Precoding Mapping Optimization for Magnetic Recording Channels

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We study the problem of designing a rate-1 block-precoder to minimize bit/symbol error rate when storing a given source on a magnetic recording channel. A block-precoder of length b-bits is defined by a permutation π on 2^b blocks. We show that the problem of finding a permutation for the block-precoder that minimizes bit/symbol error rate is equivalent to solving the quadratic assignment problem, a known combinatorial optimization problem that is NP-complete. We exploit the symmetry group of the b-dimensional hypercube to reduce the search space, allowing a branch-and-bound technique to find the optimal 5-bit precoders. We also implement a local search algorithm that can find good precoders for larger blocklengths. We design precoders for MTR-constrained user bits and unconstrained parity bits with a reverse-concatenation architecture, and we evaluate the resulting SNR gains in a turbo equalization scheme.

Index Terms—**Error-correction codes, magnetic recording, modulation coding, precoding, reverse concatenation (RC).**

I. Introduction

In many magnetic recording applications, low-density parity check (LDPC) codes are used as an outer code in a turbo equalization scheme, where the partial response channel is treated as the inner code. In high linear density regimes, the performance of iterative detection-decoding schemes has been shown to be improved by the use of precoders [1] and modulation codes [2]. In this paper, we study the problem of designing rate-1 block-precoders that minimize the error-rate as seen by the outer code. We show that designing optimal precoders is equivalent to solving a combinatorial optimization problem. For small block lengths, we derive good precoders that give noticeable SNR gains over the $1/(1 \oplus D)$ precoder studied in [1] with negligible increase in the complexity. Our setup allows us to design precoders that improve raw bit/symbol error rates for both parity bits as well as the MTR-constrained bits [3].

This paper is organized as follows. In Section II, we formulate the system model that is used to define a block-precoder and quantify its performance. In Section III, we describe the various approaches we take to find good precoders. In Section IV, we show how the proposed precoders may be incorporated in the encoding and decoding architectures used for magnetic recording applications. Section V shows the performance of the proposed precoders over magnetic recording channels through simulation results.

II. System Model

Let $\bar{X} = \{X_k\}_{k\in\mathbb{N}}$ denote the outputs of a source that has alphabet $\mathbb{F}_2^b$, for some $b \in \mathbb{N}$ such that X_k are independent and identically distributed according to the probability distribution function $\mathfrak{p}_X$ for all k. This source is transmitted on a magnetic recording channel, modeled as a partial response (PR) channel with data-dependent colored noise, and the received sequence is passed to the bit-wise MAP sequence detector with noise prediction. Let the output from the sequence detector be denoted by $\bar{Y} = \{Y_k\}_{k\in\mathbb{N}}$, with $Y_k \in \mathbb{F}_2^b$. We denote the channel from $\bar{X}$ to $\bar{Y}$ as the equivalent channel $\mathcal{W}$ as shown in Fig. 1.

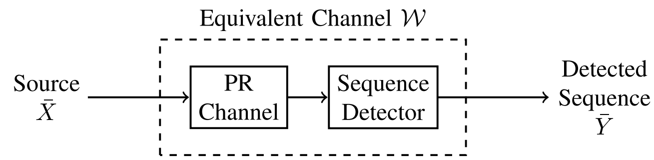

Fig. 1. Equivalent channel from source $\bar{X}$ to detected sequence $\bar{Y}$.

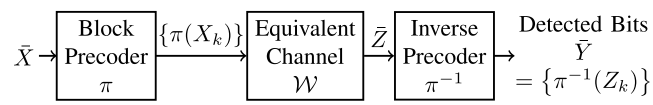

Fig. 2. Channel model with block-precoder π.

Let $\mathbf{P} = [p_{i,j}]$ denote the equivalent channel transition probability matrix, where

$$p_{i,j} = \mathbf{Pr}\,(Y_k = j \mid X_k = i) \cdot \mathfrak{p}_X(i) \tag{1}$$

for $1 \le i, j \le 2^b$. Although $\mathbf{P}$ is often not available in closed form, it can be calculated numerically using Monte Carlo techniques. Let $\mathbf{H} = [h_{i,j}]$ denote the Hamming distance matrix, where $h_{i,j}$ denotes the Hamming distance between binary sequences i and j, for $i, j \in \mathbb{F}_2^b$. Then, the bit error rate, $d(\bar{X}, \bar{Y})$, can be expressed as

$$d(\bar{X}, \bar{Y}) \triangleq \frac{1}{b}\langle \mathbf{H}, \mathbf{P}\rangle = \frac{1}{b}\sum_{i,j} h_{i,j}\, p_{i,j} \tag{2}$$

where $\langle\cdot,\cdot\rangle$ denotes the Frobenius product of two matrices.

The block-precoder is defined by a permutation $\pi : \mathbb{F}_2^b \mapsto \mathbb{F}_2^b$. Fig. 2 shows the system model with the precoder. The data $\bar{X}$ is grouped into b-bit blocks and each block is precoded using a rate-1 encoder given by π.

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At the decoder, the output of the equivalent channel, denoted by $\bar{Z}$, is then inverse precoded to get $\bar{Y} = \{Y_k\}$, where each $Y_k = \pi^{-1}(Z_k)$. Note that the precoder is different from an interleaver; the former replaces b-bit blocks with other blocks, while the latter shuffles the bit sequence. The bit error rate of the scheme with the precoder, $d_\pi(\bar{X}, \bar{Y})$, is given by

$$d_\pi(\bar{X}, \bar{Y}) \triangleq \frac{1}{b}\sum_{i,j} h_{i,j}\, p_{\pi(i),\pi(j)} = \frac{1}{b}\langle \mathbf{H}, \Pi\mathbf{P}\Pi^{\mathsf{T}}\rangle \tag{3}$$

where Π will be used to denote the permutation matrix associated with the permutation π.

III. Precoder Design

The objective of the precoder design is to find a precoder that minimizes the bit error rate. The optimal precoder, denoted by π^*, is given by

$$\pi^* \triangleq \arg\min_{\pi \in \mathcal{S}_{2^b}} d_\pi(\bar{X}, \bar{Y}) = \arg\min_{\pi \in \mathcal{S}_{2^b}} \sum_{i,j} h_{i,j}\, p_{\pi(i),\pi(j)} \tag{4}$$

where $\mathcal{S}_{2^b}$ is the symmetric group of permutations on 2^b elements. Searching for the optimal precoder by enumeration of all possible $(2^b)!$ permutations is infeasible, even for small values of b. In fact, the optimization problem in (4) is a known combinatorial optimization problem, called the Koopmans–Beckmann quadratic assignment problem (QAP) [4]–[6]. The QAP is in general an NP-complete problem. In addition, any method that gives an ϵ-approximate solution for QAP is NP-complete [7]. Methods for solving this problem can be divided into two classes: 1) exact algorithms and 2) suboptimal algorithms. We discuss these in the following sections.

A. Exact Algorithms

Naive enumeration of all $(2^b)!$ permutations gives us the exact solution, but it is only feasible to perform when $b \leq 3$. For $b = 4$, the search space consists of over 20 trillion permutations as candidates for the solution. This search space can be reduced by noting that for our problem the matrix $\mathbf{H}$ has a special structure since it consists of the Hamming distance between all b-bit blocks. The symmetry group of a b-dimensional hypercube defines a set of permutations $\mathcal{I} \subset \mathcal{S}_{2^b}$ with cardinality $|\mathcal{I}| = b! \cdot 2^b$ such that for all $\pi \in \mathcal{I}$, $\Pi \cdot \mathbf{H} \cdot \Pi^{\mathsf{T}} = [h_{\pi(i),\pi(j)}] = \mathbf{H}$. Then, for any permutation $\pi'' \in \mathcal{S}_{2^b}$ there are $|\mathcal{I}|$ distinct permutations $\pi' \in \mathcal{I}$ such that $d_{\pi'' \circ \pi'^{-1}}(\bar{X}, \bar{Y}) = d_{\pi''}(\bar{X}, \bar{Y})$. This reduces the search space for the optimization problem in (4) from $(2^b)!$ to $(2^b)!/(b! \cdot 2^b)$.

After reducing the search space, we used a branch-and-bound technique where a given QAP was divided into multiple QAPs with fewer degrees of freedom by partial assignment, and the Gilmore-Lawler lower bounds on the achievable cost were obtained for these reduced QAPs [8], [9]. Details of the techniques used are available in [5].

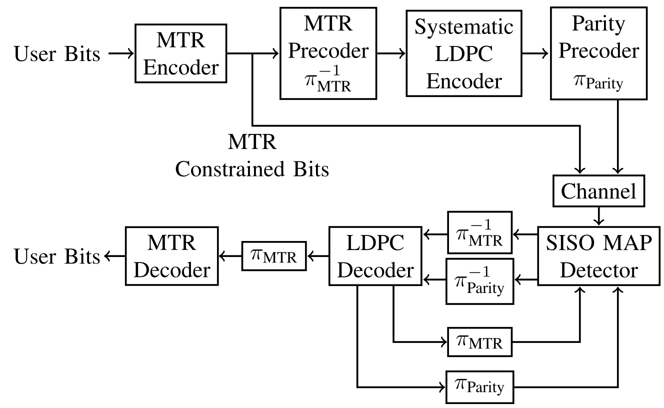

Fig. 3. RC architecture with separate block-precoders, π_{MTR} and π_{Parity}, for MTR-constrained user bits and unconstrained parity bits, respectively.

B. Heuristic Algorithms

The branch-and-bound algorithm described above let us find optimal block-precoders for up to $b = 5$ bits. QAP problems of higher dimensions are not practical to solve optimally because of the higher running time requirements. This necessitates the use of heuristic algorithms that provide good solutions in a reasonable time, although they do not provide any guarantees of optimality. Many types of heuristic algorithms have been proposed for solving QAPs [5]. We used the tabu search algorithm, first introduced in [10]. Tabu search is a local search algorithm that starts with an initial permutation and iteratively tries to improve it by finding a better permutation in its neighborhood. The algorithm maintains a tabu list where it lists forbidden transpositions on the current solution. Transpositions are moved out of the list after some iterations. Tabu search was used to find good precoders for up to $b = 6$ bits. For $b \leq 5$, these precoders were confirmed to be optimal using exact algorithms.

IV. Block-Precoders in Magnetic Recording

A. Precoding in Reverse Concatenation Architecture

We consider the reverse concatenation (RC) architecture [11] of magnetic recording channels, where the user bits are encoded using an MTR-constrained code [3] and parity bits are calculated on the MTR-constrained bits using a systematic error correction code. We design separate block-precoders for the MTR-constrained bit sequence and unconstrained parity bits that may be used independently. Fig. 3 shows the block diagram of the RC architecture with the block-precoders. For error-correction, we consider LDPC codes decoded using sum-product algorithm decoding. For the detector, a soft-input soft-output MAP detector like SOVA [12] is used with a trellis that represents the possible MTR-constrained sequences at the output of a PR channel. The SOVA is modified to appropriately encode the survivor and loser paths using the chosen block-precoder. Fig. 4(a) and (b) shows the channel models observed by the MTR-constrained bits and the parity bits between the LDPC encoder and the LDPC decoder in the RC architecture. These diagrams can be compared with the generic channel model presented earlier in Fig. 2. For all simulations, we used

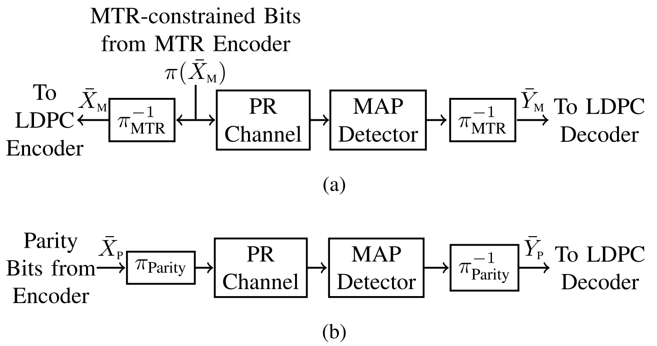

Fig. 4. Schematic diagrams showing the effect of the precoders in the RC architecture on (a) the MTR-constrained user bits, (b) the unconstrained parity bits.

$h(D) = 6 + 11D + 5D^2$ as the PR target and assume 70% jitter noise.

B. Minimizing Symbol Error Rate

An extension of the system model presented in Section II is one where sequences $\bar{X}, \bar{Y}$ have alphabet $(\mathbb{F}_q)^m$ where q is some power of a prime. Such a system is of importance for the use of non-binary LDPC codes in magnetic recording applications [13] where minimizing the raw symbol error rate is of concern. For such a model the elements of matrix $\mathbf{H} = [h_{i,j}]$ are defined as the Hamming distance between non-binary sequences i and j, for $i, j \in (\mathbb{F}_q)^m$. Using this $\mathbf{H}$ matrix, the cost function $d_\pi(\bar{X}, \bar{Y})$, defined according to (3), is proportional to the symbol error rate. The optimal precoder that minimizes the symbol error rate is then given by the optimization problem in (4) and was found to be different from the block-precoder optimized for minimizing the bit error rate.

V. Simulation Results

A. Precoders for Parity Bits

Consider a bit sequence $\bar{X}_{\text{P}}$ generated using an independent uniformly distributed (i.u.d.) source with alphabet $(\mathbb{F}_2)^b$. This models the distribution of the parity bits of a linear block code. This bit sequence is transmitted through the channel shown in Fig. 4(b) and the average bit error rate, $d(\bar{X}_{\text{P}}, \bar{Y}_{\text{P}})$, is calculated by simulation. Fig. 5 shows the raw bit error rates for the 6-bit block-precoder and the $1/(1 \oplus D)$ precoder, where $+ - +$ is the dominant error event without any precoding. The proposed block-precoder has a gain of 0.5 dB over the scheme with no precoding and 0.2 dB compared to the $1/(1 \oplus D)$ precoder at low SNRs. Though the $1/(1 \oplus D)$ precoder reduces the effect of the $+ - +$ error, the proposed block-precoder can reduce the effect of other high weight error events as well.

As discussed in Section IV-B, a non-binary source $\bar{X}_{\text{P}}$ may be of interest. We consider block-precoders that map 3 symbols of $\mathbb{F}_4$ to other symbols. The precoder that minimizes the symbol error rate is found using the techniques described in Section III. Fig. 6 shows the symbol error rates for the best 3-symbol block-precoder we found and for the bit-wise use of the $1/(1 \oplus D)$ precoder, where the source $\bar{X}_{\text{P}}$ randomly generates i.u.d. symbols from $\mathbb{F}_4$. As shown in the Fig. 6, the

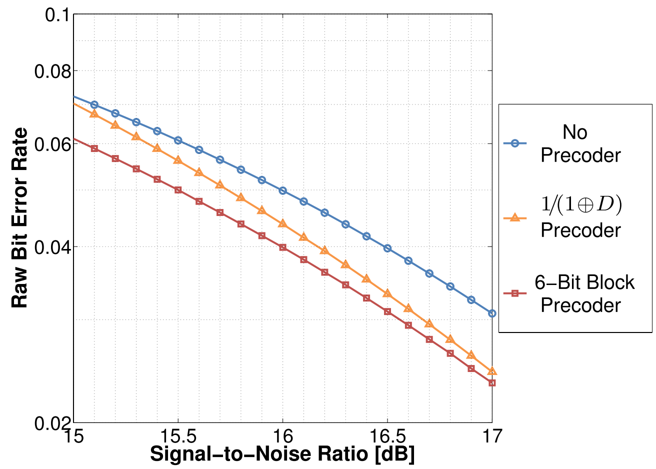

Fig. 5. Raw bit error rate at the output of the detector with various precoders for an i.u.d. source $\bar{X}$.

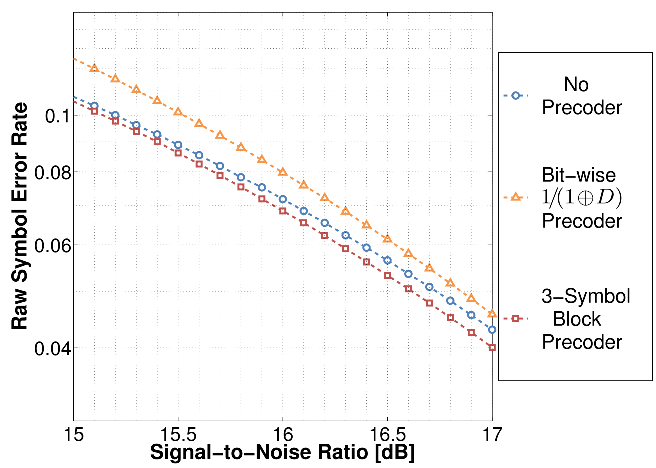

Fig. 6. Raw symbol error rate at the output of the detector with various precoders for an i.u.d. source $\bar{X}$ with alphabet $\mathbb{F}_4$.

$1/(1 \oplus D)$ precoder increases the symbol error rate compared with the scheme with no precoding.

B. Precoders for MTR-Constrained Bits

We now consider optimizing a different precoder for the MTR-constrained bits. MTR $(j = 3; k = \infty)$ constrained bits are generated using a high-rate finite-state encoder [1]. These bit sequences represent $\pi(\bar{X}_{\text{M}})$ in the channel model in Fig. 4(a). We find good block-precoders that have low error rates. Figs. 7 and 8 show the bit error rate for the best 6-bit precoder and the symbol error rate for the best 3-symbol precoder, respectively. Note that the gain for the MTR-constrained bits is lower than the gains for the parity bits since there are no error events with high weight that are dominant in this case, unlike the $+ - +$ error events for the latter. However, the MTR-constrained bits form a large fraction of the codeword bits for a high-rate code. Therefore, even smaller gains for them using precoding are useful.

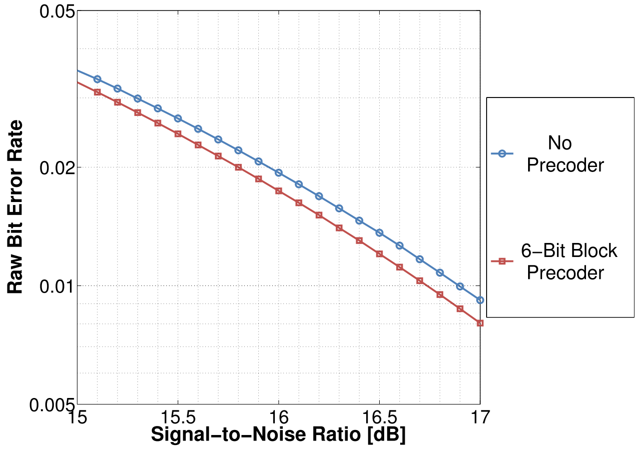

Fig. 7. Raw bit error rate at the output of the detector with a 6-bit precoder for a MTR-3 constrained sequence.

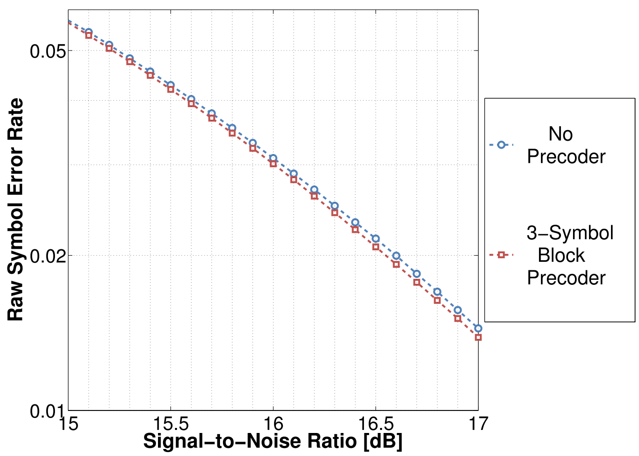

Fig. 8. Raw symbol error rate at the output of the detector with a 3-symbol precoder for MTR-3 constrained sequence where each symbol consists of 2 bits.

TABLE I

SNR GAINS FOR THE PROPOSED BLOCK-PRECODERS FOR MTR-CONSTRAINED BITS AND PARITY BITS AT VARIOUS CHANNEL BIT DENSITIES COMPARED WITH NO-PRECODING SCHEME

Channel Bit Density	SNR Gains (in dB)			
	MTR Bits		Parity Bits	
	BER	SER	BER	SER
1.151	0	0	0.043	0
1.319	0.001	0	0.130	0
1.532	0.052	0.007	0.514	0.052

C. SNR Gain Versus Channel Bit Density

Table I shows the SNR gains for the proposed block-precoders at various channel bit densities, for the channel models given in Fig. 4(a) and (b) for MTR-constrained bits and parity bits, respectively. These SNR gains are relative to no-precoding scheme and are measured at raw bit/symbol error rates 0.01. It is clear that the SNR gains increase as the channel bit density increases.

D. Precoders in RC Architecture

We consider transmission of a coded sequence over the channel model described in Section IV-A. A rate 0.9 $(3, 31)$-regular binary LDPC code of blocklength 4818 is used

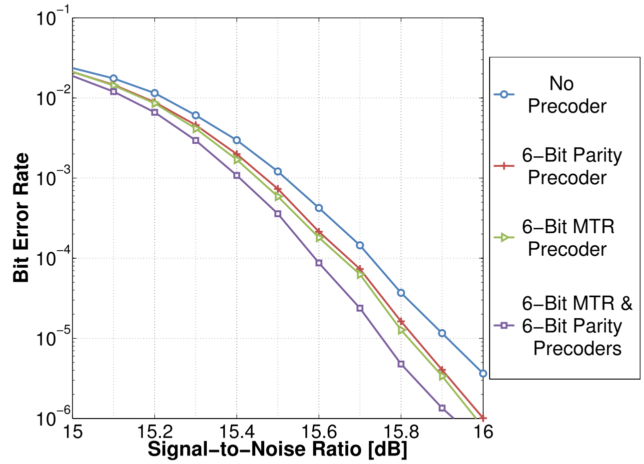

Fig. 9. Bit error rate on MTR-constrained user bits with various precoders and a rate 0.9 binary LDPC code decoded using turbo equalization.

in the RC architecture along with turbo equalization at channel bit density 1.532. We performed up to five global iterations, with 20 iterations of sum-product algorithm decoding in each global iteration. We measured the bit error rate on the MTR-constrained bits with and without a precoder on the MTR-constrained bits and/or parity bits. Fig. 9 shows the bit error rates for each of these cases, showing that the SNR gain is $\sim$0.1 dB for each precoder. This confirms that the proposed precoders give gains similar to those observed in raw bit error rates, even when multiple rounds of the detector and decoder are performed in turbo equalization.

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