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Incorporation of Robustness Properties into the Observer Based Anti-Windup Scheme in the Case of Actuator Uncertainties

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***Abstract*— Saturation is a very common nonlinearity in control systems and may produce serious performance deterioration or even loss of stability. To cope with saturation, several anti-windup (AW) schemes have been developed over a long time. Unfortunately, they are based on the assumption that there is a static nonlinearity between the output of the controller and the plant input, which, in many situations, is not the case, because of an actuator dynamics. Against this background we provide a design procedure for the design of the AW-compensator that guarantee stability of the observer based anti-windup to face unmodeled actuator dynamics and guarantee a certain level of performance. This mixed performance method is later extended for systems with unmeasurable actuator outputs by the use of an unknown input observer (UIO). The effectiveness of the presented algorithm is demonstrated on an engine test-bench simulator.**

I. Introduction

As result of physical limitations, the output of actuators is always limited in amplitude and rate, such as maximum or minimum torque in an engine or the maximum safe pitch rate in an aircraft. Such limits must be taken in account in the control design, otherwise the controller output will be different from the plant input, leading to wrong update of the controller states and to consequences ranging from performance deterioration over large overshoots and sometimes even to limit cycles or stability loss. Therefore, this phenomenon – usually called "controller windup" – has a paramount practical relevance and therefore many existing techniques address this problem of actuator constraints, *e.g.* the "Model Predictive Control" (MPC) [1] or even the traditional "Anti-Windup (AW) Compensator" [2].

Among the many contributions to handle input constraints for this class of problems, we recall the recent surveys of Galeani [3], Tarbouriech and Turner [4] about early and recent anti-windup research. The observer-based anti-windup design goes back to the publications of Åström and Hägglund [5] and Åström and Rundqwist [6]. All these methods, however, are based on the assumption that the actual plant input, i.e. the saturation output, is available together with the control output. In practice, however, this will be frequently not the case, as many saturation occur inside the process. To cope with this problem, other approaches have been developed which rely on robustness to cope with poorly known actuators.

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In the case of actuator or plant uncertainties there are only a few contributions, such as the approach of Teel [7], Sofrony [8], Turner [9] or Galeani [10], which consider the robustness of the anti-windup compensator in the design procedure.

In this paper based on the Integral-Quadratic-Constraints (IQCs) framework we extend the observer-based anti-windup design procedure to handle actuator uncertainties and present a design procedure that allows tuning the AW for performance requirements. To this end two weighting matrices are introduced in the performance criteria. In addition some nicely interpretable rules are provided for choosing the weighting matrices. In the case, where the true plant input can't be measured the closed-loop system is extended with an unknown input observer (UIO). To the best of our knowledge, we are not aware of any work in the literature dealing with a mixed performance AW-design, jointly tackling both, unmeasurable actuator outputs and dynamic actuator uncertainty. All these algorithms are tested on an engine test-bench simulation example.

The paper is structured as follows: first we introduce the observer based anti-windup compensator and present some robust stability considerations in the case of actuator uncertainties based on the IQC-framework. Afterwards an UIO is introduced to keep the performance in the case, when the output of the actuator isn't available for measurement. Finally the method is tested on a test-bench simulator.

II. Observer Based Anti-Windup Design

For reasons of global stability, throughout the paper the plant P of order n is assumed to be stable, and that the controller (A_c, B_c, C_c, D_c) stabilizes the system when the saturation is not active. The plant is described by the standard equations:

$$P\begin{cases}\dot{x}_p = A_p \cdot x_p + B_P \cdot sat(u_c) \\ y_p = C_p \cdot x_p\end{cases} \tag{1}$$

where $x_p \in \mathbb{R}^n, u_c \in \mathbb{R}^m$ and $y_p \in \mathbb{R}^p$.

A simple and effective way to handle input constraints is by adding a term of the form $L\cdot[sat(u_c) - u_c]$ to the controller dynamics which leads to the observer based anti-windup compensator (static AW) [11]:

$$\tilde{C}\begin{cases}\dot{\hat{x}}_c = A_c \cdot \hat{x}_c + B_c \cdot e + L[sat(u_c) - u_c] \\ u_c = C_c \cdot \hat{x}_c + D_c \cdot e\end{cases} \tag{2}$$

where (A_c, B_c, C_c, D_c) is the state-space realization of the controller and L is the desired feedback matrix of the anti-windup compensator (see Fig. 1).

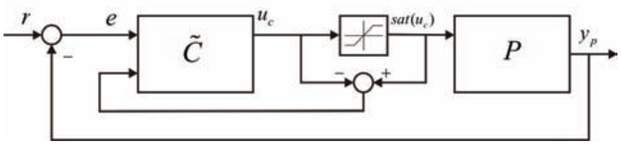

Fig. 1 Observer-based anti-windup scheme

In equation (2) e represents the tracking error, $sat(u_c)$ the actual input to the linear plant and L the anti-windup gain. In this context it has to be mentioned, that an additional direct feedthrough term $\tilde{L}\cdot[sat(u_c)-u_c]$ (see *e.g.* [12]) in the output equations of the controller (2) can improve the performance in the nominal case but may cause problems in the robust case and therefore is not considered in this work.

III. IQC-Framework

Integral quadratic constraints (IQCs) provide a unified framework for the robustness analysis of feedback interconnections of LTI plants and perturbation blocks Fig. 2. Although a unifying IQC-framework was introduced by Megretzki and Rantzer [13], the main ideas originate from Yakubovich (*e.g.* [14]).

The purpose of the IQCs in this paper is to exploit structural information about the saturation function $sat(\cdot)$ and the actuator uncertainty and use this information to guarantee robust stability of the observer-based AW-compensator. To this end for observer-based AW synthesis the frequency dependent equations for robust-stability based on IQCs are transformed by the Kalman-Yakubovich-Popov (KYP) lemma to Linear Matrix Inequalities (LMIs). In the case of the actuator uncertainty the LMIs offers a tradeoff between robust-stability and robust performance.

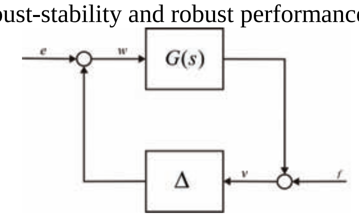

Fig. 2 Basic feedback configuration

IQCs are used for stability analysis of a feedback interconnection (see Fig. 2)

$$\begin{aligned} v &= G(s)w + f \\ w &= \Delta(v) + e \end{aligned} \tag{3}$$

where $G(s)\in RH_\infty$ and Δ is a causal operator. This feedback interconnection of G and Δ is well-posed if the map $(v,w)\mapsto(e,f)$ defined by (3) has a causal inverse on L_{2e}. The interconnection is stable if, in addition the inverse is bounded, i.e., if there exists a constant $C>0$ such that

$$\int_0^T (|v|^2+|w|^2)dt \le C\int_0^T (|f|^2+|e|^2)dt \tag{4}$$

for any $T\ge 0$ and for any solution of (3). In the linear case this is equivalent to $I-G\Delta$ is causally invertible.

The causal uncertainty block Δ satisfies an IQC with a matrix-valued function $\Pi(i\omega)=\Pi(i\omega)^*$ that is bounded on the imaginary axis $\mathbb{C}^0$ if:

$$\int_{-\infty}^{\infty}\begin{pmatrix}\widehat{\Delta v}(i\omega)\\ \hat{v}(i\omega)\end{pmatrix}^* \Pi(i\omega)\begin{pmatrix}\widehat{\Delta v}(i\omega)\\ \hat{v}(i\omega)\end{pmatrix} d\omega \ge 0 \quad \forall v\in L_2[0,\infty) \tag{5}$$

where $\hat{}$ denotes the Fourier transform. If the uncertainty Δ defined by an IQC according to (5) includes all available structure information on Δ, the stability of the feedback interconnection (3) can be proven with the following theorem.

Theorem 1: ([13]) Let $G(s)\in RH_\infty$ and Δ a bounded causal operator and assume that:

1) for every $\tau\in[0,1]$, the interconnection of $\tau\Delta$ and G is well-posed
2) for every $\tau\in[0,1]$, the IQC defined by Π is satisfied by $\tau\Delta$
3) there exists $\varepsilon>0$ such that

$$\begin{bmatrix} G(i\omega)\\ I\end{bmatrix}^* \Pi(i\omega)\begin{bmatrix} G(i\omega)\\ I\end{bmatrix} \le -\varepsilon I \quad \forall\omega\in\mathbb{R} \tag{6}$$

Then, the feedback interconnection (3) is stable.

Thanks to the KYP lemma we are able to transform the frequency domain criterion (6) into equivalent conditions on the system matrices in the realization of the transfer function G and Π [15]. In our case we are using a realization of $G=C(\imath\omega I-A)^{-1}B+D$ and a constant multiplier $\Pi(i\omega)=M$ and therefore we get:

$$\begin{aligned} &\begin{bmatrix} I\\ G(i\omega)\end{bmatrix}^* \Pi(i\omega)\begin{bmatrix} I\\ G(i\omega)\end{bmatrix} \\ &=\begin{bmatrix} I\\ C(i\omega I-A)^{-1}B+D\end{bmatrix}^* M\begin{bmatrix} I\\ C(i\omega I-A)^{-1}B+D\end{bmatrix} \le -\varepsilon I \end{aligned} \tag{7}$$

Then (7) is equivalent to the existence of $P=P^T>0$ such that:

$$\begin{bmatrix} I & 0\\ A & B\\ \hline 0 & I\\ C & D\end{bmatrix}^T \left[\begin{array}{cc|cc} 0 & P & 0 & 0\\ P & 0 & 0 & 0\\ \hline 0 & 0 & Q & S\\ 0 & 0 & S^T & R\end{array}\right]\begin{bmatrix} I & 0\\ A & B\\ \hline 0 & I\\ C & D\end{bmatrix} < 0; M=\begin{bmatrix} Q & S\\ S^T & R\end{bmatrix} \tag{8}$$

In order to show stability a constant multiplier matrix M according to (8) is derived in the case of the deadzone function $dz(\cdot)$ and a combination of the actuator uncertainty and $dz(\cdot)$.

IV. Systematic Design Method of the Anti-Windup Observer Gain Matrix L

In this section a systematic method to design the feedback-gain L of the AW-compensator is presented, which satisfies the following specifications:

1) The stability of the closed-loop system against input magnitude saturation is guaranteed.
2) The AW-compensator gain L should be designed such that the weighted difference in the controller states between the linear unsaturated and the nonlinear saturated case is minimized.

Therefore to transform this nonconvex optimization problem into a convex one, replace in Fig. 1 $sat(u_c)=u_c-dz(u_c)$. As a result the closed-loop system has the following structure (see Fig. 3):

$$\begin{aligned} &\dot{x} = Ax + (B_w + B_L L)w_1 \\ &z_1 = Cx, x=\begin{bmatrix} x_p & x_c\end{bmatrix}^T, z_1 = u_c, w_1 = dz(z_1) \end{aligned} \tag{9}$$

and A, B_w, B_L and C can easily be computed from the system equation (1) and the controller equation (2), where the transformation of the saturation function is used.

In order to use the IQC-framework for the design of L a multiplier and a certain performance criteria is necessary. Therefore to derive the constant multiplier matrix for $dz(\cdot)$ the sector condition in the sector $[0 \quad K_\kappa]$ is used:

$$\int_0^\infty \begin{bmatrix} w_1 \\ z_1 \end{bmatrix}^T \underbrace{\begin{bmatrix} -2I & K_\kappa \\ K_\kappa & 0 \end{bmatrix}}_{M} \begin{bmatrix} w_1 \\ z_1 \end{bmatrix} dt \geq 0 \tag{10}$$

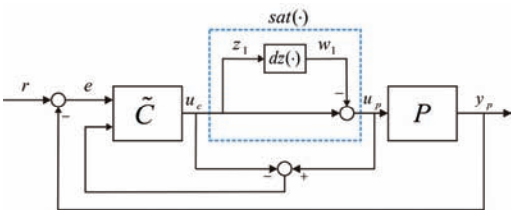

Fig. 3 Observer-based AW with deadzone function

The usage of the constant multiplier M of (10) in (11) is equivalent to the application of the multivariable circle criterion [13]. Hence for stability the following LMI has to be fulfilled:

$$\begin{bmatrix} I & 0 \\ A & B_w + B_L L \\ \hline 0 & I \\ C & 0 \end{bmatrix}^T \left[\begin{array}{cc|cc} 0 & Q & 0 & 0 \\ Q & 0 & 0 & 0 \\ \hline 0 & 0 & -2I & K_\kappa \\ 0 & 0 & K_\kappa & 0 \end{array}\right] \begin{bmatrix} I & 0 \\ A & B_w + B_L L \\ \hline 0 & I \\ C & 0 \end{bmatrix} < 0 \tag{11}$$

It has to be mentioned that in (11) the bilinear matrix inequality (BMI) can be transformed due to a congruence transformation $diag(Q^{-1}, I)$ and a variable transformation $Q^{-1} = X$ to the first LMI in (14). If K_κ is equivalent to the identity matrix I, global stability can be guaranteed by the LMI in (11). Since we want to reach the performance of the linear unsaturated controller after saturation as fast as possible, for the performance criteria the weighted difference in the controller states between the linear unsaturated $[x_p \quad x_c]^T$ and saturated nonlinear $\begin{bmatrix} \hat{x}_p & \hat{x}_c \end{bmatrix}^T$ case is chosen:

$$\begin{aligned} \dot{\bar{\xi}} &= A\bar{\xi} - (B_w + B_L L) R_c w_1 \\ z_p &= x_c - \hat{x}_c = Q_c \underbrace{[0 \quad I]}_{C_p} \bar{\xi} \end{aligned} \tag{12}$$

where

$$\begin{gathered} \xi = \begin{bmatrix} x_p \\ x_c \end{bmatrix}, \hat{\xi} = \begin{bmatrix} \hat{x}_p \\ \hat{x}_c \end{bmatrix}, \bar{\xi} = \begin{bmatrix} x_p - \hat{x}_p \\ x_c - \hat{x}_c \end{bmatrix}, R_c > 0, Q_c > 0 \\ T_{z_p, w_1} = Q_c C_p (sI - A)^{-1} (B_w + B_L L) R_c \end{gathered} \tag{13}$$

and the weighted H_2-norm is minimized. Q_c is necessary for the weighting of the discrepancy in the controller states and R_c can be used to consider the more critical saturated input to the process for the performance criteria. The overall resulting LMI for the design of L has the following form:

$$\begin{aligned} & \min \gamma \\ & s.t. \\ & \begin{bmatrix} XA^T + AX & B_w + B_L L + XC^T K_\kappa \\ * & -2 \cdot I \end{bmatrix} < 0, X > 0 \\ & \begin{bmatrix} AK + KA^T & KQ_c^T C_p^T \\ * & -\gamma \cdot I \end{bmatrix} < 0, \begin{bmatrix} K & (-B_w - B_L L) R_c \\ * & Z \end{bmatrix} > 0 \\ & trace(Z) < \gamma, K > 0, \gamma > 0; \end{aligned} \tag{14}$$

In (14) the first two inequalities are responsible for stability of the closed-loop system and the remaining inequalities are for ensuring a certain H_2-performance criteria, where K represents the observability gramian. The goal of the AW-compensator is to recover the desired linear controller performance after saturation as fast as possible. Hence the tuning of the AW-compensator is done by just choosing the weighting matrices Q_c and R_c in (13). The selection of the weights is intuitive: high values in R_c are chosen for the critical actuators. The values in Q_c are used to selectively weight those controller states errors which are considered to be more critical than the others. To find out the critical states, the difference between the linear unsaturated and the nonlinear saturated controller states is considered. If the LMI in (14) is fulfilled and the performance is satisfying you are done, otherwise you have to adapt the weights Q_c and R_c. Notice that the degree of freedom in the choice of the weighting matrices is important to enforce the feasibility of (14) and a desired performance.

V. Robust Observer Based Anti-Windup

In the case of actuator uncertainties (Fig. 4) the observer-gain L has to fulfill the specifications:

1) The stability of the closed-loop system against input magnitude saturation and some unknown norm-bounded actuator dynamics has to be guaranteed.
2) The weighted difference in the controller states between the linear unsaturated and the nonlinear saturated case is minimized.

To be robust in the face of the uncertainty, a multiplier fulfilling (5) and (6) has to be derived. Therefore the actuator dynamics is modeled by a multiplicative uncertainty:

$$\Delta_{Act}(s) = I + \tilde{\Delta}_{Act}(s) \tag{15}$$

where $\tilde{\Delta}_{Act}(s) \in RH_\infty$ with $\left\| \tilde{\Delta}_{Act}(s) \right\|_\infty < \gamma$. In the case of the norm bounded actuator uncertainty Δ_{Act}, the closed-loop system has the following structure:

$$\begin{aligned} \dot{x} &= Ax - (B_w + B_L L) w_1 + (B_w + B_L L) w_2 \\ z_1 &= Cx \\ z_2 &= z_1 + w_1 \end{aligned} \tag{16}$$

where

$$x = \begin{bmatrix} x_p \\ x_c \end{bmatrix}, z_1 = u_c, z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} u_c \\ u_c + \Delta u_c \end{bmatrix}, w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \Delta z_1 \\ dz(z_2) \end{bmatrix} \tag{17}$$

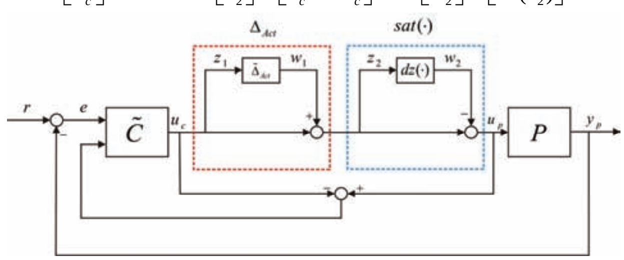

Fig. 4 Unmodelled actuator dynamics

The multiplier of the $dz(\cdot)$ and Δ_{Act}, can be derived firstly using the sector condition of $dz(\cdot)$ in the sector $[0 \quad K_\kappa]$:

$$\int_0^\infty (w_2^T K_\kappa (w_1 + z_1) + (w_1 + z_1)^T K_\kappa w_2 - 2 w_2^T w_2) dt \geq 0 \tag{18}$$

and secondly bounding the actuator uncertainty by the H_∞-norm $\|\tilde{\Delta}_{Act}(s)\|_\infty < \gamma$:

$$\int_0^\infty (\gamma^2 z_1^T z_1 - w_1^T w_1)dt > 0 \qquad (19)$$

As a result of combining (18) and (19), the constant multiplier matrix has the following structure:

$$\int_0^\infty \left(\begin{bmatrix} w_1 \\ w_2 \\ z_1 \end{bmatrix}^T \begin{bmatrix} -I & K_\kappa & 0 \\ K_\kappa & -2I & K_\kappa \\ 0 & K_\kappa & \gamma^2 I \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ z_1 \end{bmatrix} \right) dt \ge 0 \quad \forall w_1, w_2 \in \mathbb{R} \qquad (20)$$

Hence the closed-loop system (16) with the norm-bounded actuator uncertainty is stable if:

$$\begin{bmatrix} * \end{bmatrix}^T \left[\begin{array}{cc|ccc} 0 & Q & 0 & 0 & 0 \\ Q & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -I & K_\kappa & 0 \\ 0 & 0 & K_\kappa & -2I & K_\kappa \\ 0 & 0 & 0 & K_\kappa & \gamma^2 I \end{array} \right] \begin{bmatrix} I & 0 & 0 \\ A & -B_w - B_L L & B_w + B_L L \\ \hline 0 & I & 0 \\ 0 & 0 & I \\ C & 0 & 0 \end{bmatrix} < 0 \qquad (21)$$

where [*] is used for expressions that can be derived by symmetry. After applying the Schur complement, a congruence transformation $diag(Q^{-1}, I, I, I)$ and a variable transformation $Q^{-1} = X$, the BMI in (21) can be transformed to the first LMI in (22). Similar to (12) again the H_2-norm from the input w_2 to the error in the controller states $\bar{\xi}$ is minimized. Then the design of the AW-compensator is a tradeoff between robust stability and performance:

$$\begin{gathered} \min \alpha_1 \gamma_\Delta + \alpha_2 \gamma_p \\ s.t. \\ \begin{bmatrix} XA^T + AX & -B_w - B_L L & B_w + B_L L + XC^T K_\kappa & XC^T \\ * & -I & K_\kappa & 0 \\ * & * & -2I & 0 \\ * & * & * & -\gamma_\Delta \end{bmatrix} < 0 \\ X > 0; \gamma_\Delta > 0; \gamma = \sqrt{\frac{1}{\gamma_\Delta}} \\ \begin{bmatrix} AK + KA^T & K \cdot Q_c{}^T C_p{}^T \\ * & -\gamma_p \cdot I \end{bmatrix} < 0; \begin{bmatrix} K & (-B_w - B_L L)R_c \\ * & Z \end{bmatrix} > 0 \\ K > 0; \gamma_p > 0; trace(Z) < \gamma_p \end{gathered} \qquad (22)$$

In (22) the LMIs one to three are responsible for the closed-loop stability and the remaining LMIs for the inclusion of the H_2-performance criterion. Thanks to this design procedure it is possible to ensure stability of the closed-loop system for all norm bounded uncertainties $\tilde{\Delta}_{Act}(s)$ with $\|\tilde{\Delta}_{Act}(s)\|_\infty < \gamma$. At the same time it is also possible to evaluate the maximum tolerable level of the actuator dynamic uncertainty:

$$\|\Delta_{Act}(s)\|_\infty \le 1 + \|\tilde{\Delta}_{Act}(s)\|_\infty \qquad (23)$$

VI. Unknown Input Observer to Cope with Actuator Uncertainties

In real life applications it is often the case that the true plant input, the output of the actuator is not available for measurement. Therefore an unknown input observer (UIO) is introduced to estimate the true input of the plant u_p , such that the AW-compensator designed by (22) can still be used.

It has to be mentioned, that the UIO is only necessary if an unknown dynamics is present in the actuator. Due to this scheme, the performance in the case of actuator uncertainties and not measurable plant inputs can be kept similarly to the case where the plant input is available. To estimate the true plant input we assume to have a linear system in the form:

$$P \begin{cases} \dot{x}_p = A_p \cdot x_p + B_p \cdot u_p \\ y_p = C_p \cdot x_p \end{cases} \qquad (24)$$

with A_p stable and we want to reconstruct u_p from y_p. For simplicity we consider u_p as a constant, *i.e.* $\dot{u}_p = 0$ and therefore u_p can be added as a state to the plant P :

$$P_{ext} \begin{cases} \begin{bmatrix} \dot{x}_p \\ \dot{u}_p \end{bmatrix} = \underbrace{\begin{bmatrix} A_p & B_p \\ 0 & 0 \end{bmatrix}}_{A_{ext}} \cdot \underbrace{\begin{bmatrix} x_p \\ u_p \end{bmatrix}}_{x} \\ y_p = \underbrace{\begin{bmatrix} C_p & 0 \end{bmatrix}}_{C_{ext}} \cdot \begin{bmatrix} x_p \\ \hat{u}_p \end{bmatrix}, u_p = \underbrace{\begin{bmatrix} 0 & I \end{bmatrix}}_{C_{u_p}} \cdot \begin{bmatrix} x_p \\ u_p \end{bmatrix} \end{cases} \qquad (25)$$

Of course, there are other possibilities for input deconvolution in literature [16]. However our choice is motivated by the application example, which is presented in the paper, section VII. Using an observer P_{obs} with the state-space description:

$$P_{obs} \begin{cases} \dot{\hat{x}} = A_{ext} \cdot \hat{x} + L_{UIO} \cdot (y_p - \hat{y}_p) \\ \hat{y}_p = C_{ext} \cdot \hat{x}, \hat{u}_p = C_{u_p} \cdot \hat{x} \end{cases} \qquad (26)$$

and the following error dynamics equation:

$$\begin{aligned} \dot{\xi}_{obs} &= \dot{x} - \dot{\hat{x}} = A_{ext} \cdot x - A_{ext} \cdot \hat{x} - L_{UIO} \cdot (y_p - \hat{y}_p) \\ \dot{\xi}_{obs} &= (A_{ext} - L_{UIO} \cdot C_{ext}) \cdot \xi_{obs} \end{aligned} \qquad (27)$$

which converges to zero if $(A_{ext} - L_{UIO} \cdot C_{ext})$ is Hurwitz, the estimated plant input converges to true plant input:

$$u_p - \hat{u}_p = C_{u_p} \cdot \xi_{obs} \qquad (28)$$

As a result the true input to the plant can be estimated and is equivalent to the state $\hat{u}_p$. Now using an observer $(H_2, H_\infty, etc...)$ a virtual plant input $\hat{u}_p$ is available for the anti-windup design.

Although there is an additional dynamic system in the closed-loop system which has to be considered for stability the performance can be kept similarly. To prove the stability of the extended version with the UIO, we take the L designed by (22) and derive a LMI feasibility problem according to the IQC-framework.

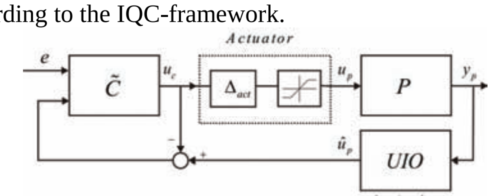

Fig. 5 Extended observer-based AW-scheme with UIO for actuator uncertainties (u_p …true plant input, $\hat{u}_p$ …estimated plant input)

VII. Application: Engine Test-Bench

A. Plant Description

The following Fig. 6 shows the combustion engine test-bench system, consisting of a dynamometer, which is simulating the load, a common connection shaft and the engine itself. The control objective for the test-bench system is to follow simultaneously a reference trajectory of the engine torque and the engine speed using the control variables accelerator pedal α and the torque of the dynamometer T_D. The accelerator pedal can be approximated by a time delay, a magnitude saturation and a second order low-pass filter. To consider the delay time of the accelerator pedal it is modeled with a pade-approximation and added to the model. The dynamometer is modeled similar except the dead-time element. The resulting MIMO-control consists of two inputs and two outputs. The mathematical model has the following structure:

$$\begin{aligned} \Delta\dot{\varphi} &= \omega_E - \omega_D \\ \dot{\omega}_D &= \theta_D^{-1}(c\Delta\varphi + d(\omega_E - \omega_D) - T_D) \\ \dot{\omega}_E &= \theta_E^{-1}(T_E - c\Delta\varphi - d(\omega_E - \omega_D)) \\ \dot{T}_E &= -\rho(\omega_E,\alpha)T_E + \rho(\omega_E,\alpha)T_{E,Stat} \end{aligned} \qquad (29)$$

where the constaints of the actuators can be summarized:

$$\alpha_{\max} = 100\%, \alpha_{\min} = 0\%, T_{D,\max} = 295\,Nm, T_{D,\min} = -295\,Nm \qquad (30)$$

For a detailed description of the engine test-bench, see [17].

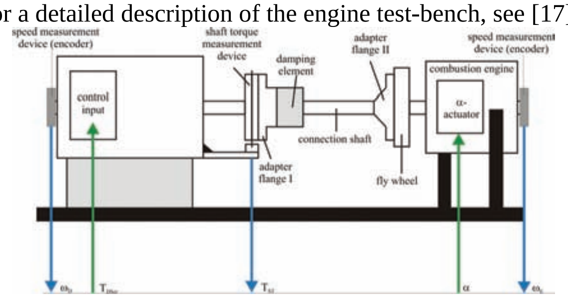

Fig. 6 Engine test-bench

The unconstrained H_∞-controller of order 20 with integral action is designed to get offset tracking and offers a satisfying performance.

B. Magnitude Saturation + Weighting Matrices Q_c, R_c

In the first case the advantage of using weighting matrices in the AW-design procedure should be clarified. As it can be seen in Fig. 7, minimizing a performance criterion leads to an undesired shift in the performance weighting. To this end the introduction of the weighting matrices allows to tune the AW-compensator. The output response in the shaft-torque tracking can be improved significantly, while the engine speed-tracking is kept close to the design without weighting matrices. It is to mention that due to the rules in section IV it is easy to get this result.

C. Magnitude Saturation + Unmodelled Actuator dynamics + Unknown Input Observer

In this example the robustness of the proposed method is tested by extending the nominal plant with an unmodeled dynamics of the accelerator pedal $\Delta_{act,\alpha}(s)$ and the dynamometer $\Delta_{act,T_D}(s)$.

$$\Delta_{act,\alpha}(s) = \frac{\omega^2_{n,\alpha}}{s^2 + 2\varsigma_\alpha \omega_{n,\alpha} s + \omega^2_{n,\alpha}}, \Delta_{act,T_D}(s) = \frac{\omega^2_{n,T_D}}{s^2 + 2\varsigma_{T_D}\omega_{n,T_D} s + \omega^2_{n,T_D}} \qquad (31)$$
$$\omega_{n,\alpha} = 500\frac{rad}{s}, \zeta_\alpha = 0.4, \omega_{n,T_D} = 1000\frac{rad}{s}, \zeta_{T_D} = 0.4$$

This unmodeled actuator dynamics drives the saturated plant to a limit cycle at the shaft torque tracking. The output due to our robust AW-design can cancel the oscillations in the actuators and hence in the shaft-torque tracking, although the performance degrades. In the case where the actuator output is not available, we add the unknown input observer to the closed-loop system to estimate the true input. In order to get a good estimate of the virtual plant input a high process noise has to be assumed in the weighting of this channel. As it can be seen in Fig. 8, the performance can be kept comparable to the case where plant input can be measured.

D. Brief Summary of the Results

The results confirm that it is possible to robustify the design of the AW-compensator, although the performance is not completely satisfying yet. An additional direct feedthrough term $\tilde{L}\cdot[sat(u_c) - u_c]$ in the output equations of the controller (1) may improve the performance, and therefore should be considered in future work. However it is possible to get rid of the oscillations. The allowable norm bounded uncertainties are summarized in TABLE I. The

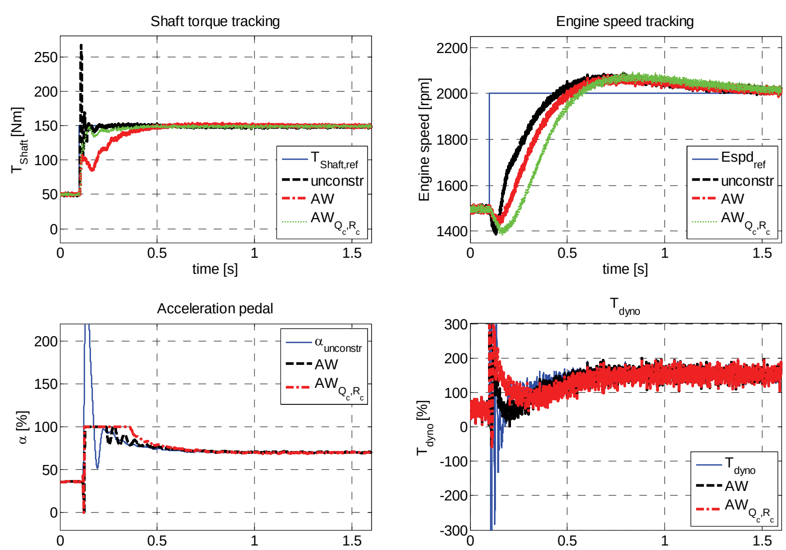

Fig. 7 Engine test-bench with input magnitude saturation; unconstrained controller… *unconstr*, AW without weighting matrices… *AW*, AW-design including weighting matrices… AW_{Q_c,R_c}

table shows that the maximum robustness is reached with the robust AW-design procedure. Due to the performance-robustness tradeoff it would be possible to get a more robust AW-compensator.

TABLE I
COMPARISON OF THE ROBUSTNESS OF THE DIFFERENT AW-COMPENSATORS

Description	Abbreviation	Allowable norm bounded actuator uncertainty γ
AW without weighting	AW	1.82
AW + weighting	AW_{Q_c,R_c}	1.58
AW + weighting + robust	$AW_{Q_c,R_c,\Delta_{Act}}$	2.21

VIII. CONCLUSION

The main result of the paper is the derivation of a robust mixed performance design procedure based on the IQC-framework for the observer-based AW. It guarantees closed-loop stability for all norm bounded actuator uncertainties and a certain level of performance. The presented algorithm shows a good performance in the case of input magnitude constraints and actuator uncertainties. If the proposed scheme is augmented with an unknown input observer in the case of not measurable plant inputs, the performance can be kept on a satisfying level. Furthermore straightforward rules for the selection of weighting matrices in the design procedure of the observer-based AW-compensator are provided, which helps to improve the performance and are really necessary for real-life application examples.

In summary, even though the proposed scheme has a simple structure, it works quite well and therefore it is highly recommended for real applications, which has been shown on the verified engine test-bench simulator.

The developed algorithms should be tested on our real-life engine test-bench. For future work the direct feedthough term in the controller output should be considered.

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REFERENCES

[1] E. F. Camacho and C. Bordons, *Model Predictive Control*, Second ed: Springer Verlag, 2004.

[2] H. A. Fertik and C. W. Ross, "Direct digital control algorithm with anti-windup feature," *ISA Transactions*, pp. 317-328, 1967.

[3] S. Galeani, S. Tarbouriech, M. Turner, and L. Zaccarian, "A Tutorial on Modern Anti-Windup Design," in *European Control Conference* Budapest, 2009, pp. 306-323.

[4] S. Tarbouriech and M. Turner, "Anti-windup design: an overview of some recent advances and open problems," *IET Control Theory and Applications*, pp. 1-19, 2009.

[5] K. J. Åström and T. Hägglund, *Automatic Tuning of PID Controllers*: Instrument Society of America, 1988.

[6] K. J. Åström and L. Rundqwist, "Integrator windup and how to avoid it," in *American Control Conference*. Pittsburgh, 1989.

[7] A. R. Teel and N. Kapoor, "The L2 anti-windup problem: Its definition and solution," in *European Control Conference*, 1997.

[8] J. Sofrony, M. C. Turner, and I. Postlethwaite, "Anti-windup synthesis using Riccati equations," *International Journal of Control*, pp. 112-128, 2007.

[9] M. C. Turner, G. Herrmann, and I. Postlethwaite, "Incorporating Robustness Requirements Into Antiwindup Design," *IEEE Transactions on Automatic Control*, pp. 1842-1855, 2007.

[10] S. Galeani and A. R. Teel, "On a performance-robustness trade-off intrinsic to the natural anti-windup problem," *Automatica*, pp. 1849-1861, 2006.

[11] N. Kapoor, A. R. Teel, and P. Daoutidis, "An Anti-Windup Design for Linear Systems with Input Saturation," *Automatica*, pp. 559-574, 1998.

[12] E. F. Mulder, M. V. Kothare, and M. Morari, "Multivariable anti-windup controller synthesis using linear matrix inequalities*," *Automatica*, pp. 1407 - 1416, 2001.

[13] A. Megretski and A. Rantzer, "System Analysis via Integral Quadratic Constraints," *IEEE Transactions on Automatic Control*, pp. 819-830, 1997.

[14] V. A. Yakubovich, "Frequency conditions for the absolute stability of control systems with several nonlinear or linear nonstationary blocks," *Automation and Remote Control*, 1967.

[15] U. Jönsson, "Lecture Notes on Integral Quadratic Constraints," 2001, pp. 53.

[16] A. Saberi, A. Stoorvogel, and A. A. Sannuti, *Filtering Theory*: Birkhäuser, 2007.

[17] E. Gruenbacher, L. d. Re, H. Kokal, M. Schmidt, and M. Paulweber, "Online Trajectory Shaping Strategy for Dynamical Engine Test Benches," presented at IEEE International Symposium on Intelligent Control, 2006.

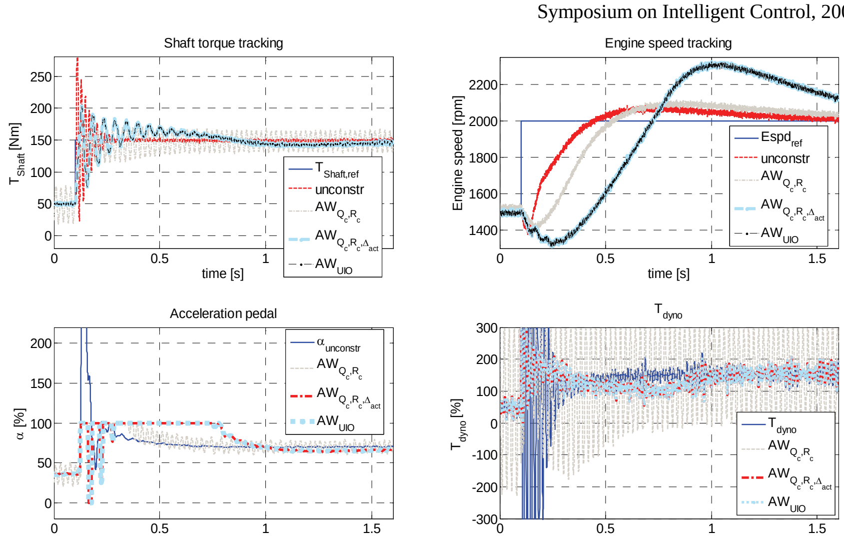

Fig. 8 Engine test-bench with input magnitude saturation and unknown actuator dynamics; unconstrained controller… *unconstr* , weighted AW… AW_{Q_c,R_c} , robust weighted AW… $AW_{Q_c,R_c,\Delta_{act}}$, robust weighted AW in combination with an UIO… AW_{UIO}