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- Technical Paper -

ANALYSIS OF SHEAR CRACKING BEHAVIOR IN PARTIALLY PRESTRESSED CONCRETE BEAMS

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ABSTRACT

This paper presents an analytical simulation of shear cracking behavior of partially prestressed concrete beams using a numerical model, based on the Modified Compression Field Theory (MCFT). It is shown that this model can predict experimental results of the load displacement relationship, flexural cracking load, diagonal cracking load, stirrup strain, principal tensile strain, shear crack width, final failure crack pattern and failure mode consistently and satisfactorily.

Keywords: Modified Compression Filed Theory (MCFT), partially prestressed concrete (PPC), principal tensile strain, stirrup strain, shear crack width

1. INTRODUCTION

Cracking in concrete structures is unavoidable due to low tensile strength of concrete. The wide cracks allow water to penetrate into the structures and may cause corrosion of embedded steel reinforcements, which adversely affect the long-term durability performance. In fact, for concerned durability of the structures, controlling cracking is the most desirable issue. An extensive experimental program and a numerical study were carried to investigate the shear cracking behavior of Partially Prestressed Concrete (PPC or PRC) beams. PPC is generally defined as a combination of the prestressed and non-prestressed reinforcement in a concrete beam [4]. A PPC beam has a mechanism of shear cracking behavior that is more complex than the flexural cracking due to having an axial compression than that of reinforced concrete (RC) beams [2]. These complex mechanisms arise due to shear cracks which are not perpendicular to the beam axis and giving inclination to stirrups at the shear crack – stirrup intersection.

The numerical simulation was carried out using Modified Compression Field Theory (MCFT) based numerical model. The MCFT is a general model for the load deformation behavior of two-dimensional cracked reinforced concrete subjected to shear. It models the concrete considering the stresses in principal direction summed with reinforcing stresses assuming only axial stress. The most important assumption in the model is that the cracked concrete in reinforced concrete can be treated as a new material with empirically defined stress-strain behavior. The strains used for these stress-strain relationships of the numerical model are average strains, that is, they lump together the combined effects of local strain at cracks, strain between cracks, bond slip, and crack slip [3]. The calculated stresses are also average stresses in that numerical model implicitly include stresses between cracks, stresses at cracks, interface shear on cracks, and dowel action

In this paper, an attempt is made to evaluate the MCFT based Response-2000 numerical model to predict the shear cracking behavior of partially prestressed concrete beams. The numerical results obtained from the model were verified with experimental results. Furthermore, the numerical results obtained from the numerical model are based on the CEB-FIP (European Committee for Concrete – International Federation for Prestressing, 1990) model code [5] crack spacing model.

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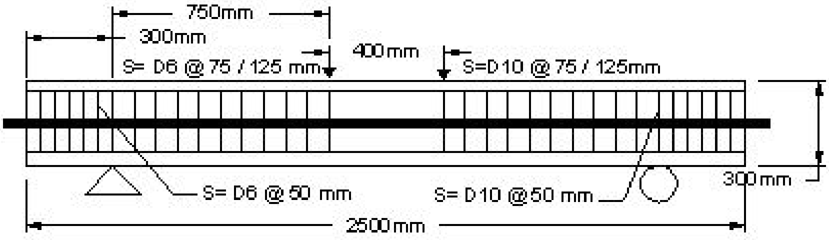


Fig. 1: Layout of the specimen

Table 1: Experimental variables

Specimen	f_c' (MPa)	Effective depth to prestressing bar (mm)	$\sigma_{c,ps}$ (MPa)	Stirrup spacing (mm)
RC-1	36.9	-	-	75
RC-2	37.5	150*	0	75
PRC-1	39.7	150	2.5	75
PRC-2	39.2			125
PRC-3	39.7		5.0	75
PRC-4	.37.8			125
PRC-5	38.6	200	2.5	75
PRC-6	40.5	150**		75

* no prestressing
** two external prestressing bars were used
$\sigma_{c,ps}$: compressive stress in concrete due to prestressing

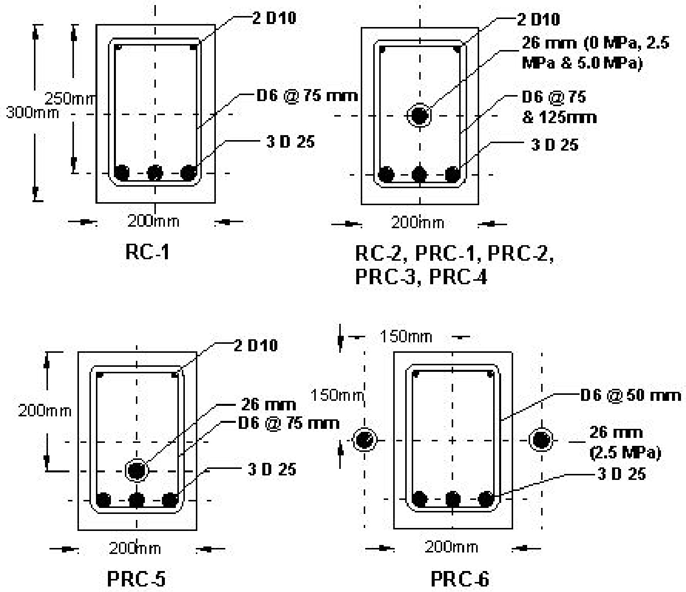


Fig. 2: Cross sectional details of the specimens

Table 2: Properties of reinforcements

Type of bar	ϕ (mm)	Type	f_y (MPa)	f_u (MPa)	Es x 10^3 (MPa)
Deformed bar	D6	SD 345	447.9	-	262.1
	D10		428.7	-	180.4
	D25	USD 685	720.0	-	206.0
PC bar	26	SBPR 1080/1230	1224.0	1277.0	200.0

The CEB-FIP crack spacing model used in the Response-2000 numerical model to determine the crack spacing in the RC beams was modified to determine the crack spacing in the PPC by taking account the effect of providing a prestressing bar for PPC beams. The crack spacing model (CEB-FIP) depends on the crack control characteristics of both the longitudinal and the transverse (stirrups) reinforcement. The proposed model takes the prestressing tendon into account to calculate the average crack spacing of the PPC.

2. EXPERIMENTAL PROGRAM

In order to investigate the influence of prestressing force on shear crack width in PPC beams, the following extensive experimental program was carried out. Table 1 shows the description of eight specimens tested under the static four point monotonic loading. Test specimens consisted of two RC and six PPC beams as describe in Table 1. The shear-span to effective depth ratio was maintained at 3.0 for all specimens. The compressive stress in concrete due to prestress, stirrup ratio, and position of PC tendon were the main experimental parameters. The layout of specimens and typical cross sectional details are shown in Figs. 1 and 2 respectively. In these specimens, stirrups of D6 were provided in left span while those of D10 were provided for right span of the beams to ensure the main diagonal crack occurs in the left span. Therefore, shear crack widths were accurately monitored on the left span of the beam using a digital microscope with an accuracy of 0.001 mm. Contact gage points were pasted on the two sections of the shear span to measure the strain in three directions. The strains measured were used to calculate the principal tensile strain based on Mohr's principal

3. NUMERICAL SIMULATION

3.1 Numerical method

The numerical simulation was carried out using Response-2000 model, which was developed based on the Modified Compression Field Theory (MCFT). Experimental variables and actual compressive strength of concrete in the test specimens were used in the numerical model for the simulation. The numerical simulation in the model combines a plane section analysis for flexure with the modified compression field theory for shear that accounts for strain compatibility and uses the tensile and compressive stress-strain relationships for diagonally cracked concrete [6]. In this method, the spacing of shear crack is accounted to determine shear cracking load. The crack spacing is a function of the spacing of the longitudinal and transverse reinforcement as described in the section 4.1 as crack spacing model. The numerical simulation was performed at a section located at a distance of 225 mm (approximately at a distance 'd' and at the section along stirrup location) from the face of the loading point in the shear span. The shear-to-moment ratio is 1.905 at the selected section. Although the ACI 318-02 code limits the yield stress of shear reinforcement to 400 MPa, the experimentally measured value of yield stress 450 MPa was used in numerical simulation of the model.

3.2 Comparison with experimental and numerical results

3.2.1 Load stirrup strain

Figs. 3a and 3b show the relationship between the load and stirrup strain. From the Figs. 3a and 3b it was emphasized that, there is a satisfactory agreement between the numerical results obtained from the Response-2000 model and the experimental results. Furthermore, the numerical prediction of the shear cracking load agrees well with experimental value (Fig. 3a). In PRC beams, the compressive stress in concrete due to prestressing $\sigma_{c,ps}$, was increased the shear cracking load and it yields a smaller stirrup strain than the RC-1. The increasing rate is almost identical in RC and PRC beams. This implies that the $\sigma_{c,ps}$ has an influence on increasing diagonal cracking load. But thereafter the occurrence of diagonal cracks, the stirrup strains are not affected by the $\sigma_{c,ps}$.

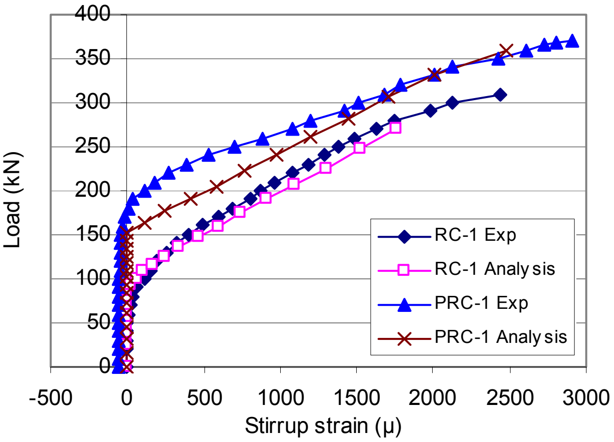


Fig. 3a: Load stirrup strain relationship

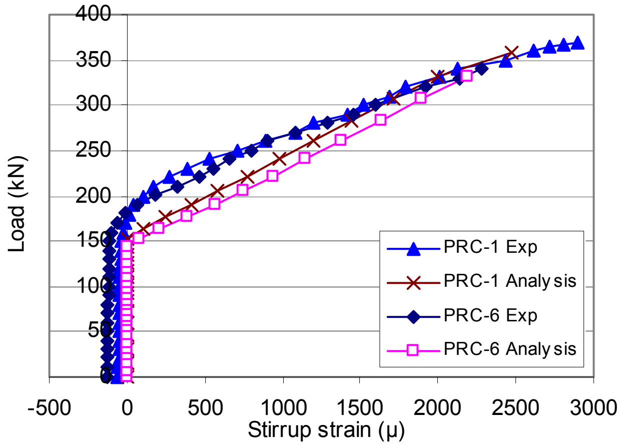


Fig. 3b: Load-stirrup strain relationship

Fig. 3b shows the load-stirrup strain relationship due to the effect of axial compression with and without PC tendon. It is interesting to note that such an effect did not appear significantly on shear cracking load or yielding rate of stirrup strain. Fig. 3b revealed that the numerical prediction also well agrees with experimental trend. It can be clearly observed that the MCFT based Response-2000 can show a close agreement with experimental results. Therefore, such a model would be useful to predict the response of beams with new parameters.

3.2.2 Load-principal tensile strain

Response-2000 numerical model was used to obtain the principal tensile strains. In the experiment, principal tensile strain was measured at the critical section of the shear span by using the contact gage. Because the shear crack propagate across the section, electric strain gages would not be possible to use to measure concrete strain in cracked beams.

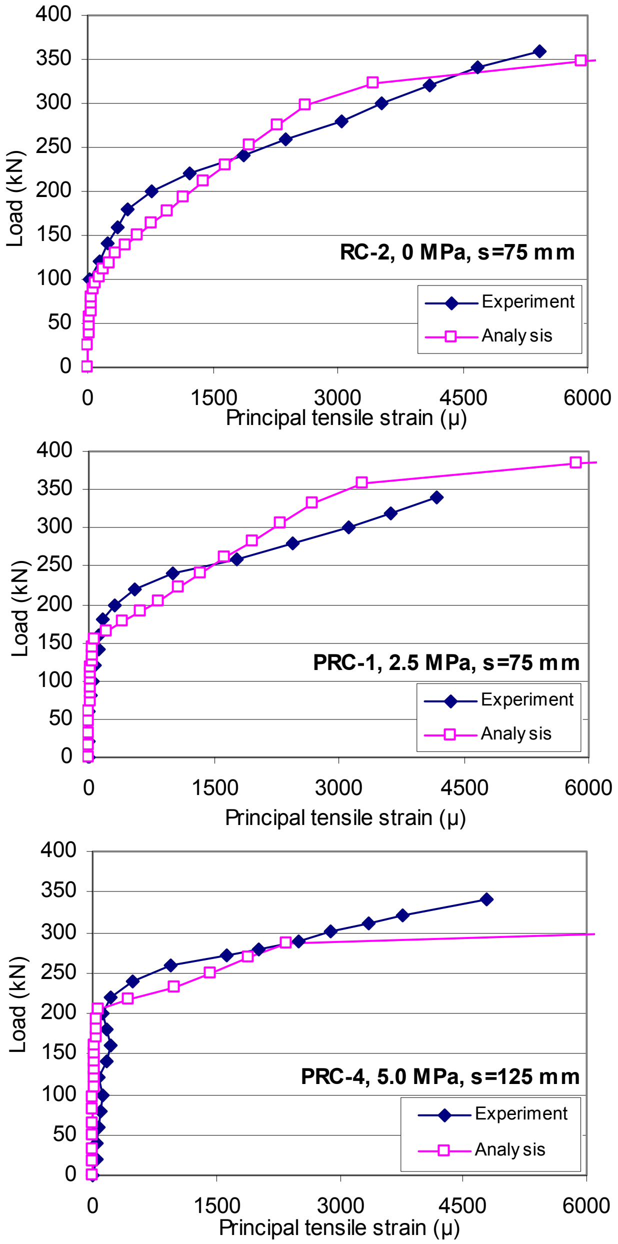


Fig. 4: Load-principal tensile strain relationship

Fig. 4 shows the load-principal tensile strain relationship for the RC-2, PRC-1 and PRC-4 beams.

It is clear that the experimental results and predicted results from the numerical simulation of MCFT based Response-2000 model showed a better correlation. In Fig. 4, it revealed that principal tensile strain starts to increase with load. The load at the starting of rapid increments in principal tensile strain can be considered as the shear cracking load. The shear crack usually forms normal to the direction of the principal tensile stress. Comparing PRC-1 and PRC-4 beams revealed that the increasing compressive stress of concrete due to prestressing ($\sigma_{c,ps}$) causes significant increase in shear cracking load. Furthermore, RC-2 specimen having a higher rate of increasing in principal tensile strain compared with PRC specimens. This implies that the effect of axial compression by prestressing is caused to reduce the increasing rate of principal tensile strain and there by reducing the shear crack width. Therefore RC-2 specimen proved that there is no any effect from providing prestressing bar only to reduce the shear crack width by enhancing the opening in longitudinal direction at the centroid of the concrete section.

4. CALCULATION OF SHEAR CRACK WIDTH

Crack width (w) can be taken as the product of the average crack spacing ($s_{m\theta}$) and the principal tensile strain (ε_1). The average spacing of diagonal cracks (Fig. 6) is calculated using the following equation and it converts the calculated crack spacing into the two orthogonal directions to an estimated diagonal spacing.

Thus, $w = \varepsilon_1 s_{m\theta}$ (1)

4.1 Crack spacing model

The spacing of the inclined cracks will depend on the crack control characteristics (Fig. 5) of both the longitudinal and the transverse reinforcements. Therefore it is suggested in CEB-FIP model code, that the crack spacing can be taken as the;

$$s_{m\theta} = \frac{1}{\dfrac{\sin\theta}{s_{mx}} + \dfrac{\cos\theta}{s_{mv}}} \qquad (2)$$

Where θ is diagonal crack angle (Fig. 6), s_{mx} and s_{mv} are the crack spacing indicative of the crack control characteristics of the longitudinal reinforcements (Fig. 7b) and transverse reinforcement (Fig. 7a) respectively. Thus, s_{mx} is the average crack spacing that would result if the member was subjected to longitudinal tension while s_{mv} is the average crack spacing that would result if the member subject to a transverse tension. These crack spacing can be estimated from the CEB-FIP code crack spacing expression. The CEB-FIP expression was intended to calculate shear crack spacing on the surface of the member.

The crack spacing, s_{mx} and s_{mv} are estimated using the formulas given by the CEB-FIP Model Code (1990);

$$s_{mv} = 2\left(c_v + \frac{s_v}{10}\right) + 0.1\frac{d_{bv}}{\frac{A_v}{b_w s}} \tag{3}$$

$$s_{mx} = 2\left(c_x + \frac{s_x}{10}\right) + 0.1\frac{d_{bx}}{\frac{A_x + A_{px}}{A_c}} \tag{4}$$

in which;

d_{bx} -diameter of longitudinal reinforcement (mm)

d_{bv} - diameter of transverse reinforcement (mm)

c - distance to reinforcements from the centre line of the section considered (*v, x*) (mm)

s - bar spacing in horizontal direction (*x*) and transverse direction (*v*) (mm)

A_x - longitudinal reinforcement area (mm^2)

A_v - stirrup area (mm^2)

A_{px} - prestressing steel area (mm^2)

A_c - concrete area (mm^2)

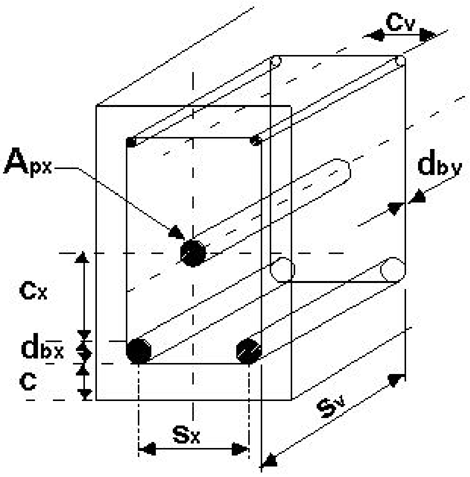


Fig. 5: Parameters influencing crack spacing

Based on this crack spacing model to calculate the vertical crack spacing due to the axial tension, s_{mx}, was modified by introducing prestressing steel area, A_{px}, for PPC beams. Principal tensile strain was calculated based on the strain measured in the three directions (rectangular strain-gage rosette) using a contact gage. The proposed crack spacing model was further verified by comparing the numerical results with the experimental results of RC and PPC beams. For the comparison of the proposed crack spacing model, RC-2, PRC-1 and PRC-4 beams were taken to identify the different combination of parameters used in the crack spacing model. RC-2 beam consists of prestressing bar without prestressing. The difference among the PRC-1 and PRC-4 are stirrup ratio and compressive stress in concrete due to prestressing force. This is because, the parameters considered in the crack spacing model is mainly stirrup ratio, diameters of reinforcing steel and prestressing steel and, the spacing of steel bars. Fig. 8 shows that the proposed crack spacing model produce a good agreement with the experimental results. The measured crack widths were taken at the critical location of the shear span which was the same section used to calculate the principal tensile strain from the numerical simulation.

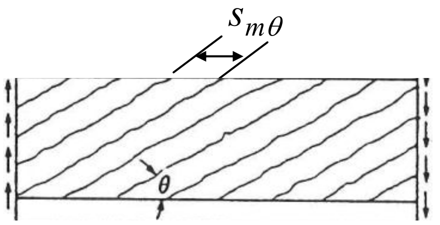
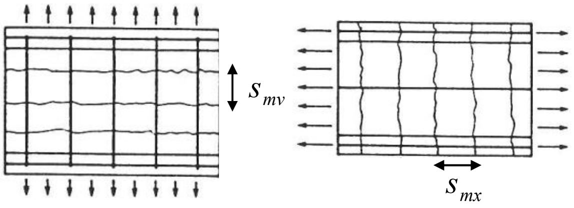


Fig. 6: Diagonal crack spacing



(a) (b)

Fig. 7: (a) Horizontal cracks due to transverse tension
(b) Vertical cracks due to axial tension

From Fig. 8 it can be clearly observed that the calculated shear crack width by the numerical model gives well with experimental results until the stirrup yielding at that location. The load when crack width starts to increase can be considered as the shear cracking load in the view point of load-shear crack width relationship. The inclined cracking shear can be defined as the shear necessary to cause a principal tensile strain equal to the tensile strength of the concrete.

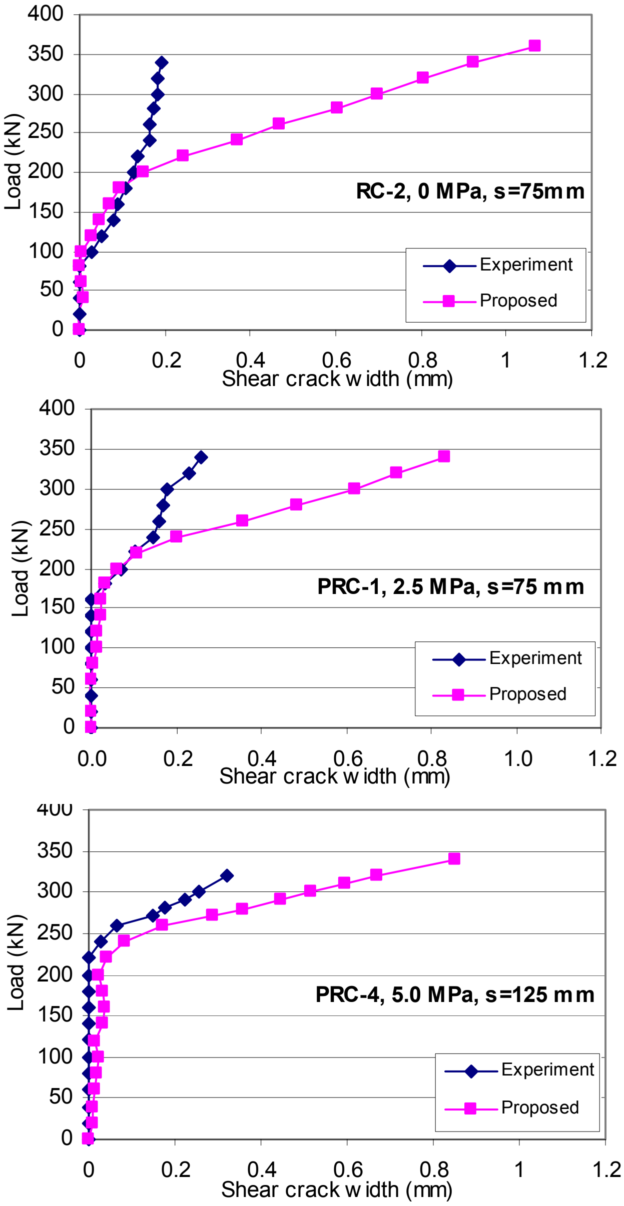


Fig. 8: Load-shear crack width relationship

However, if the member is subjected to shear stress, the principal stress directions are inclined towards the longitudinal axis of the member. The crack forms at a location where significant shear stress exists and inclined to the member axis. The shear cracking mechanism is more complicated in PRC beams than the RC beams. In PPC beams, the neutral axis location and the effective section properties depend not only on the geometry of the cross section and the material properties as far as RC beams, but also PC tendon area and axial compression by prestressing. Therefore in PPC beams maximum web shear crack would be expected to occur at or below the centroid of the cross section considered.

5. CONCLUSIONS

The numerical simulations were carried out on a PPC beam using the MCFT based numerical model and the proposed crack spacing model was presented in the paper. Based on this study following conclusion can be drawn:

1. The above mentioned numerical model is able to describe the behavior of PPC beams to estimate stirrup strain, and principal tensile strain with reasonable accuracy.
2. Proposed crack spacing model can be used to predict shear crack width in PPC beams.
3. Based on the relationships found, shear crack width in PPC beams can be defined with respect to the principal tensile stresses which expected the maximum at or below the centroid of the critical section considered in the shear span.
4. Based on the numerical simulation and experimental results revealed that the higher $\sigma_{c,ps}$ caused an increase in the shear cracking load. Furthermore it is caused to decrease the rate of increase in principal tensile stress and thereby reducing the shear crack width.

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