Optimum Resource Allocation for Relay Networks with Differential Modulation

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***Abstract*—In this paper, we investigate the resource allocation in a differentially modulated relay network. In addition to the energy optimization, we also consider location optimization to minimize the average symbol error rate (SER). The closed-form solution is derived for the single-relay case, and formulas allowing numerical search are provided for multiple-relay cases. Analytical and simulated comparisons confirm that the optimized systems provide considerable improvement over the unoptimized systems, and that the minimum SER can be achieved via the joint energy-location optimization.**

***Index Terms*—Differential phase shift keying, resource management, relays, cooperative system.**

I. INTRODUCTION

RELAY networks rely on virtual antenna arrays to provide diversity gains without imposing antenna packaging limitations [8]. To reduce communication overhead and transceiver complexity, cooperative diversity schemes relying on noncoherent or differential modulations have recently been introduced to obviate the need for channel state information (CSI) [3], [10], [11].

Optimum resource allocation emerges as an important direction for the improvement of the performance and energy efficiency in relay networks. In the context of coherent relay networks with full CSI, the optimum power allocation was developed in [6] for Gaussian parallel relays, the optimum allocation of energy and bandwidth in multihop links is considered in [4], and more recently, the opportunistic relay selection was introduced in [2]. In this paper, we consider relay systems with differential modulation that does not require any CSI. Instead of focusing only on the energy allocation, we formulate the relay location optimization as a parallel problem to the energy allocation optimization. The resource allocation will be optimized based on an upper bound of the overall symbol error rate (SER) of a relay network employing the decode-and-forward (DF) protocol. For single-relay scenario, we derive a closed-form optimum solution. For multi-relay cases where the relays are placed together, we provide formulas allowing numerical searches for the optimum solution. We will also show that, though the optimizations can be carried out independently, the joint energy and location optimization gives the optimum SER performance.

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II. SYSTEM MODEL

Consider a network with one source node s, L relay nodes $\{r_k\}_{k=1}^{L}$, and one destination node d. The DF relaying protocol is considered, in which the relay nodes de-modulate the signal from the source, then re-modulate and forward to the destination. We assume that obstacles disable a direct transmission resulting in no direct link between the source node and the destination node.

With the nth phase-shift keying (PSK) modulated symbol being $s_n = e^{j2\pi c_n/M}$, $c_n \in \{0,\ldots,M-1\}$, the corresponding transmitted signal from the source is $x_n^s = x_{n-1}^s s_n$ with initial value $x_0^s = 1$. Each symbol duration is partitioned into two segments. During the first segment, the source broadcast the symbol via a common channel to all relays, and the received signal at the kth relay is given by

$$y_n^{r_k,s} = \sqrt{\mathcal{E}_s} h_n^{r_k,s} x_n^s + z_n^{r_k},\ k = 1,2,\ldots,L\,, \tag{1}$$

During the second segment, the relays form estimates $\hat{s}_n^{r_k}$, differentially modulate them as $x_n^{r_k}$, and forward to the destination through their distinct channels. The received signal at the destination corresponding to each relay node is given by

$$y_n^{d,r_k} = \sqrt{\mathcal{E}_{r_k}} h_n^{d,r_k} x_n^{r_k} + z_n^d,\ k = 1,2,\ldots,L. \tag{2}$$

In (1) and (2), $\mathcal{E}_i$ is the energy per symbol at node i, the fading coefficient $h_n^{i,j}$ and noise z_n^i are zero-mean complex Gaussian with variance $\sigma^2_{h_{i,j}}$, $\forall i,j \in \{s,r_k,d\}$, and $\mathcal{N}_0$, respectively. Accordingly, the received instantaneous signal-to-noise ratio (SNR) between the transmitter j and the receiver i is $\gamma_{i,j} = (|h_n^{i,j}|^2\mathcal{E}_j)/\mathcal{N}_0$, and the average received SNR is $\bar{\gamma}_{i,j} = (\sigma^2_{h_{i,j}}\mathcal{E}_j)/\mathcal{N}_0$.

Given that the channel is slowly time-varying and the fading coefficient remains nearly invariant over two consecutive symbols, the conditional distribution of the received signal y_n is complex Gaussian with mean $y_{n-1}s_n$ and variance $2\mathcal{N}_0$. Hence, the log likelihood function (LLF) of y_n is $l_m^{i,j}(y_n) = \Re\{y_n y_{n-1}^* e^{-j2\pi m/M}\}$, where $i,j \in \{s,r_k,d\}$ and $m \in \{0,1,\ldots,M-1\}$. Then, the differential demodulator at the kth relay and the destination nodes are given by

$$\begin{aligned}
\hat{s}_n^{r_k} &= e^{j2\pi m'/M} : m' = \arg\max_m l_m^{r_k,s}(y_n^{r_k,s}) \\
&= \arg\max_m \Re\{(y_n^{r_k,s})^* y_{n-1}^{r_k,s} e^{j2\pi m/M}\} \\
\hat{s}_n^{d} &= e^{j2\pi m'/M} : m' = \arg\max_m \sum_{k=1}^{L} l_m^{d,r_k}(y_n^{d,r_k}) \\
&= \arg\max_m \sum_{k=1}^{L} \Re\{(y_n^{d,r_k})^* y_{n-1}^{d,r_k} e^{j2\pi m/M}\}.
\end{aligned} \tag{3}$$

With no CSI assumed at either the relays or the destination node, the decision rule in (3) turns out to be the differential detection with postdetection equal gain combining (EGC) [9, Ch. 9].

III. Error Performance Analysis

Let us denote the average SER at the kth relay as P_{e,r_k}. For differential M-ary PSK (DMPSK) signaling, the $s-r_k$ link SER P_{e,r_k} can be obtained as in [9, Ch. 8]. At the destination, provided that the symbol s_n is correctly demodulated at all relays, the conditional SER $P_{e,d}$ can be obtained using the results in [7, Appendix C]. Based on these, we formulate an upper bound on the average SER P_e at the destination as follows:

Proposition 1 *With any given P_{e,r_k} and $P_{e,d}$, an upper bound on P_e can be found as :*

$$P_e \leq \bar{P}_e = 1 - \prod_{k=1}^{L}(1-P_{e,r_k})(1-P_{e,d}). \tag{4}$$

This is an upper bound since the cases where the $r_k - d$ link error corrects the $s-r_k$ link error are ignored. Fig. 1 plots the SER bound as a function of both $\bar{\gamma}_{d,r_k}$ and $\bar{\gamma}_{r_k,s}$ when $L=2$, where we assume that all $r_k - d$ and $s-r_k$ links have the same SNR; i.e., $\bar{\gamma}_{d,r_k} = \bar{\gamma}_{d,r}$ and $\bar{\gamma}_{r_k,s} = \bar{\gamma}_{r,s}, \forall k$. Notice that the surface flattens along the $\bar{\gamma}_{d,r}$ axis, but keeps descending along the $\bar{\gamma}_{r,s}$ axis. These observations suggest that the overall error performance of the DF based cooperative system depends more on the $s-r$ link than the $r-d$ link. Such unbalanced effects of the relay links confirm that optimum resource allocation is critical in achieving the optimum error performance.

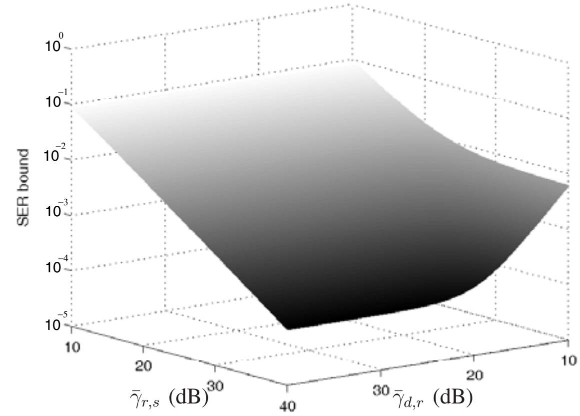

Fig. 1. SER bound versus $\bar{\gamma}_{d,r}$ and $\bar{\gamma}_{r,s}$ ($L=2, M=2, \bar{\gamma}_{d_k,r} = \bar{\gamma}_{d,r}$ and $\bar{\gamma}_{r_k,s} = \bar{\gamma}_{r,s}, \forall k$).

IV. Optimum Resource Allocation

In this section, we will investigate the effects of energy allocation and relay location on the SER performance. For analytical tractability, we consider a network topology where all relay nodes are located at the same distance from the source and the destination nodes; that is, $D_{s,r_k} = D_{s,r}$ and $D_{r_k,d} = D_{r,d}$, $\forall k$. Even with such a topology, resource optimization among relays is possible as in [2]. However, in the absence of CSI, it is reasonable to assign equal energies to all relays $\mathcal{E}_{r_k} = \mathcal{E}_r$, $\forall k$.

To carry out the optimization, we will also exploit the relationship between the variance of the channel fading coefficient and the inter-node distance: $\sigma^2_{h_{i,j}} = C \cdot D_{j,i}^{-\nu}, \;\; i,j \in \{s,r,d\}$, where ν is the path loss exponent of the wireless channel and C is a constant which we henceforth set to 1 without loss of generality.

A. Energy Allocation Optimization

Problem Statement 1 *For any given source, relay and destination node locations ($D_{s,r}$ and $D_{r,d}$), and the total energy per symbol $\mathcal{E}$, determine the optimum energy allocation $\mathcal{E}_s^o$ and $\mathcal{E}_r^o$ which minimize $\bar{P}_e$ in Eq.* (4) *while satisfying the total energy constraint : $\mathcal{E}_s + L\mathcal{E}_r = \mathcal{E}$.*

Clearly, the energy constraint is equivalent to the SNR constraint: $\rho = \rho_s + L\rho_r$, where $\rho := \mathcal{E}/\mathcal{N}_0$, $\rho_s := \mathcal{E}_s/\mathcal{N}_0$, and $\rho_r := \mathcal{E}_r/\mathcal{N}_0$. To gain some insight, we consider a single-relay setup with DBPSK and establish the following result:

Proposition 2 *For a single-relay setup with $L=1$, at given $s-r$ and $r-d$ distances $D_{s,r}$ and $D_{r,d}$, and under the total energy constraint, the optimum energy allocation ρ_s^o should satisfy Eq.* (5)[1] and correspondingly, $\rho_r^o = \rho - \rho_s^o$.

Although (5) is an exact solution for any $\mathcal{E}$ and $\mathcal{N}_0$ values and $s-r$ and $r-d$ distances, its complex form does not provide much insight. Under the high SNR assumption, we neglect the constant terms in (5) and obtain an approximate solution for the optimum energy allocation

$$\rho_s^o = \frac{D_{r,d}^{-\nu/2}}{D_{s,r}^{-\nu/2} + D_{r,d}^{-\nu/2}} \cdot \rho \Leftrightarrow \mathcal{E}_s^o = \frac{D_{r,d}^{-\nu/2}}{D_{s,r}^{-\nu/2} + D_{r,d}^{-\nu/2}} \cdot \mathcal{E}\ . \tag{6}$$

Interestingly, this solution coincides with the optimum power allocation obtained by minimizing the outage probability [5, (8)] and by minimizing the error probability bound of the coherent (de-)modulation [1, (8)].

From (6), it readily follows that the optimum energy allocation ratio between the source and the relay nodes satisfies

$$\frac{\mathcal{E}_s^o}{\mathcal{E}_r^o} = \left(\frac{D_{s,r}}{D_{r,d}}\right)^{\nu/2}. \tag{7}$$

Eq. (7) reveals explicitly that the optimum energy allocation heavily hinges upon the inter-node distances. In addition, the path loss exponent of the wireless channel, ν , also affects the optimum energy allocation.

B. Relay Location Optimization

Problem Statement 2 *For any given transmit energies at the source and relay nodes ($\mathcal{E}_s$ and $\mathcal{E}_r$), and the path loss exponent ν of the wireless channel, determine the optimal location of the relays, $D_{s,r}^o$, which minimizes $\bar{P}_e$ in Eq.* (4) *while satisfying $0 < D_{s,r}^o < D_{s,d}$.*

Considering the single-relay setup with DBPSK and applying the high-SNR approximation, we establish the following result:

[1]This solution can be readily obtained using the Lagrange multiplier. The detailed proof is omitted due to the space limit.

$$\rho_s^o = \sqrt{\frac{2D_{r,d}^{-2\nu} + D_{s,r}^{-\nu} D_{r,d}^{-\nu}(6D_{r,d}^{-\nu}\rho + 5) + 2D_{s,r}^{-2\nu}(2D_{r,d}^{-2\nu}\rho^2 + 3D_{r,d}^{-\nu}\rho + 1)}{4D_{s,r}^{-\nu} D_{r,d}^{-\nu}(D_{s,r}^{-\nu} - D_{r,d}^{-\nu})^2}} - \frac{2D_{r,d}^{-\nu}\rho + 3}{2(D_{s,r}^{-\nu} - D_{r,d}^{-\nu})}\,, \tag{5}$$

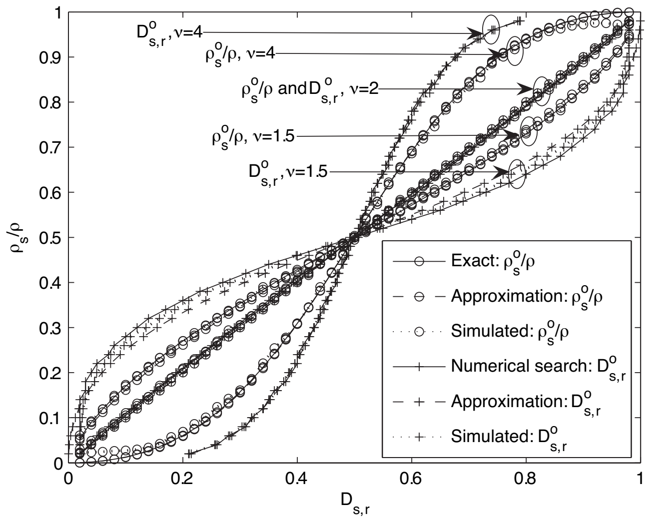

Fig. 2. Optimum energy allocation and relay location ($\rho = 10$dB, $L = 1$, $\nu = 1.5, 2, 4$).

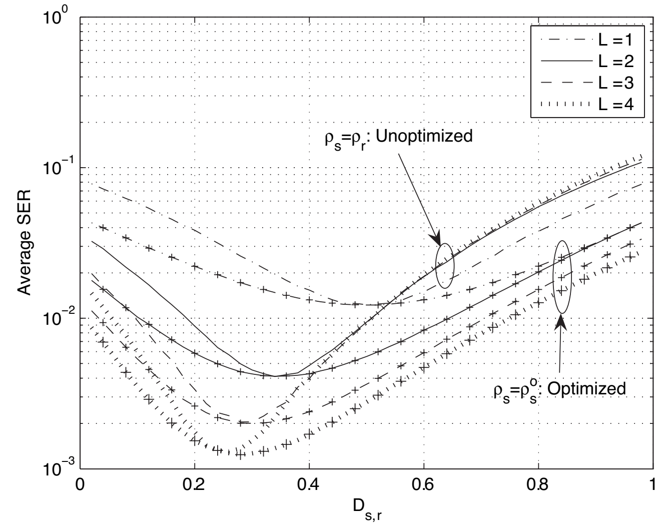

Fig. 3. SER comparison between relay systems with and without energy optimization ($\rho = 10$dB, $\nu = 4$).

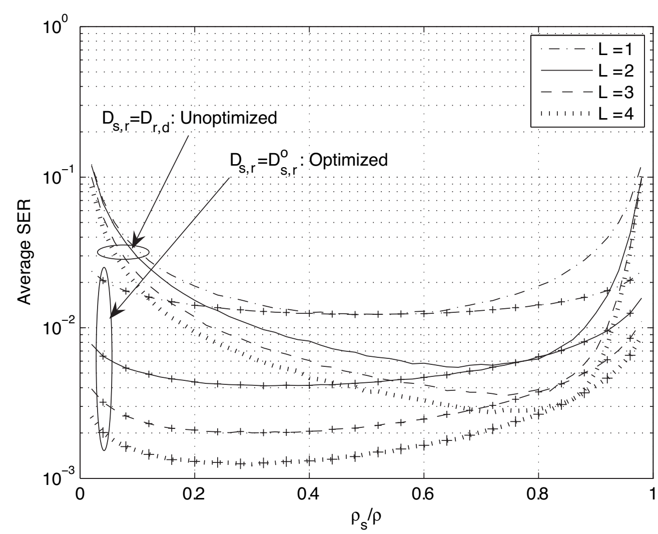

Fig. 4. SER comparison between relay systems with and without location optimization ($\rho = 10$dB, $\nu = 4$).

Proposition 3 *For a single-relay setup with $L = 1$ and the $s-d$ distance $D_{s,d}$, and given the prescribed transmit energy $\mathcal{E}_s$ and $\mathcal{E}_r$, the optimum location of the relay is*

$$D_{s,r}^o = \frac{\rho_s^{1/(\nu-1)}}{\rho_s^{1/(\nu-1)} + \rho_r^{1/(\nu-1)}} \cdot D_{s,d}\,, \tag{8}$$

and accordingly, $D_{r,d}^o = D_{s,d} - D_{s,r}^o$.

Alternatively, (8) can be represented as

$$\frac{D_{s,r}^o}{D_{r,d}^o} = \left(\frac{\rho_s}{\rho_r}\right)^{1/(\nu-1)} = \left(\frac{\mathcal{E}_s}{\mathcal{E}_r}\right)^{1/(\nu-1)}\,. \tag{9}$$

Eq. (9) bears a very similar form as its counterpart for the optimum energy allocation in (7). In fact, when the path loss exponent $\nu = 2$, they are identical. However, for general ν values, (7) and (9) are quite different. Such a discrepancy is actually very reasonable, because (7) and (9) result from two distinct optimization problems.

C. Discussions and Extensions

The optimum energy allocation and relay location are depicted in Fig. 2 when $\rho = 10$dB and $L = 1$, where lines with circle mark represent the optimum energy allocation and lines with plus mark represent the optimum relay location, respectively. As mentioned before, these curves overlap when $\nu = 2$. For all cases, Fig. 2 shows that the approximated values are nearly identical to the exact values and the Monte Carlo simulations. For general L values, the path loss exponent ν renders it impossible to derive the closed-form optimum solution, even with the high SNR approximation. In such cases, one can resort to the numerical search using the SER bound in Proposition 1.

It is worth stressing that, the direct link is ignored in our considerations. The relay location optimization is independent of the direct link, because the relay link(s) and the direct link are parallel for any given energy distribution. The energy allocation, however, can be considerably affected by the inclusion of the direct link, especially when the relay is close to the source node. In the extreme case, where $D_{s,r}$ approaches zero and SNR is sufficiently high, uniform energy allocation becomes optimum.

As aforementioned, the energy and location optimizations can be performed individually. However, the minimum SER can only be achieved through joint energy and location optimization. For $L = 1$, Eqs. (7) and (9) suggest that, when $\nu = 2$, any relay location is equally good in terms of minimizing SER, as long as the energy allocation satisfies (7); whereas, for other ν values, it is always optimum to place the relay at the midpoint of the $s - d$ link. For general L values, the joint optimization can be carried out in an iterative manner: i) find the optimum relay location, and ii) optimize the energy distribution based on the updated location. Our simulations show that the SER function is generally convex, which ensures convergence. In addition, we will see that the uniform energy allocation is a very good starting point for the iterative optimization.

V. Simulations

Figs. 3 and 4 depict the SER of the relay system with and without energy and location optimizations, respectively. The system parameters are: $\rho = 10$dB, $L = (1, 2, 3, 4)$, $D_{s,d} = 1$ and $\nu = 4$. In the system without energy optimizations, a uniform energy allocation $\rho_s = \rho_r = \rho/(L+1)$ is considered. In the system without location optimization, the relays located at the midpoint of the source-destination link are considered. From these figures, we observe that the optimized system universally outperforms the unoptimized system with the same total energy.

Notice that the minima of the energy-optimized SER curves almost coincide with those of the unoptimized system. This implies that (near-)optimum SER can be achieved even with the uniform energy allocation across the source and relay nodes, provided that the relay location is carefully selected. However, the minima of the location-optimized SER curves are generally far from those of the upoptimized system (see Fig. 4). These indicate that placing the relay nodes at the midpoint *cannot* achieve the minimum SER even with careful energy allocation. In addition, these observations suggest that the uniform energy allocation is an ideal starting point for the iterative energy-location optimization.

VI. Conclusions

In this paper, we investigated the optimum energy distribution and optimum location of relays in a system employing differential demodulation. Our simulations confirm that both the energy and location optimizations provide considerable SER advantages. Without optimization, the system with more relays may at times underperform the system with less or no relays. We also showed that the minimum SER can be achieved by the joint energy-location optimization, and that the location optimization is generally more critical than the energy optimization.

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