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Economic Theory

General equilibrium, incomplete markets and sunspots: A symposium in honor of David Cass

Guest editors' introduction

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This special edition of *Economic Theory* honors David Cass, one of *ET*'s founding editors, on the 30th anniversary of his joining the faculty at the University of Pennsylvania's Economics Department. The contributions to this volume are, for the most part, from Dave's students or co-authors, and we hope they communicate both to Dave and to the economics profession generally the high regard those of us who have trained under Dave's tutelage or worked with him on research have for him.

Most scientists would be happy to have had one major, influential idea over the course of their careers. Dave Cass has had three, and is still going strong.

His first major contribution to economics was the characterization of optimal growth trajectories in his thesis work under Hirofumi Uzawa's supervision. The celebrated Cass criterion for optimal time paths in the one good growth model quickly followed. The essence of this work is the search for price characterizations of efficiency for dynamic time paths, an effort that directly pointed the way to the subsequent full dynamic decentralization of the neoclassical optimal growth model, a fact that permits its use for modeling a wide range of business cycle and other macroeconomic phenomena. Accordingly, Dave is rightly honored, in conjunction with Tjalling Koopmans, not only as one of the founding fathers of optimal growth theory but also of dynamic modern macroeconomics, where the neoclassical growth model, in all its variations and interpretations, remains the fundamental underlying paradigm for all of dynamic macroeconomic analysis.

Dave's second contribution – the notion of a so-called sunspot equilibrium in dynamic economies which he developed jointly with Karl Shell – is also the stuff of legend, and grew out of his long and productive collaboration with Karl at Penn. The early impetus for Dave's interest in this topic stemmed from work he did with Manny Yaari on overlapping generations models, and from his early acquaintance with Bob Lucas at Carnegie Mellon and Lucas's seminal work on rational expectations in dynamic economic models. To quote from the interview with Dave by Spear and Wright in *Macroeconomic Dynamics*

> I wasn't so interested in macro, but what struck me, and this is related to some of my later work, was the assumption that [Lucas] made to solve for equilibrium, that the state variables were obvious . . . . Bob and I had some long discussions, and I would say, "Well Bob, why is this the actual state space in this model?" That question came up . . . after I came to Penn. At some point Karl [Shell] and I started talking about that and we developed what we called the idea of sunspots. (Spear and Wright [21])

In addition to raising troubling questions about what the right state space was for dynamic stochastic economies, the notion of sunspot equilibrium raised a number of deep questions about the overall determinacy of economic equilibria and the role of the welfare theorems in the occurrence or non-occurrence of sunspot equilibria. These questions spawned a large literature on determinacy in dynamic economies in which the welfare theorems broke down. These include overlapping generations models, growth models with externalities or taxes, and models in which asset markets were incomplete. All were shown to allow the existence of sunspot equilibria. And, in a suitable twist of intellectual fate, macroeconomists have more recently begun to explore the question of whether sunspots can provide a more plausible source of fluctuations in dynamic equilibrium models than the conventional aggregate productivity disturbances.

Dave's third major contribution to economic theory was his work on general equilibrium with incomplete markets, work which grew out of his exploration of the question of existence of sunspot equilibria in models with incomplete asset markets. Dave's follow-on work on existence and determinacy of general equilibrium in models with incomplete asset markets spawned another large literature which has come to be known simply as GEI.

The earliest work on market incompleteness goes back to Arrow in the 1950's, Diamond in the mid-'60's and a number of related papers in the finance literature between the late 1950's and early '70's (Geanakoplos [10] provides an excellent survey of this literature). The canonical GEI model was formulated by Radner in the early 1970's (Radner [19]) in a paper which also pointed up one of the fundamental puzzles about models with incomplete markets: the possible loss of dimensionality in the span of the asset payoffs as prices vary.

This potential for non-existence of equilibrium (which was formally developed in Hart's [12] counterexamples to existence of equilibrium) left the literature in limbo for almost a decade, until Dave's work on existence in economies with purely financial assets pointed the way out. As Geanakoplos notes

> Suddenly in the middle 1980s the pure theory of GEI fell into place. In two provocative and influential papers, Cass [4,5] showed that the existence of equilibrium could be guaranteed if all the assets promise delivery in fiat money, and he gave an example showing that with such financial assets there could be a multiplicity of equilibrium. Almost simultaneously Werner [22] also gave a proof of existence of equilibrium with financial assets, and Geanakoplos and Polemarchakis [11] showed the same for economies with real assets that promise delivery in the same consumption good. (Geanakoplos [10])

This work was followed very quickly by results showing that the non-existence problem pointed out by Hart was not generic, and led ultimately to the generic existence results of Duffie and Shafer [8], and again spawned a new literature looking positively at the welfare implication of market incompleteness, and normatively at issues of asset engineering.

In the course of making these contributions, Dave has worked with a large group of coauthors, including (to date): Y. Balasko, L. Benveniste, G. Chichilnisky, A. Citanna, R. Green, M. Majumdar, T. Mitra, M. Okuno, A. Pavlova, H. Polemarchakis, K. Shell, P. Siconolfi, S. Spear, J. Stiglitz, A. Villanacci, H.-M. Wu, M. Yaari, and I. Zilcha. Dave's graduate students (to date) include S. Chae, A. Citanna, J. Donaldson, R. Forsythe, F. Kydland, Y. W. Lee, M. Lisboa, A. Pavlova, T. Pietra, P. Siconolfi, S. Spear, S. Suda, J-M. Tallon, and A. Villanacci. Those of us who have worked with Dave and/or under his tutelage as graduate students have benefited tremendously from his razor-sharp analytic mind, from his willingness to work at understanding problems we have posed to him, or new methodological techniques we have discovered, and (perhaps most importantly) from his no-nonsense approach to doing science. The papers we have collected here reflect Dave's influence on us as students and coauthors. Six of the papers [7, 13, 14, 16, 18, 20] look at models of incomplete markets, production or growth, and sunspot equilibrium and the interactions that arise as a result of these market frictions. Three additional papers [1, 2, 15] look at issues in general equilibrium theory that Dave would find familiar. The remaining papers don't bear directly on aspects of Dave's work, but are indicative of the types of scientific thinking about economics that Dave's training and influence has spawned. We hope *ET*'s readers will find the volume interesting.

Putting together a volume like this is never easy, and we owe a large measure of thanks to *ET*'s editor, Roko Aliprantis, and *ET* editorial assistant, Beverly Cohen, for all of their help and support in making this volume possible.

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