Gold Rush Dynamics of Private Equity*

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Abstract

We develop a model with learning that likens private equity waves to gold rushes. Fund managers differ in talent, the stock of potential target firms is depletable, and investment profitability is inferred from past outcomes. The model produces waves with endogenous transitions from boom to bust. Supply and demand are inelastic, and supply comoves with investment valuations. Performance differences are persistent, first-time funds underperform the industry, and funds raised during booms are less likely to see follow-on activity. Entry and past industry performance are positively related, while contemporaneous entry and performance are inversely related. Finally, the time-series (cross-sectional) relationship between fund size and fund performance is negative (positive and concave).

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1 Introduction

Empirical evidence suggests that commitments to and investments by the private equity industry are highly cyclical. This has been documented for the venture capital industry by Gompers and Lerner (2000) and Lerner (2002), and for the buyout industry by Kaplan and Stein (1993) and Kaplan and Strömberg (2008). To give a recent example, global buyout volume in early 2007 amounted to \$527.7 billion. By mid-2008, this amount had shrunk to \$124.7 billion. Such boom-bust cycles suggest that the private equity business is inherently transient, expanding when opportunities for profitable control investments arise, and contracting when such opportunities deteriorate.

We develop a simple model which captures this transient nature. The basic idea is to liken private equity waves to gold rushes. As the name indicates, gold rushes start with the discovery of gold. Following a discovery, gold-diggers begin settling nearby in the hope of making a fortune. Over time, more crowd into the area until all claims are staked. At last, when the gold reserves dry up, the gold-diggers retire or migrate to the next discovery.[1] This paper draws an analogy between gold discoveries and the emergence of private equity investment opportunities, gold-diggers and private equity partnerships, claims and investments, and gold and investment returns.

The model produces waves which endogenously transition from boom to bust. Moreover, the dynamics of entry, prices and returns *within* a wave match a wide range of empirically documented patterns: the notion that changes in private equity demand elicit a sluggish response in private equity supply and vice versa,

[1] An example is the Klondike Gold Rush. In August 1896, gold was discovered in the Klondike river. By the summer of 1897, the nearby town of Dawson had grown to a population of 3,500, and steamships in San Francisco and Seattle unloaded about one and half million dollars worth of gold. Within half a year, the population of Dawson rose to over 30,000. In the summer of 1899, the gold rush was officially over.

the procyclicality of capital inflow and investment valuations, persistent performance differences across partnerships, the underperformance of first-time funds, the positive relationship between entry and past returns as well as the negative relationship between entry and subsequent returns.[2]

In our model, different partnerships repeatedly decide whether to enter (or to exit) the market for private equity. Each partnership's decision to enter the market depends on its specific ability to generate value and the general quality of the available investment opportunities. While the latter quality is unknown, it can be inferred from past investment outcomes. This learning process creates an intertemporal link between past and current investment decisions (as e.g., in Veldkamp, 2005; or Van Nieuwerburgh and Veldkamp, 2006). Finally, we assume that the stock of investment opportunities is depletable.

An endemic feature of the model is that expansions in private equity activity follow a wave pattern. Initial investments are triggered by a latent exogenous shock that affects the profitability of private equity engagements in a stock of firms (e.g., buyout targets or start-up ventures). Because the extent of the shock is unknown, only few partnerships form at the outset. When actual profitability is low, these pioneers earn modest or disappointing returns, and investment activity subsequently stagnates or subsides. In contrast, when actual profitability is high, their returns are promising and new partnerships enter. As the industry grows, the true profitability is revealed at a faster rate, which in turn accelerates entry. This feedback loop between learning and entry fuels the build-up of the wave. The countereffect is that the influx of new partnerships precipitates the decline in investment opportunities. This depletion ultimately induces exit.

[2]These empirical patterns are documented by Gompers and Lerner (1999, 2000), Kaplan and Schoar (2005) and Hochberg et al. (2008). The reported performance patterns in private equity stands in stark contrast to the evidence in the mutual fund industry (Malkiel, 1995; Berk and Green, 2004) and the investment management industry (Busse et al., 2008).

Compared to the full information setting, the supply of private equity is inelastic because partnerships gradually learn about the profitability of investing during the wave. The *speed of learning* depends on the degree of investment specificity and the degree of surprise. The greater the idiosyncratic risk of an investment, the less informative is its outcome about the profitability of other investments. Similarly, if the market deems large profits unlikely, it more cautiously interprets successful investments as a sign of general profitability. The *speed of entry* depends on the distribution of talent among (potential) partnerships. For example, a pyramid structure with "few at the top, and many at the bottom" induces slow entry when expectations are low but fast entry when expectations are high. Slow learning coupled with a talent pyramid can lead to waves with slow starts, explosive booms and sudden ends.

Since the market's expectations jointly determine entry and valuation, aggregate entry and valuation levels comove. This is consistent with the evidence in Kaplan and Stein (1993) and Gompers and Lerner (2000). Fundamentals are not affected by learning. Therefore, the increase in valuation levels does not imply that investments become more profitable. On the contrary, as valuation increases relative to fundamentals, average fund profitability declines during a wave. This decline is reinforced by the influx of less talented partnerships.

The assumed heterogeneity in talent entails persistent differences in performance across partnerships. Furthermore, a partnership's talent influences its time of entry and exit. For any given market expectations, only the more talented partnerships enter the market. This translates into a last-in-first-out principle of entry and exit: the least talented partnerships are the last to enter the market when expectations increase and, by the same token, the first to exit the market when expectations decrease. Thus, at any point in time, the latest entrants (i.e., the first-time funds) underperform the industry. Moreover, as the least talented partnerships enter when market expectations are high and are prone to exit early, our model implies that partnerships started during boom times are less likely to raise follow-on funds. Both of these results are consistent with the empirical findings in Kaplan and Schoar (2005).

Finally, the model provides a rationale for the positive relationship between entry and past performance, and for the negative relationship between entry and subsequent performance, also documented by Kaplan and Schoar (2005). High past performance raises the market's current expectations, which in turn attracts new partnerships. At the same time, it raises prices which lowers future performance. Even though it seems as if the new partnerships mistime their entry, such patterns are rational when investors learn about profitability from past outcomes. In such a setting, *actual* profitability can diverge from *perceived* profitability.

Because our baseline model assumes a uniform and constant fund size, it does not explain the positive and concave relationship between fund size and fund performance across partnerships; nor the negative relationship between fund size and fund performance for consecutive funds of the same partnership (Kaplan and Schoar, 2005). Both of these relationships arise naturally when the model is extended to allow for variable fund size.

The occurrence of waves has been studied in the context of mergers and acquisitions by Jovanovic and Rousseau (2002), Shleifer and Vishny (2002) and Rhodes-Kropf and Robinson (2008); and in the context of venture capital markets by Inderst and Müller (2004) and Michelacci and Suarez (2004). None of these papers explore the role of learning. Furthermore, unlike our framework, their predictions about waves are based on comparative statics and hence do not pertain to endogenous dynamics within a wave.

We are not the first to study the impact of learning on financial decisions. For instance, learning models have been used to explain financial innovations (Persons and Warther, 1997), stock market returns (e.g., Timmermann, 1993, 1996; Veronesi, 1999), and going public decisions (Pastor et al., 2006; He, 2007). Finally, contemporaneous work by Hochberg et al. (2008) and Glode and Green (2008) incorporates learning into a model of the private equity market. In both models, fund investors (limited partners) learn about the ability of fund managers (general partners). By contrast, in our model, fund managers learn about market conditions which affect the profitability of private equity investments.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the equilibrium entry strategies. Section 4 analyzes the dynamics of the model. Section 5 extends the model to allow for variable fund size. Section 6 concludes the paper.

2 Model Setup

We develop a simple model in which private equity partnerships decide to enter or exit the market depending on their beliefs about the profitability of investing. Consider a risk-neutral economy in discrete time, $t \in \{0, 1, \ldots, \infty\}$, with a fixed population of $\mathcal{N}$ firms. Initially, each firm is run by an incumbent manager, and its discounted dividend value under the incumbent manager is normalized to 0.

In period 0, the economy experiences a productivity shock. The shock makes each firm, if appropriately reorganized, improvable. A firm's value after reorganization, V, is gamma-distributed with shape parameter $\alpha > 0$ and scale parameter $1/\beta > 0$. The mean of the gamma distribution, $\overline{V} = \alpha/\beta$, reflects the expected reorganization value. We assume that α is commonly known, whereas

β is unobserved. Since a lower β translate into a higher expected reorganization value, this means that the market is uncertain about the magnitude of the shock. The market's initial beliefs about the value of β are also represented by a gamma distribution, with known shape and scale parameters $\tau > 0$ and $1/\gamma > 0$ respectively.[3]

We preclude the possibility that the incumbent managers can generate the value improvement, e.g. by procuring consulting services or tapping the labor market. Instead, let there be $\mathcal{M}$ outside management teams (partnerships) who can carry out this task provided that they set up the necessary operations and make a control investment in a firm.[4]

In every period $t \geq 1$, each partnership decides whether or not to enter the market for the duration of that period. To enter, a partnership must raise and operate a fund imposing a per-period fixed cost (e.g., search activities, due diligence, negotiations, legal expenses). The cost is independent of investment outcomes but varies across partnerships, which are ordered according to their costs: $C_1 < C_2 < \cdots < C_{\mathcal{M}}$. For later use, we define a continuously increasing function $C(\cdot)$ with $C(i) = C_i$ for all $i \in \{1, 2, \ldots, \mathcal{M}\}$. This function reflects the talent distribution among the partnerships and is commonly known. To ensure interior equilibria, let $C(1) = 0$ and $C(\mathcal{M}) = \infty$.[5]

Once a partnership operates a fund, it seeks to invest in firms. Time constraints put a limit on the number of investments that a fund can undertake

[3] We choose the gamma distribution because it rules out negative value improvements and allows for a tractable Baysian analysis. Notwithstanding, our qualitative results hold for any stochastic setting with parameter uncertainty where high past observations lead agents to increase their expectations about the mean of the underlying probability distribution.

[4] Private equity funds often enforce changes in the governance of their portfolio firms (Gertner and Kaplan, 1996; Acharya and Kehoe, 2008; Cornelli and Karakas, 2008). Acharya and Kehoe (2008) report that one-third of CEOs in buyout targets are fired in the first 100 days.

[5] The formulation of heterogeneity in terms of cost is not to be taken too literally. The same qualitative results are obtained when costs are uniform and partnerships instead differ in their ability to improve target firms. We choose the cost formulation because it makes the analysis more tractable.

per period. For simplicity, we assume that a fund can at most invest in one firm. (Alternative limits are discussed in section 5.1.) In every period, each active fund is paired with a potential target firm. Once paired, they negotiate the price at which the partnership can purchase (a stake in) the firm. Negotiations are modeled as Nash bargaining with ω $(1-\omega)$ denoting the bargaining power of the fund (firm). If a negotiation fails, the involved parties part and neither is paired again in the ongoing period. Otherwise, the fund purchases and reorganizes the firm. A reorganized firm harbors no further potential for improvement.

$M_t \leq \mathcal{M}$ and $N_t \leq \mathcal{N}$ denote respectively the number of funds and of potential targets in period t. If $M_t > N_t$, we adopt the convention that the most efficient funds are paired with a firm first.

The timing of the model is as follows. In period 0, the market learns about the occurrence of the shock but does not observe its magnitude, i.e. β. In each subsequent period $t \geq 1$, events unfold in the below order:

1. Everyone enters the period with beliefs $\overline{V}_t = E_t(\overline{V})$.

2. All partnerships decide whether to raise a fund for this period.

3. Funds are matched with a firm and bargain over the purchase price P_t.

4. Funds that have successfully negotiated the price invest in their targets.

5. The targets are reorganized and their new value becomes public.

6. Everyone updates their beliefs.

3 Competitive Equilibrium

The key decisions in the model are the partnerships' repeated choices whether or not to raise a fund. Let $a_t^i \in \{1, 0\}$ denote partnership i's decision in period t, where $a_t^i = 1$ if the partnership decides to raise a fund, and $a_t \equiv (a_t^1, \ldots, a_t^{\mathcal{M}})$. We assume competitive behavior and rational expectations. That is, each individual partnership ignores its own impact on aggregate variables but its expectations about these variables is ex ante correct.

In each period t, the history of all previous investment outcomes is commonly known. This history has a direct influence on the payoffs from t onward (only) through their impact on the state variables $\overline{V}_t$ and N_t. Given a state $(\overline{V}_t, N_t)$, partnership i chooses a_t^i to maximize the sum of its discounted expected future per-period profits:

$$\Pi^i(a_t, \overline{V}_t, N_t) = \mathrm{E}_t[\sum\nolimits_{\tau=t}^{\infty} \delta^{\tau-t} \pi_\tau^i(a_\tau, \overline{V}_\tau, N_\tau) \left| \overline{V}_t, N_t \right.]$$

where $\pi_t^i(a_t, \overline{V}_t, N_t)$ is i's period-t profit, and $\delta \in [0, 1]$ is a discount factor.

We restrict attention to Markov strategies which depend only on the current state (e.g., Maskin and Tirole, 2001). In a Markov equilibrium, the optimal entry strategies and profits can be written as $a_t^* = a_t(\overline{V}_t, N_t)$ and $\Pi^i(a_t^*, \overline{V}_t, N_t)$. Hence, given optimal future behavior, we can decompose $\Pi^i(a_t, \overline{V}_t, N_t)$ into the current profit and a "future" value:

$$\pi_t^i(a_t, \overline{V}_t, N_t) + \delta \mathrm{E}_t[\Pi^i(a_{t+1}^*, \overline{V}_{t+1}, N_{t+1}) \left| \overline{V}_t, N_t \right.].$$

Importantly, i's decision today affects the future only through its impact on the aggregate variables $\overline{V}_{t+1}$ and N_{t+1}. In a competitive equilibrium, partnerships ignore this (intertemporal) impact. That is, they treat entry decisions in different periods as *independent* options. As a result, they behave as if they were myopic. Intuitively, each partnership perceives the impact of its current investment on future market conditions as so small that its only decision criterion is the immediate profit. This simplifies the equilibrium analysis, and distinguishes our approach from a real options framework.[6]

The dynamic properties of the competitive Markov equilibrium are the focus of our paper. The key driver of these dynamics is a feedback loop between entry decisions and market conditions. Entry depends on how market conditions evolve, and vice versa. Accordingly, we first analyze entry decisions for given market conditions, and then the market conditions for a given history of entry decisions.

3.1 Entry

To determine entry in t for a given state $(\overline{V}_t, N_t)$, we must first determine the outcome of the ensuing bargaining stage. Since a partnership behaves quasi-myopically, its threat point in bargaining is the risk-free return P_t/δ. By contrast, a firm's threat point in bargaining is the expected payoff from returning to the market in the hope of being acquired in the future. For simplicity, we assume that a firm that has been in negotiations before is certainly approached by entrants in the next period. As the literature on search markets, we further assume that the firm's payoff from a future match is the payoff from a successful deal, i.e. the future "inside" option. The firm's *current* outside option is therefore $\delta E_t[P_{t+1}]$.

Given these threat points, the Nash bargaining outcome solves $\max_{P_t}(\overline{V}_t -$

[6] The assumption of competitive behavior has two main consequences: On the one hand, partnerships with negative expected current profits do not take into account the possibility of active *experimentation*, and hence become adaptive learners (cf. Van Nieuwerburgh and Veldkamp, 2004; Veldkamp, 2004). On the other hand, partnerships with positive expected current profits discard the possibility of *procrastinating* entry until they have learned more from information produced by other investments.

$P_t - P_t/\delta)^{\omega}(P_t - \delta E_t[P_{t+1}])^{1-\omega}$. This yields $P_t = \frac{\delta(1-\omega)}{1+\delta}\overline{V}_t + \omega\delta E_t[P_{t+1}]$. To get a closed-form solution, we conjecture an equilibrium in which the price is a *linear* function of $\overline{V}_t$ such that $P_t = k\overline{V}_t$. By the law of iterated expectations, it then follows that $E_t[\overline{V}_{t+1}] = \overline{V}_t$ and $E_t[P_{t+1}] = E_t[k\overline{V}_{t+1}] = k\overline{V}_t = P_t$. In other words, *if* P_t is a linear function of $\overline{V}_t$, it is a martingale. Conversely, if P_t is a martingale, the Nash bargaining solution is indeed linear in $\overline{V}_t$. Formally, substituting $E_t[P_{t+1}] = P_t$ into the bargaining solution yields

$$P_t = \frac{\delta(1-\omega)}{(1+\delta)(1-\omega\delta)}\overline{V}_t.$$

Thus, $k = \frac{\delta(1-\omega)}{(1+\delta)(1-\omega\delta)}$ is a rational equilibrium outcome. Consistent with intuition, a more patient firm (lower δ) bargains for a higher price ($\partial k/\partial\delta > 0$). Also, since a failure to agree is inefficient, all matches result in a successful trade.

Turning to entry, since a partnership behaves quasi-myopically, it raises a fund if the current expected profit from entry exceeds the current period's outside option which we normalize to 0. That is, a partnership enters if $C_i \leq \overline{V}_t - P_t = \frac{1-\omega\delta^2}{(1+\delta)(1-\omega\delta)}\overline{V}_t$ and is sure to be matched with a firm. Since this is true for all types, there exists a cut-off cost C_{i^*} such that all types with $C_i \leq C_{i^*}$ raise a fund. In this case, i^* is equivalent to the total number of funds M_t. It is defined by $C(i^*) = \frac{1-\omega\delta^2}{(1+\delta)(1-\omega\delta)}\overline{V}_t$ if $i^* < N_t$; and by $i^* = N_t$ otherwise.

Lemma 1 *There exists a competitive Markov equilibrium in which all partnerships with $C_i \leq C(M_t)$, where*

$$C(M_t) = \min\left\{\frac{1-\omega\delta^2}{(1+\delta)(1-\omega\delta)}\overline{V}_t, C(N_t)\right\}, \tag{1}$$

enter the market in period t.

The intuition behind this equilibrium is straightforward. More talented partnerships are more inclined to enter so that, in every period, the relatively "best" partnerships raise a fund. Furthermore, the comparative statics of $k\overline{V}_t$ show that M_t is increasing in both $\overline{V}_t$ and ω but decreasing in δ. That is,

Corollary 1 *The number of funds is larger if the expected reorganization value is higher, funds' bargaining power is stronger, and firms are more impatient.*

The number of funds is also weakly increasing in the (remaining) number of potential target firms. Though the number only matters when it becomes a binding constraint ($N_t \leq M_t$). In section 5.2, we discuss possible channels for market congestion, which cause the depletion of investment opportunities to have a more continuous effect on entry and exit.

3.2 Market Conditions

Lemma 1 characterizes the equilibrium outcome for a given state process $\{\overline{V}_t, N_t\}$. We now turn to the determination of this process. The target stock N_t monotonically decreases as more and more investments are completed. More specifically, if M^t denotes the number of investments consummated prior to t, the target stock at the beginning of period t is $N_t = \mathcal{N} - M^t$.

Past investment also allows market participants to make inference about the true β, i.e., to learn about the magnitude of the shock. In this respect, the revenue generated by each reorganization represents a noisy signal about $\overline{V}$. We assume that reorganization revenues are observable to other market participants. The assumption is not to be taken literally, as private equity partnerships in practice are known to be secretive about their returns. But it parsimoniously captures the notion that information about superior profitability – at least informally – leaks to other potential target firms and to other investors who are interested in starting their own partnership. The information spillover is crucial for the dynamics as it creates a intertemporal link between past performance and future market entry.

Let v_j denote the revenue generated by investment j. A history of investment outcomes is $\mathcal{H}_t = \{v_j\}_{j=1}^{M^t}$, and the historic average is $\overline{v}^t = \sum_{i=1}^{M^t} v_j/M^t$. Conditional on a history $\mathcal{H}_t$, the posterior distribution of $\overline{V}$ is *inverse* gamma with shape and scale parameters $\tau + M^t\alpha$ and $\alpha(\gamma + M^t\overline{v}^t)$ respectively. (Details of the Bayesian updating process are provided in the Appendix.) In period t, the market's expectations about the reorganization value are equal to the mean of the inverse gamma distribution, $\overline{V}_t = E(V\,|\mathcal{H}_t)$, or more precisely

$$\overline{V}_t = \frac{\alpha(\gamma + M^t\overline{v}^t)}{\tau + M^t\alpha - 1}. \tag{2}$$

The conditional expectation contains all distributional parameters except β, about which inference is being made. Recall that α is the known shape parameter of the V-distribution, whereas τ and $1/\gamma$ are the parameters of the distribution representing the market's initial (period-0) beliefs about the true β.

Lemma 2 $\{\overline{V}_t\}$ *converges to* $\overline{V}$ *as* $M^t \to \infty$*, and* $\overline{V}_t$ *is ceteris paribus*

- *increasing in* $\overline{v}^t$*,*
- *increasing in* M^t *iff* $\overline{v}^t > \alpha\gamma/(\tau - 1)$*,*
- *increasing in* α *and* γ *but decreasing in* τ*.*

Consistent with intuition, the current expectations increase with the historic average. Good past outcomes indicate that the reorganization value is high. If the historic average is sufficiently high relative to the initial expectations, current expectations also increase in the number of past investments. Otherwise, the opposite relationship holds. The reason is that more observations increase the precision of the estimate (in either direction).

Finally, current expectations are higher when the initial expectations $\overline{V}_0 = E(\alpha/\beta\,|\mathcal{H}_0)$ are high, which explains why they are increasing in α and decreasing in $E(\beta) = \tau/\gamma$. Throughout our analysis, we assume that $\overline{V}_0$ is relatively small. This is meant to capture that, absent positive experiences, the market is sceptical about the prospects of reorganization.

4 Dynamics

We now study entire equilibrium paths to characterize the dynamics of aggregate investment activity, prices and returns. A conceptual difficulty is that, even for a given β, the economy evolves stochastically so that there is no *unique* equilibrium path. To describe "typical" properties of an equilibrium path, we characterize the path which is obtained when every reorganization generates the mean revenue $\overline{V}$. We refer to this particular path (somewhat incorrectly) as the "trend" path, and index it with o.

It is important to bear in mind that the agents in the model are not aware that the deviations from the mean are zero. Hence, they update their beliefs as if the reorganization revenues were genuinely random. More specifically, given that $\overline{v}^t = \overline{V}$ for all t, market expectations on the trend path evolve according to

$$\overline{V}_t^o = \frac{\alpha(\gamma + M^t\overline{V})}{\tau + M^t\alpha - 1}. \tag{3}$$

The expectations monotonically converge to $\overline{V}$ as M^t goes to infinity. The speed of convergence decreases for large absolute values of τ and γ (keeping their ratio constant). Thus, one may interpret a large value of $\tau = \bar{z}\gamma$ for constant $\bar{z}$ as a low "signal-to-noise" ratio.

4.1 Waves

In $t = 0$, the economy receives news about the occurrence of the shock and forms prior expectations about the expected reorganization value. For entry to occur, these expectations must exceed $C_1/(1-\alpha)$ so that at least partnership 1 finds it worthwhile to raise a fund (Lemma 1). Otherwise, there is no initial entry and consequently no learning that can serve as an impetus for future entry.

Given entry, the evolution of investment activity is primarily determined by the true realization of $\overline{V}$. If $\overline{V}$ is small, the initial reorganizations generate modest revenues, and investment activity remains low. Indeed, for $\overline{V} < C_1/(1-\alpha)$, the revenues disappoint the market and investment activity subsides.

By contrast, if $\overline{V}$ is very large, the market becomes increasingly optimistic on the trend path because the investments are more profitable than expected. This attracts new partnerships, which in turn causes the target stock to diminish faster. Both learning and depletion have monotonic, yet countervailing consequences for future investment activity. When the number of funds reaches the number of remaining targets, investment climaxes and then collapses.

The ultimate decline in investments is rather extreme on the trend path. Yet, it epitomizes the wave pattern inherent in any equilibrium path. Even on stochastic paths, investment booms endogenously transition to sudden busts.

Proposition 1 *Expansions in investment activity follow a boom-bust pattern.*

In reality, productivity shocks occur more than once. In most cases, the shocks are probably small with little effect on aggregate investment. In a few cases, however, the shocks may be large, and investment activity may blossom

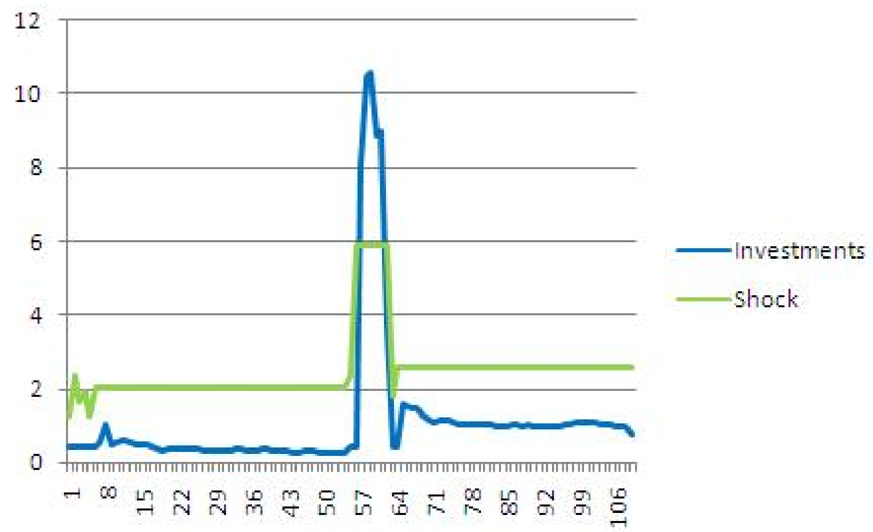

Figure 1: Long-run pattern

into a full-fledged wave extending over several years. Such waves are observable ex post but unpredictable ex ante [like the technological revolutions in Pastor and Veronesi (2008)]. To illustrate such a long-run pattern, we simulate equilibrium paths for a large number of shocks $\{\beta_k\}$ drawn from a gamma distribution with a high mean τ/γ (so that $\overline{V}_0$ is low). Figure 1 depicts a representative sequence of shocks with the investment activity that followed in their wake. As expected, long periods of relative inactivity are interrupted by a rare large wave.

The cyclicality of private equity investment is well documented (Kaplan and Stein, 1993; Lerner, 2002; Acharya et al., 2007; Kaplan and Strömberg, 2008). For instance, venture capital activity expanded during the biotechnology boom in the early 1990s and during the information technology boom in the late 1990s. Similarly, the buyout industry experienced high activity in the 1980s and in the mid-2000s.

The specific shape of a wave depends on the speed of learning and the talent distribution. When learning is slow (high $\tau = \bar{z}\gamma$), optimism develops more

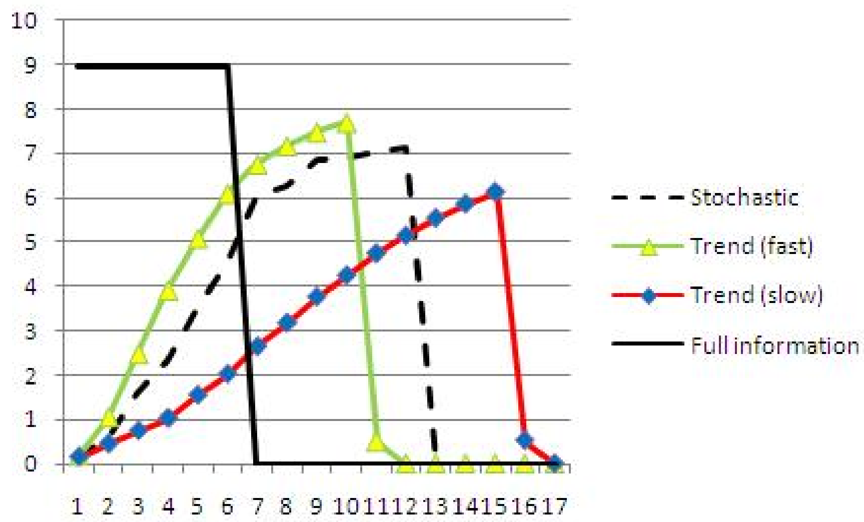

Figure 2: Different speeds of learning.

slowly. Similarly, when top talent is scarce (high $C' > 0$ and $C'' < 0$), many potential partnerships need to become more optimistic before they enter. When slow learning and scarce talent are combined, the wave incubates slowly, then mushrooms explosively, and crashes in the end. The explosive growth period is the result of a feedback loop between learning and entry: optimism induces entry, which in turn accelerates learning and further fuels the optimism. The crash occurs because, once the wave reaches its climax, the high activity level rapidly depletes the remaining target stock. The overall magnitude of the wave depends on the shock $\overline{V}$ and on the initial target stock $\mathcal{N}$.

In Figure 2, we depict four different paths following a large shock ($V \gg \overline{V}_0$). The horizontal line is the investment path when $\overline{V}$ is immediately observable. The two solid lines represent two trend paths differing in the speed of learning. Finally, the dashed line depicts a stochastic path corresponding to the trend path with faster learning.

The comparison of the different paths bears on the notion that supply and demand in the market for private equity are inelastic (Gompers and Lerner, 1999). Demand inelasticity is hard-wired into the model. Because demand arises due to exogenous productivity shocks, it does not respond to changes in supply. By contrast, the supply inelasticity is endogenous. Supply responds sluggishly to demand shocks because talent is scarce and potential partnerships want to learn about profitability before they enter the market. Accordingly, supply is less elastic when learning is more cautious and talent is scarcer.

4.2 Valuation and Entry

A wave is triggered by a shock, but it unfolds gradually because the market must learn about its magnitude over time. As the market grows more optimistic about the expected reorganization revenues, it raises the valuation of potential targets for reorganization. As a result, funds have to pay increasingly higher prices to invest in these firms, which in conjunction with Corollary 1 implies that

Proposition 2 *Entry and valuation levels increase together.*

Kaplan and Stein (1993) document that buyout prices during the wave in the 1980s rose relative to fundamentals. Gompers and Lerner (2000) find similar results using a large data set comprising private equity investments in different stages and industries from 1987 to 1995. Specifically, they report that capital inflows into the private equity industry coincide with higher valuations of the funds' new investments. Both papers argue that such increases were driven by fund competition rather than by improved investment prospects, suggesting that too much capital was chasing too few attractive investment opportunities.

Proposition 2 can explain the observed pattern despite the absence of fund competition. (Recall that the sharing rule k is time invariant. We introduce fund

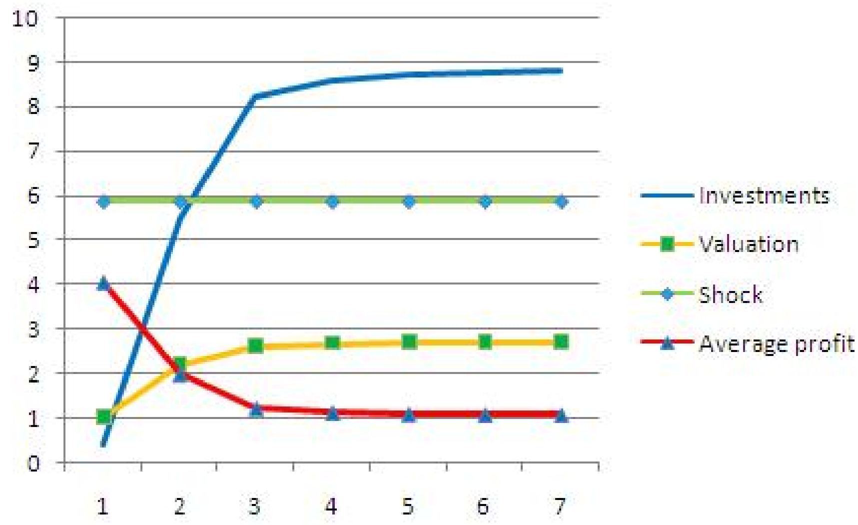

Figure 3: Entry, valuation and average profit.

competition in Section 5.2.) In our model, more entry and higher valuations are *jointly* caused by learning about the reorganization value. Yet, neither effect coincides with a concurrent or subsequent increase in the actual reorganization value.[7] Figure 3 illustrates these relationships for a trend path.

On a stochastic path, prices mean-revert around a positive trend, as revenues are random draws from a distribution with mean $\overline{V}$. Relative to its trajectory on the trend path, the price may overshoot or undershoot. Empirically, potential target firms should thus exhibit short-run return reversals and a positive long-run momentum during a wave. Although detectable ex post, the return predictability cannot be exploited by investors (e.g., Lewellen and Shankin, 2002).[8]

Over time, learning also reduces uncertainty so that prices become less volatile. With respect to end-of-period prices, this is also true in our model. Though, a

[7] If the shock to profitability is a shock to future cash flows, the increase in valuation levels corresponds to an increase in valuation multiples, such as the price-earnings ratio.

[8] Other papers showing that learning about parameters of stock price or return distributions can generate return predictability and excess volatility are Stulz (1987), Lewis (1989), Wang (1993), Timmermann (1993, 1996), Veronesi (1999), and Brennan and Xia (2001).

less literal interpretation leads to more subtle volatility implications. Suppose that the investment revenues *within* a period are observed sequentially. In that case, prices change more often in periods of high activity. That is, price volatility measured at a higher frequency may increase over time.

4.3 Performance and Entry

Despite learning, the market's expectations $\overline{V}_t$ typically diverge from the true expected reorganization value $\overline{V}$. When bringing the model predictions to real data, this distinction is crucial as empirical fund returns reflect the true investment profitability, as opposed to *subjective* expectations about the profitability. Predictions about fund performance must therefore be based on $\overline{V}$ as opposed to $\overline{V}_t$. On the trend path, the true expected profit of partnership i's period-t fund is $\pi_i^o(t) = \overline{V} - P_t^o - C_i$. Accordingly, the true expected average fund profit in period t is $\bar{\pi}_i^o(t) = \overline{V} - P_t^o - \overline{C}_t^o$ where $\overline{C}_t^o = \sum_{i=1}^{M_t^o} C_i / M_t^o$.

4.3.1 Industry

For a given shock, the true expected reorganization value $\overline{V}$ remains constant throughout a wave. But as discussed above, the trend path predicts a monotonic increase in the price P_t^o. This implies a general decline in average fund profits. This is reinforced by a decrease in average talent (i.e., an increase in $\overline{C}_t^o$) as the industry expands, since new entrants are always less talented than incumbent partnerships (Lemma 1).

Proposition 3 *Average fund performance tends to decrease during a wave.*

The line marked with triangles in figure 3 shows the evolution of average fund profits on a trend path. The decrease in average profits is steeper than the increase in prices because of the declining average talent. Recall that the decline in average performance does not rely on increased fund competition, but is purely a result of learning and heterogeneity in talent.

The decline in fund profitability across vintages appears to be at odds with the empirical finding that first-time funds tend to underperform the industry (Kaplan and Schoar, 2005). This is not the case if the empirical comparison between first-time and later-time funds is made in the cross-section; nor is it the case for the performance of consecutive funds relative to the industry. The next section elaborates on both points. However, it should be noted that our model cannot explain systematic increases in the *absolute* performance of consecutive funds by the same partnership.[9]

4.3.2 Partnerships

While average profitability declines, performance differences among active partnerships are persistent. That is, a partnership that has outperformed the industry likely continues to outperform the industry in subsequent periods. This follows directly from the assumed heterogeneity in talent, and is consistent with the empirical evidence (Kaplan and Schoar, 2005).

A result more unique to our model is that a partnership's talent and its time of entry are related. As already pointed out, entering partnerships are less talented than incumbent ones. By the same token, exiting partnerships are less talented than remaining ones. Entry and exit thus follow the last-in-first-out principle: the least talented are the latest to enter when market conditions improve, and the earliest to exit when the conditions deteriorate. Figure 4 illustrates this for

[9] Hochberg et al. (2008) provide a possible explanation for this phenomenon. They study a model in which investors in private equity funds (limited partners) may not be able to fully assess the talent of the fund managers (general partners). The investment terms in a first-time fund may be chosen such that the fund cannot fully exploit its profit potential, whereas the terms are relaxed as investors learn about the manager's talent.

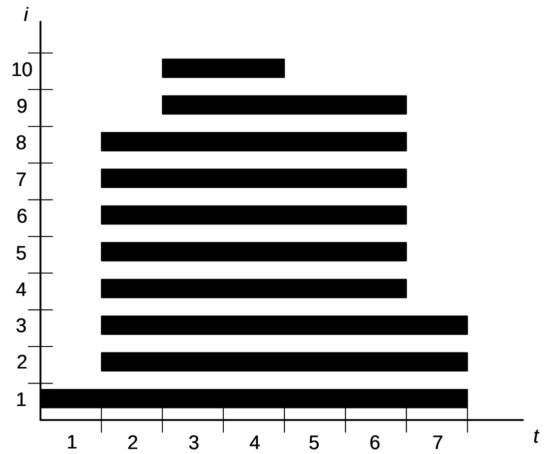

Figure 4: Last in, first out

the case of ten partnerships and a stochastic path that lasted for seven periods.

This entry pattern endogenously creates a cross-sectional relation between a partnership's age and its relative performance.

Proposition 4 *Younger partnerships perform worse and are less likely to raise a follow-on fund.*

The result highlights that a cross-sectional relationship between performance and experience need not (solely) be driven by experience gains, i.e., "learning-by-doing". Rather, it may reflect a causal relationship between a partnership's underlying ability and its timing of entry and exit. Empirically, the last-in-first-out pattern of our model predicts that many transient partnerships enter during a wave, while the partnerships which are left at the end are those that have been around from the beginning.

Since less talented partnerships enter the industry when aggregate activity and valuation levels are generally high (Proposition 4), the model also matches the empirical finding in Kaplan and Schoar (2005) that

Corollary 2 *Entrants in boom times are less likely to raise a follow-on fund.*

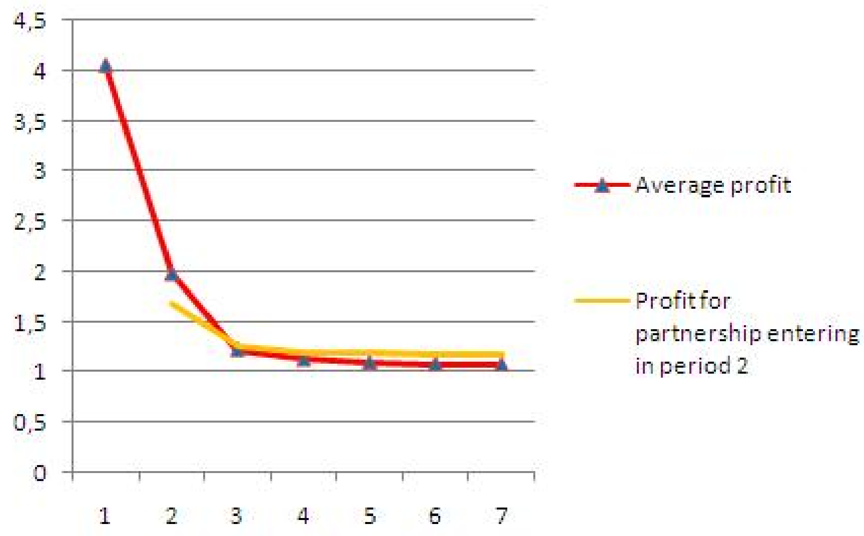

Figure 5: First-time funds.

Proposition 4 is a potential explanation for why first-time funds (i.e., young partnerships) in cross-sectional comparison underperform the industry. At the time a partnership i enters the market with its first fund, it belongs to the least talented partnerships in the industry. That is, first-time funds in our model are run by less talented managers than contemporaneous later-time funds. However, as the boom continues, even less talented partnerships enter in subsequent periods, so that i's quality relative to the industry improves over time. Thus,

Proposition 5 *Relative to the industry, a partnership's performance during a wave tends to improve across consecutive funds.*

Figure 5 illustrates this result by comparing the average fund profit on a trend path with the profit of a partnership that enters the market in period 2. While the fund is below average in period 2, it is better than the average fund in the industry from period 3 onwards.

4.3.3 Lagged correlations

Kaplan and Schoar (2005) also study the relationship between capital inflow into the private equity industry and fund returns. They find that capital inflow is positively related to past industry performance, and that capital inflow decreases subsequent performance. Their conclusion is that high performance attracts new funds, and that these funds perform worse.

It is straightforward to see that such relationships are in general attainable on the trend path where an industry expansion goes together with a decline in fund profits. That is, our model in general exhibits dynamics in which high past performance is followed by high future entry and low(er) future performance.

The relationships are even stronger on a stochastic path where average revenues are random and follow a mean-reverting process. To see this, consider the case $\overline{V}_1 = \overline{V} > 0$ where the market's initial expectations happen to be correct. As the market observes the average per-period revenues $\{\overline{v}_t\}$, it adapts its expectations $\{\overline{V}_t\}$. This learning implies a positive correlation between $\{\overline{v}_t\}$ and $\{\overline{V}_{t+1}\}$ (Lemma 2). Because the average revenues are independent draws from a gamma distribution with mean $\overline{V}$, the sequence $\{\overline{V}_t\}$ will be mean-reverting around $\overline{V}$. Moreover, as M_t and P_t are increasing in V_t (Lemma 1), and $\overline{C}_t$ is increasing in M_t, the sequences $\{M_t\}$, $\{P_t\}$ and $\{\overline{C}_t\}$ will also be mean-reverting and comove with each other (Figure 6).

Taken together, these relationships in conjunction with the mean-reversion imply the following. First, average revenues $\{\overline{v}_{t-1}\}$ are positively correlated with lagged investment activity $\{M_t\}$. Second, (true expected) average fund profits $\{\overline{V} - P_t - \overline{C}_t\}$ are countercyclical to investment activity $\{M_t\}$. And third, actual average fund profits $\{\overline{v}_t - P_t - \overline{C}_t\}$ have a negative autocorrelation. Intuitively, this means that

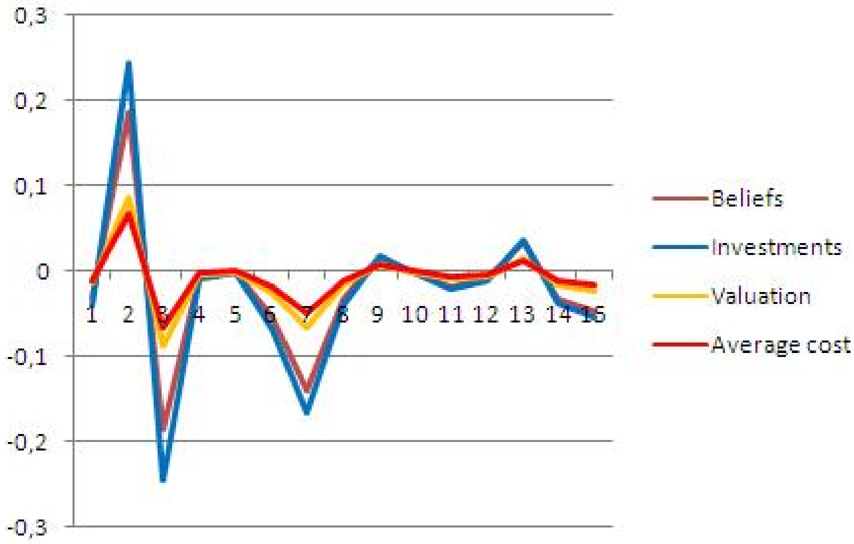

Figure 6: Mean-reversion patterns

Proposition 6 *Current entry increases in past average performance, while current performance decreases in current entry.*

Figure 7 illustrates these patterns. At first glance, one might be tempted to interpret them as "bad timing" by partnerships that enter the market when profitability drops, while being absent when it is high. However, as we show, such patterns arise naturally in a model with rational learning, where changes in perceived profitability and in actual profitability need not necessarily move in the same direction.

5 Extensions

5.1 Fund Size

Kaplan and Schoar (2005) also study the relationship between fund size and fund profitability and report two distinct findings: the relationship is positive and

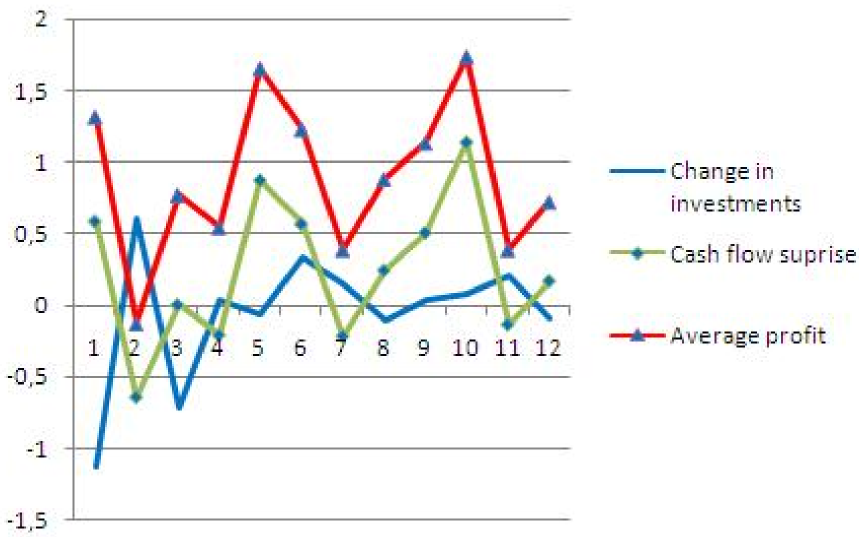

Figure 7: Lagged correlations.

concave across different partnerships, whereas it is negative across funds from the same partnership. Our baseline model is mute on this issue as it assumes a uniform and constant fund size. In this section, we extend the model to allow for variable fund size and show that the above relationships between size and profitability arise naturally.

For simplicity, suppose that $\mathcal{M} = 2$. Each partnership $i \in \mathcal{M}$ can now undertake as many investments as desired. However, we assume that the fund's per-period cost of operating a fund is increasing and convex in the number of considered investments. More specifically, let $C_{it}(M_{it}) = (M_{it} + C_i)^2$ where C_i is a constant that reflects the (inverse) talent of partnership i, and M_{it} is the number of investments undertaken by partnership i in period t.[10]

As long as $M_t \leq N_t$ is not a binding constraint, the number of investments

[10]The results also hold for $C_{it}(M_{it}) = M_{it}^2 + C_i$. In this case, a fund's marginal cost per investment is the same across all partnerships. By contrast, under the cost function in the text, a fund's marginal cost per investment decreases in the partnership's talent.

chosen by partnership i in period t satisfies $C_{it}(M_{it}) = (1-k)\overline{V}_t$. This yields

$$M_{it} = \sqrt{(1-k)\overline{V}_t} - C_i.$$

Since $C_1 < C_2$, this immediately implies that the fund of partnership 1 is larger than the fund of partnership 2. That is, fund size increases with talent.

A fund's profitability can be measured by its true expected profit *per investment*

$$\frac{M_{it}(\overline{V} - P_t) - C_{it}(M_{it})}{M_{it}} = \overline{V} - P_t - \frac{(1-k)\overline{V}_t}{\sqrt{(1-k)\overline{V}_t} - C_i}$$

which is decreasing in C_i. Thus, the larger (and more talented) fund earns a higher return per investment. The reason is that the average cost per investment is lower for the more talented fund, whereas the true expected revenue per investment $\overline{V} - P_t$ is the same for both funds. Rewriting the expected profit per investment as $\overline{V} - P_t - (1-k)\overline{V}_t/M_{it}$ and differentiating twice with respect to M_{it} furthermore shows that the relationship between fund size and fund profitability is concave.

Proposition 7 *Within the cross-section of funds, performance is increasing and concave in fund size.*

This is consistent with the first of the two findings mentioned above. Intuitively, for given market expectations, the more talented partnership raises a larger fund. Fund size and fund profitability are therefore jointly driven by the partnership's talent, and hence positive correlated. This result relies on the heterogeneity among fund managers but does not exploit the dynamic properties of the model, to which we turn next.

To examine how a fund size and fund profitability evolve during a wave, consider two arbitrary points in time t'' and t' such that $\overline{V}_{t''} > \overline{V}_{t'}$. From the above analysis, it follows that (as long as $M_t \leq N_t$ is not a binding constraint) a partnership raises a larger fund in t'' than in t', i.e. $M_{it''} > M_{it'}$. Partnership i's (true expected) profit in a period t can be written as

$$\overline{V} - P_t - \frac{(M_{it} + C_i)^2}{M_{it}}.$$

Since $P_t = k\overline{V}_t$, we know that $P_{t''} > P_{t'}$. Moreover, it is straightforward to show that $(M_{it} + C_i)^2 / M_{it}$ is increasing in M_{it}. Taken together, this implies that the true expected revenue per investment $\overline{V} - P_t$ is lower in t'' (because of the higher valuation levels), while the average cost per investment is higher in t'' (because of the larger fund size). In other words,

Proposition 8 *Within the same partnership, fund performance is decreasing in fund size.*

During a wave, market expectations tend to increase over time. Proposition 8 implies that, as a result, partnerships will raise larger but less profitable funds during the course of a wave. In fact, the decrease in profitability across consecutive funds will be proportional to the increase in size. This is consistent with the second finding by Kaplan and Schoar (2005).

5.2 Congestion

INCOMPLETE.

5.3 Leverage

INCOMPLETE.

6 Conclusion

The paper presents a model of the private equity market in which heterogenous partnerships learn about investment profitability from past outcomes and the stock of potential target firms is depletable. We derive the optimal entry and exit strategies of partnerships as a function of their talent and market expectations. An endemic feature of our model is that large expansions in private equity activity occur in waves with endogenous transitions from boom to bust. In addition, the model matches a wide range of stylized facts regarding the dynamics of aggregate investment, valuation levels and fund performance *during* the course of a wave.

Bayesian Updating and Derivation of $\overline{V}_t$

Let X be a gamma distributed random variable with shape parameter α and scale parameter θ. It is convenient to define $\beta = \theta^{-1}$ as the inverse scale parameter. The expected value of X is then equal to $\alpha\theta$ or equivalently $\alpha\beta^{-1}$.

In Bayesian probability theory, a class of prior probability distributions $p(\zeta)$ are said to be conjugate to a class of likelihood functions $p(x|\zeta)$ if the resulting posterior distributions $p(\zeta|x)$ belong to the same family as the prior probability distributions.

The gamma distribution is a conjugate prior to itself whenever the likelihood function is a gamma distribution with known shape parameter α and unknown inverse scale parameter β. Thus suppose we have a random sample $\{x_i\}_{i=1}^{n}$ from the random variable X which is gamma distributed with known shape parameter α and unknown inverse scale parameter β. Then the likelihood function is a gamma distribution with known shape parameter α and unknown inverse scale parameter β. If the prior probability distribution for β is a gamma distribution with known shape and inverse scale parameters τ and γ respectively then the resulting posterior distribution belongs to the gamma distribution and has a shape parameter equal to $\tau + n\alpha$ and an inverse scale parameter equal to $\gamma + \sum_{i=1}^{n} X_i$.

In addition, if a random variable X is gamma distributed with shape parameter α and scale parameter θ then the random variable X^{-1} is inverse gamma distributed with shape parameter α and scale parameter $\theta^{-1} = \beta$. The expected value of the random variable X^{-1} is then $\beta/(\alpha - 1)$. Finally if the random variable X is inverse gamma distributed with shape parameter α and scale parameter $\theta^{-1} = \beta$ then the random variable cX, where $c \in R^{+}$, is inverse gamma distributed with shape parameter α and scale parameter $c\theta^{-1} = c\beta$.

In our particular case this is all we need to derive the conditional expectation of the magnitude of the shock. Simply let $\alpha \to \alpha$, $\beta \to \beta$, $\tau \to \tau$, $\gamma \to \gamma$, $n \to M^t$, $X_i \to v_j$ and it immediately follows that

$$V_t = E(V\,|\mathcal{H}_t) = E(\alpha\beta^{-1}\,|\mathcal{H}_t) = \frac{\alpha\left(\gamma + \sum_{i=1}^{M^t} v_j\right)}{\tau + M^t\alpha - 1},$$

where $\mathcal{H}_t = \{v_j : j \in \mathcal{M}^t\}$.

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