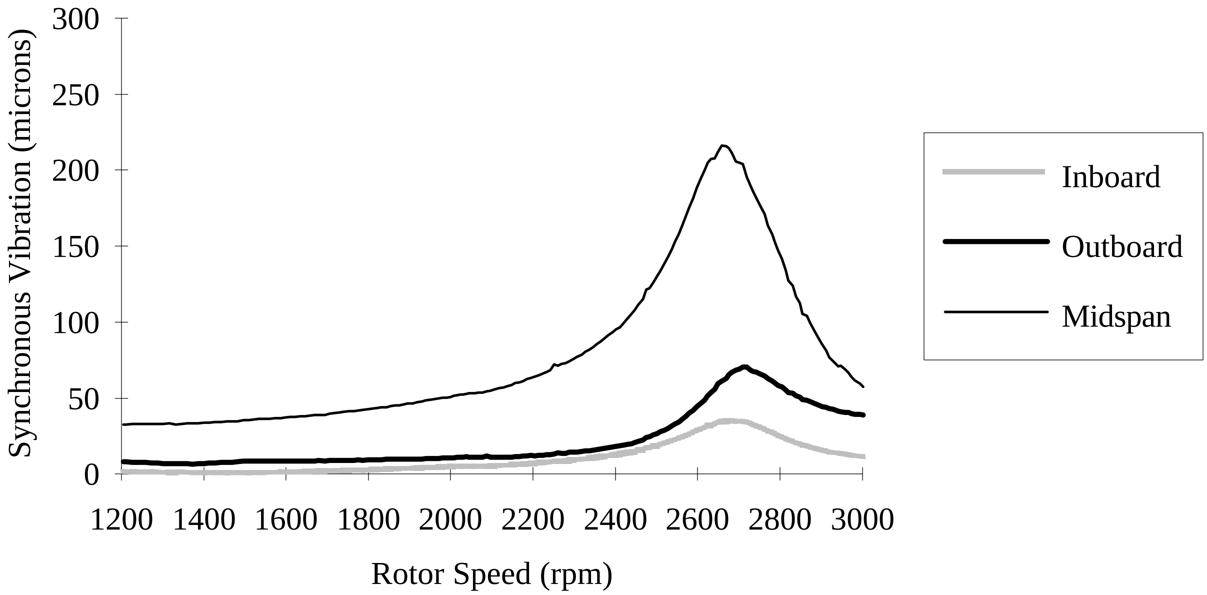

Figure 7: Synchronous vibration amplitudes without adaptive open loop control during a 30 second run-up from 1200 to 3000 rpm.

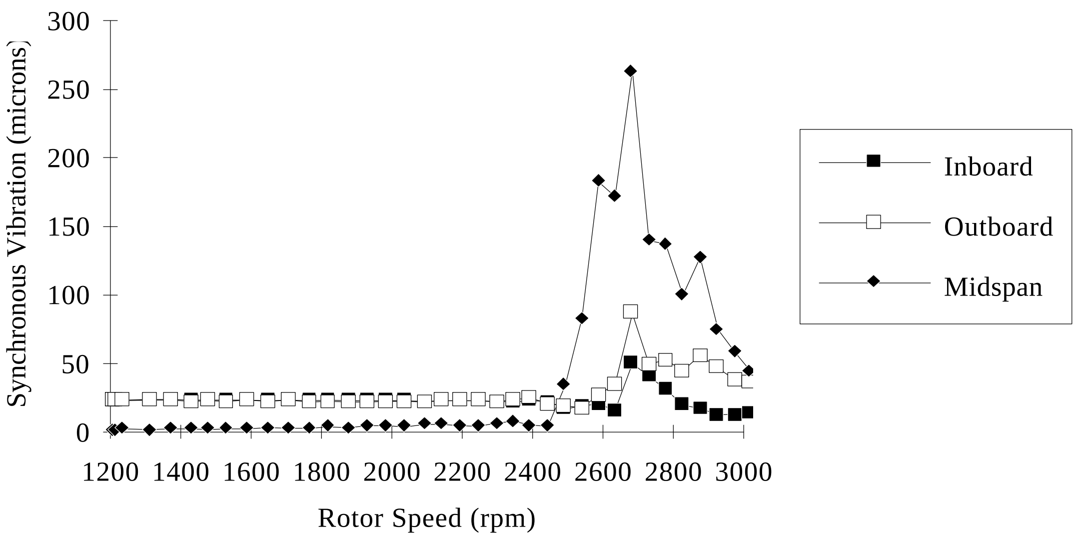

Figure 8: Synchronous vibration amplitudes with the NR-SEC algorithm during a 30 second run-up from 1200 to 3000 rpm.

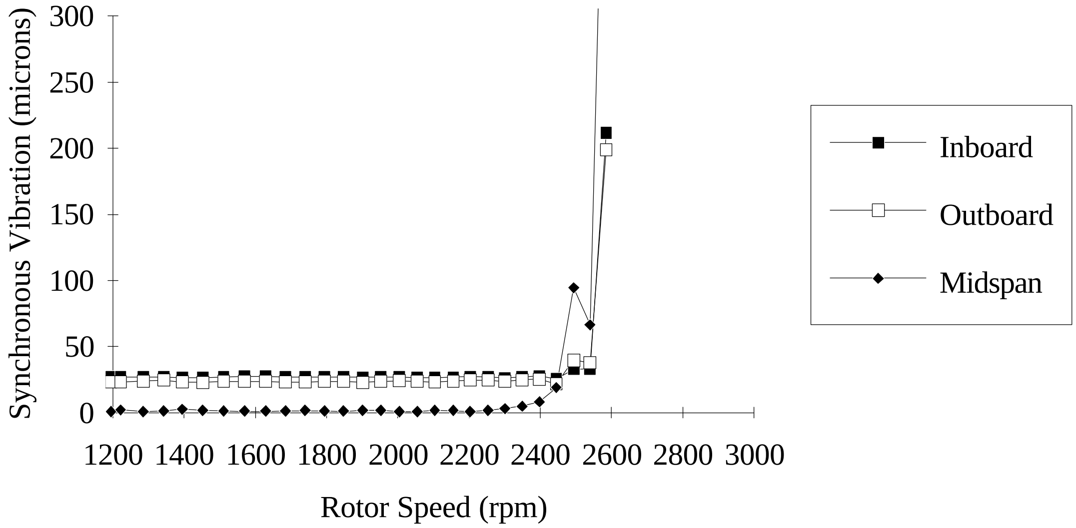

Figure 9: Synchronous vibration amplitudes with the R-SEC algorithm during a 30 second run-up from 1200 to 3000 rpm.

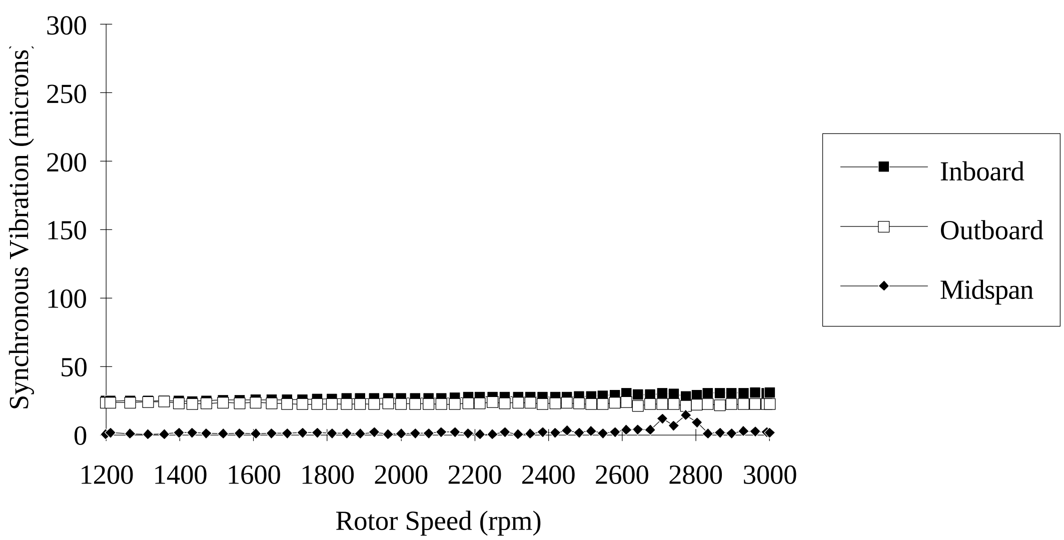

Figure 10: Synchronous vibration amplitudes with the R-GS algorithm during a 30 second run-up from 1200 to 3000 rpm.