Resource Allocation for Constrained Backhaul in Picocell Networks

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***Abstract*—Cellular network capacity and coverage can be improved by deployment of low power base stations referred to as picocells. Due to the associated deployment cost, a large number of picocells challenges the traditional approach to backhaul, where each cell has a dedicated backhaul link. This paper considers a more efficient approach, in which the backhaul is provided over a wireless channel shared among picocells. The considered backhaul network consists of multiple connector nodes (CNs) each providing backhaul to a group of picocells. A key problem in this setting is how to efficiently exploit and allocate this limited bandwidth resource among picocells. We consider joint scheduling and power allocation of backhaul transmissions based on limited bandwidth availability. We propose a backhaul scheduling approach based on traffic demands on picocells (i.e., the load of their mobile users), that maximizes the picocell utility. The approach applies to any underlying physical layer transmission scheme. We then investigate the proposed solution for an OFDM system. We first determine optimal power allocation under power and interference constraints for OFDM transmissions from multiple CNs. We then present an algorithm that performs joint scheduling and power allocation for OFDM transmissions in the backhaul channel.**

I. Introduction and Motivation

A promising approach to improve wireless network capacity and cellular coverage is to deploy a large number of low power base stations in the existing cellular infrastructure. These base stations will create cells of smaller radius referred to as picocells, resulting in high capacity and better coverage in these areas. The increased density of picocells when compared to a macrocell network raises a number of design issues. In particular, the large number of picocells requires that their operation, configuration and optimization of their transmit parameters be automated. These requirements lead to the concept of *self organizing networks (SONs)*. Picocells, forming a self-organized network after being added to an existing wireless network, should automatically recognize the neighboring base stations, determine frequency bands to operate in, allow new mobile users to connect and send data, and adapt to the wireless channel conditions.

The presence of a large number of picocells challenges also the existing solutions to backhaul communications between a pico base station and the core network. Conventionally, the backhaul communications for each base station is performed over a dedicated connection, such as fiber or a microwave. However, providing a dedicated backhaul to a large number of picocells would lead to a large deployment cost. In picocell networks, an attractive alternate solution is to deploy wireless backhaul for multiple picocells *simultaneously*. The backhaul communication is then performed over a wireless channel which is shared among picocells, thus avoiding the need for dedicated backhaul.

This approach enables a more efficient backhaul to picocells than existing solutions. However, it introduces several challenges:

1) Capacity is smaller than the capacity provided over fiber or microwave backhaul links. Consequently, the wireless backhaul capacity can be constrained compared to the rates in the wireless access network providing service to mobile users.
2) Due to fading in the wireless channel, the backhaul capacity is time-varying.
3) There is interference between picocells. Forwarding data to/from one picocell interferes with communications of other picocells.

These differences reveal that successful wireless backhaul solutions demand: 1) transmission schemes that achieve high spectral efficiency in the backhaul wireless channel, and 2) optimal allocation of the limited backhaul among picocells.

The wireless channel from multiple connector nodes (CNs) each providing backhaul to a group of picocells, is an extension of the *Gaussian MIMO interference channel* and is equivalent to the multi-cell downlink of the MIMO cellular system. The capacity region of this channel is in general unknown, but efficient transmission schemes have been extensively developed. Full degrees of freedom in the Gaussian MIMO interference channel are achieved with *interference alignment* [1], [2], also effective in reducing inter-cell interference in the downlink cellular system [3]. For 4G cellular systems, OFDM transmission has been adopted, motivating extensive research on resource allocation for these systems (see [4] and the references therein). These techniques are also of interest for the wireless backhaul systems studied in this paper. There are several important differences in the backhaul wireless system compared to the cellular downlink:

1) Users in the system (i.e., picocells) are stationary.
2) The number of picocells served by one CN is typically much smaller than in a macro cellular network ($2-10$

picocells).

3) Nodes at both ends (CNs and picocells) are equipped with multiple antenna elements (8-16).
4) The capacity per user is higher than in the cellular network.
5) The resource allocation needs to satisfy dynamic traffic demands of picocells.

The differences 1)-4) eliminate some of the challenges faced by the cellular downlink, and may allow for simpler or more efficient solutions for the wireless backhaul. For example: 1) no handoff will be needed to handle mobility; 2) due to stationarity and the smaller number of picocells, the CNs will need less extensive channel state information (CSI), and will also be able to perform simpler scheduling.

We will specifically consider the problem of power allocation and scheduling of backhaul transmissions from multiple CNs to picocells (see Fig. 1). For a considered physical layer transmission scheme, the power allocation maximizes the weighted sum rates for arbitrary weights, so that the network operates on the boundary of the rate region. The scheduling is performed to allocate the limited backhaul resource among the picocells depending on *traffic demands* of the picocells (i.e., the number of mobile users served by a picocell and their quality-of-service). The obtained service rates allow the network to operate at the point of the rate region that is closest to the traffic demands of users. We define the scheduling problem as maximization of the network utility such that the rates are in the ergodic rate region of the backhaul wireless channel. The considered scheduling problem is then a special case of the network utility maximization problem [5], [6] (see also [7], [8]). The approach of [6] has been applied to the single-cell downlink MIMO scheduling problem in [9] and to the multicell system in [10]. Joint scheduling and ARQ optimization for the multicell downlink was considered in [11]. Results in [5] show the asymptotic optimality of the *gradient scheduling algorithm*. In [12], the gradient scheduling algorithm was adopted for the single-cell uplink OFDM system scheduling. Based on its optimality, we apply the gradient scheduling algorithm to our setting. We then discuss an alternative (but not necessarily optimal) scheduling algorithm. As a viable transmission scheme for the wireless backhaul channel, we consider an OFDMA scheme. We formulate and solve a problem that determines optimal power allocation at multiple CNs to picocells under power and interference constraints. Finally, we present an algorithm that performs joint scheduling and power allocation for OFDMA transmissions in the backhaul channel to maximize picocell utility.

The remainder of the paper is organized as follows. The system model is presented in Section II. In Section III, the backhaul allocation problem is defined. In Section IV, we formulate the power allocation problem for the OFDMA backhaul channel and develop the optimal power allocation algorithm. Joint scheduling and power allocation for the OFDM backhaul system is presented in Section V. Section VI concludes the paper.

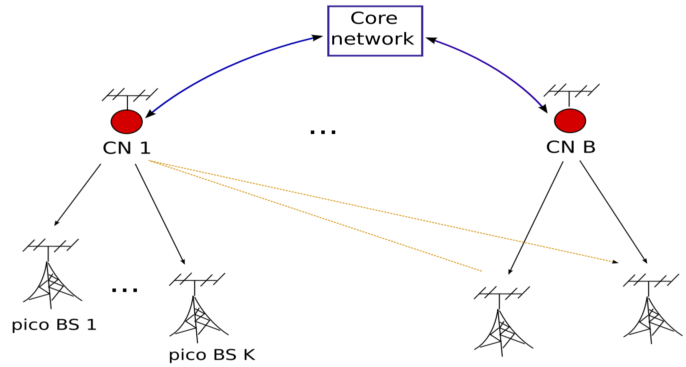

Fig. 1. Picocell network with CNs providing backhaul to the core network.

Notation. The inner product of two vectors $\mathbf{x}, \mathbf{y}$ is denoted $\mathbf{x}^T\mathbf{y}$. For a vector $\mathbf{x}$, $\|\mathbf{x}\|$ denotes the Euclidian norm of $\mathbf{x}$.

II. System Model

Consider a wireless network consisting of B connector nodes (CNs) providing backhaul to picocells over a wireless channel as illustrated in Fig. 1. Each connector node serves K picocells. Attached to each picocell is a wireless MIMO transceiver for backhaul communication. Each connector node is equipped with M_T antennas, and each picocell with M_R antennas. The received signal at picocell i is given by

$$\mathbf{y}_i = \sum_{b=1}^{B} \mathbf{H}_{ib}\mathbf{x}_b + \mathbf{z}_i \quad (1)$$

where $\mathbf{y}_i \in \mathbb{C}^{M_R \times 1}$, the transmitted signal at the CN is $\mathbf{x}_b \in \mathbb{C}^{M_T \times 1}$, and the random channel gain matrix from the CN b to picocell i is $\mathbf{H}_{ib} \in \mathbb{C}^{M_R \times M_T}$. Receiver noise $\mathbf{z}_i \in \mathbb{C}^{M_R \times 1}$ has i.i.d. components with zero mean and unit variance. A block fading channel is considered where channel gains are constant over the length of one code-block. This is a suitable channel model for the considered network where all the nodes are stationary. We assume that the channel gain matrices are known at the CN, which has an average power constraint P.

III. Backhaul Optimization

The purpose of the backhaul is to supply the data intended for users in the cellular network, to/from pico base stations. To optimize the performance of the network, the scheduling of backhaul transmissions will depend on traffic demands of the served picocells. These traffic demands will change over time. Typically, these changes will occur on a longer time scale compared to the channel dynamics. Our goal is to satisfy long-term traffic demands over blocks of N fading slots. We next consider the scheduling problem over one such block. In the case of the wireless backhaul, the long-term demands will typically be out of the ergodic rate region of the backhaul wireless channel. This motivates us to define a backhaul scheduling problem to determine and operate at the rate point in the backhaul ergodic rate region that is *closest*, in some sense, to the traffic demands in the access network.

Let $\bar{\mathbf{D}} \in \mathbb{R}^{BK}$ denote the long-term traffic demands during one block made by all picocells in the network. Similarly, $\bar{\mathbf{R}} \in \mathbb{R}^{BK}$ denotes assigned average rates over one block for all picocells. We define the optimization problem as follows:

$$\min \|\bar{\mathbf{D}} - \bar{\mathbf{R}}\|^2 \tag{2}$$
$$\text{s. t. } \bar{\mathbf{R}} \in \mathcal{C} \tag{2a}$$

where $\mathcal{C}$ is the ergodic rate region of the backhaul wireless channel. Problem (2) can be solved by considering a more general problem:

$$\max g(\bar{\mathbf{R}}) \tag{3}$$
$$\text{s. t. } \bar{\mathbf{R}} \in \mathcal{C} \tag{3a}$$

where $g : \mathbb{R}^{BK} \to \mathbb{R}$ is a strictly concave network utility function. In addition to traffic demands, $g(\cdot)$ can also incorporate network performance objectives such as total throughput, fairness, priorities, and system revenue.

To determine the rates at each scheduling instant n, we use the *gradient scheduling algorithm* that performs the following. Let $\mathcal{C}(n)$ denote the achievable rate region in slot n. At each time n choose rates $\mathbf{R}(n+1)$ such that

$$\mathbf{R}(n+1) = \arg \max_{\mathbf{R} \in \mathcal{C}(n+1)} \bigtriangledown g(\mathbf{X}(n))^T \mathbf{R} \tag{4}$$

and update

$$\mathbf{X}(n+1) = (1-\beta)\mathbf{X}(n) + \beta \mathbf{R}(n) \tag{5}$$

where $\mathbf{X}(n+1) \in \mathbb{R}^{BK}$, $\mathbf{X}(0)$ is an arbitrary chosen initial value, and $\beta > 0$ is a small chosen parameter.

As $\beta \to 0$ and the averaging interval becomes large, the average rates chosen by the algorithm converge to the optimal solution of (3) [5]. Choosing $g(\bar{\mathbf{R}}) = -\|\bar{\mathbf{D}} - \bar{\mathbf{R}}\|^2$ then gives the asymptotically optimal solution to (2).

We next propose an alternative algorithm to determine a solution to (3), as follows. At each step n we maximize the utility function $g(\cdot)$, i.e., choose $\mathbf{R}(n+1)$ that satisfies

$$\max g\left(\frac{1}{n+1}\sum_{i=1}^{n+1}\mathbf{R}(i)\right) \tag{6}$$
$$\text{s. t. } \mathbf{R}(n+1) \in \mathcal{C}(n+1) \tag{6a}$$

By using Taylor expansion up to the second order implies that (6) results in $\mathbf{R}(n+1)$ given as:

$$\max \Big(\bigtriangledown g(\bar{\mathbf{R}}(n))^T \mathbf{R}(n+1) + \frac{1}{2(n+1)}\mathbf{R}(n+1)^T \bigtriangledown^2 g(\bar{\mathbf{R}}(n))\mathbf{R}(n+1)\Big) \tag{7}$$
$$\text{s. t. } \mathbf{R}(n+1) \in \mathcal{C}(n+1) \tag{7a}$$

where we denote

$$\bar{\mathbf{R}}(n) = \frac{1}{n}\sum_{i=1}^{n}\mathbf{R}(i). \tag{8}$$

Alternatively, the higher order terms in the Taylor expansion in (7) can also be kept.

For the problem (2) the algorithm (6) then does the following. At each time n choose $\mathbf{R}(n+1)$ that minimizes the function $||\mathbf{D}(n) - \mathbf{R}(n+1)||^2$, i.e.,

$$\min ||\mathbf{D}(n) - \mathbf{R}(n+1)||^2 \tag{9}$$
$$\text{s. t. } \mathbf{R}(n+1) \in \mathcal{C}(n+1) \tag{9a}$$

and update

$$\mathbf{D}(n+1) = \mathbf{D}(n) + (\bar{\mathbf{D}} - \mathbf{R}(n+1)) \tag{10}$$

where $\mathbf{D}(0) = \bar{\mathbf{D}}$.

By neglecting the second-order terms in (7), the algorithm (6) reduces to the gradient scheduling algorithm (4) where instead of $\mathbf{X}(n)$ we use $\bar{\mathbf{R}}(n)$. Convergence of this algorithm to the optimal solution of (3) cannot be shown using the approach of [5]. However, this algorithm is interesting for practical applications for three reasons:

1) In a practical system, N may not be large enough for the gradient algorithm to experience the asymptotic behavior.
2) Algorithm (9) uses a simple running average of transmission rates.
3) For a smaller number of steps, n, (9) may outperform the gradient algorithm by taking the higher-order terms of the Taylor expansion into account. As n increases, the impact of the higher-order terms will diminish.

Omitting the indices in the optimization problem (9) yields:

$$P_2: \quad \min ||\mathbf{D} - \mathbf{R}||^2 \tag{11}$$
$$\mathbf{R} \in \mathcal{C} \tag{11a}$$

We define optimization problem P_3 as:

$$P_3: \quad \max \mathbf{w}^T \mathbf{R} \tag{12}$$
$$\mathbf{R} \in \mathcal{C} \tag{12a}$$
$$\mathbf{w} = \mathbf{D} - \mathbf{R} \tag{12b}$$

Lemma 1: P_2 and P_3 have the same optimal solution.

Lemma 1 implies that we can obtain solution to the original problem P_2 by solving the weighted sum optimization problem and choosing the weights as given by (12b).

The transmit rates in (9) are a function of the transmit powers. Solving (9) in terms of the transmit powers is often difficult. We thus use the observation of Lemma 1 and consider, in each step,

$$\max \mathbf{w}^T(n+1)\mathbf{R}(n+1) \tag{13}$$
$$\mathbf{R}(n+1) \in \mathcal{C}(n+1) \tag{13a}$$
$$\mathbf{w}(n+1) = \mathbf{D}(n) - \mathbf{R}(n+1) \tag{13b}$$

Again, by neglecting the last term, $\mathbf{R}(n+1)$, in (13b), (13) reduces to the gradient scheduling algorithm with the role of $\mathbf{X}(n)$ taken by $\bar{\mathbf{R}}(n)$. Alternatively, $\mathbf{w}(n+1)$ can be determined in each step according to (13b). To achieve this, in each step n we can use the procedure shown in the table below. Index n is omitted. We choose a parameter $\delta \in [0, 1]$, and the desired maximum number of steps, I, and perform the following:

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Initialize w^(0) = 1.
For k=1:I
Solve (13)-(13a) to obtain R^(k).
If (D − R^(k)) = αw^(k), for some α ∈ R
stop.
Otherwise:
choose w^(k+1) = (1 − δ)w^(k) + δ(D − R^(k)).
Repeat.
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IV. Power Allocation

So far, we have considered the scheduling problem which is applicable to any underlying physical layer transmission scheme. We next focus on a specific scheme of interest for the wireless backhaul - OFDMA. For simplicity, we consider the single-antenna case, i.e., $M_R = M_T = 1$.

The channel is partitioned into f subchannels. Let f_j denote the number of subchannels assigned to picocell j. The received signal (1) at picocell j communicating with CN b, $j \in b$ at frequency i is:

$$y_j(i) = h_{jb}(i)x_b(i) + \sum_{v=1, v\neq b}^{B} h_{jv}(i)x_v(i) + z_j. \tag{14}$$

The second term in (14) is inter-cell interference created to picocell j by other CNs' transmissions on frequency i. Due to orthogonalization, there is no intra-cell interference. We let

$$\gamma_{jb}(i) = |h_{jb}(i)|^2. \tag{15}$$

The achievable rate for picocell $j \in b$ at frequency i is given by:

$$R_j(i) = \log\left(1 + \frac{\gamma_{jb}(i)P_b(i)}{1 + \sum_{v=1, v\neq b}^{B} \gamma_{jv}(i)P_v(i)}\right) \tag{16}$$

where $P_b(i)$ is power allocated by CN b to frequency i. The total backhaul rate delivered to picocell $j \in b$ is

$$R_j = \sum_{i \in \mathcal{F}_j} R_j(i) \tag{17}$$

where $\mathcal{F}_j$ denotes the set of frequencies allocated to user j.

To operate on the boundary on the achievable rate region, we wish to maximize the weighted sum of rates under the power constraints. To control inter-cell interference we add a constraint limiting the allowed interference level at each picocell. The optimization problem can be formulated as:

$$\max \sum_{j=1}^{BK} \frac{w_j}{f_j} \sum_{i \in \mathcal{F}_j} \log\left(1 + \frac{\gamma_{jb}(i)P_b(i)}{1+\Delta_b}\right) \tag{18}$$

$$\text{s.t. } \frac{1}{f}\sum_{i=1}^{f} P_b(i) \leq P, \quad b = 1, \ldots, B \tag{18a}$$

$$P_b(i) \geq 0, \quad b = 1, \ldots, B, i = 1, \ldots, f \tag{18b}$$

$$\sum_{v=1, v\neq b}^{B} \gamma_{jv}(i)P_v(i) \leq \Delta_b, j = 1, \ldots, BK, i \in \mathcal{F}_j \tag{18c}$$

where b denotes a CN such that $j \in b$, and j receives on frequency i. Constraints (18a) are the power constraints. Constraints (18c) limit received interference power for each picocell, at every frequency. Because the OFDMA is considered, for each CN and frequency pair, (b, i), there is only one picocell j such that $j \in b$ and j is assigned frequency i. Therefore, problem (18) can be rewritten as

$$\max \sum_{b=1}^{B}\sum_{i=1}^{f} \beta_{bi} \log\left(1 + \frac{\gamma_{jb}(i)P_b(i)}{1+\Delta_b}\right) \tag{19}$$

$$\text{s.t. } \frac{1}{f}\sum_{i=1}^{f} P_b(i) \leq P, \quad b = 1, \ldots, B \tag{19a}$$

$$P_b(i) \geq 0, \quad b = 1, \ldots, B, i = 1, \ldots, f \tag{19b}$$

$$\sum_{v=1, v\neq b}^{B} \gamma_{jv}(i)P_v(i) \leq \Delta_b, b = 1, \ldots, B, i = 1, \ldots, f \tag{19c}$$

where j denotes a picocell in b, receiving on frequency i. Accordingly, we introduced notation $\beta_{bi} = w_j/f_j$. (19) is a convex problem with a concave objective function and linear constraints. For every (b, i), we let $t = i + f(b-1)$, and denote $p_t = P_b(i)$, $\beta_t = \beta_{bi}$ and $c_t = \gamma_{jb}(i)/(1+\Delta_b)$. Problem (19) can then be written in the form

$$\max \sum_{t=1}^{Bf} \beta_t \log(1 + c_t p_t) \tag{20}$$

$$\text{s.t. } \mathbf{A}\mathbf{p} \leq \mathbf{b} \tag{20a}$$

$$\mathbf{p} \geq 0 \tag{20b}$$

where $\mathbf{p} \in \mathbb{R}^{Bf}$ is the vector of powers to be determined. $\mathbf{A} \in \mathbb{R}^{(B+1)f \times Bf}$, $\mathbf{b} \in \mathbb{R}^{(B+1)f}$ are obtained from (19a) and (19c) and are non negative matrices.

To solve (20) we consider the dual problem

$$\max -\mathbf{b}^T\boldsymbol{\lambda} + \sum_{j=1}^{Bf}\left(\beta_j \log(\mathbf{a}_j^T\boldsymbol{\lambda}) - \frac{\mathbf{a}_j^T\boldsymbol{\lambda}}{c_j}\right) \tag{21}$$

$$\text{s.t. } \boldsymbol{\lambda} \geq 0 \tag{21a}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^{(B+1)f}$ is the vector of Lagrange multipliers and $\mathbf{a}_j$ is the j-th column of matrix $\mathbf{A}$. For $i \leq B$, λ_i is associated with the ith constraint in (19a). For $i > B$, λ_i is associated with the $i - B$th constraint in (19b). We denote the objective function in (21) as $f(\boldsymbol{\lambda})$.

We solve (21) by the gradient projection algorithm [13] that we apply to our problem as follows. We choose as a starting point $\boldsymbol{\lambda}^{(0)} = \mathbf{1}$. Alternatively, we could choose the value of $\boldsymbol{\lambda}$ obtained when only power constraints (19a) are active. Then, for $i > B$, $\lambda_i = 0$ and for $i \leq B$, λ_i can be computed from (19a) and (37). We choose the step size in the algorithm to be a constant. Specifically, in every step k, the step size equals $t^{(k)} = 1/M$ where M is chosen such that

$$\bigtriangledown^2 f(\boldsymbol{\lambda}) \preceq MI \tag{22}$$

or equivalently, that the maximum eigenvalue of the Hessian satisfies

$$\| \bigtriangledown^2 f(\boldsymbol{\lambda})\|_2 \leq M. \tag{23}$$

For objective function (21), the maximum eigenvalue of the Hessian can be bounded as

$$\begin{aligned} \| \bigtriangledown^2 f(\boldsymbol{\lambda})\|_2 &\leq \sum_{j=1}^{Bf} \frac{1}{(\mathbf{a}_j^T\boldsymbol{\lambda})^2}\|\mathbf{a}_j\|^2 \\ &\leq^{(a)} \frac{1}{\epsilon^2}\sum_{j=1}^{Bf}\|\mathbf{a}_j\|^2 \end{aligned} \tag{24}$$

where $\epsilon > 0$. (a) holds under the condition that

$$\mathbf{a}_j^T\boldsymbol{\lambda} \geq \epsilon \tag{25}$$

is satisfied for all j and $\boldsymbol{\lambda}$. Condition (25) needs to be satisfied in each step of the algorithm.

We next determine ϵ and M. We choose $\boldsymbol{\lambda}^{(0)} = \mathbf{1}$. From constraints (19a) and (19b) and definition of $\mathbf{A}$ and $\mathbf{b}$, it follows that there exist elements of $\mathbf{b}$ and $\mathbf{a}_j$ for every j that are strictly positive. We can thus write,

$$f(\boldsymbol{\lambda}^{(0)}) = \delta \tag{26}$$

for some $\delta > -\infty$. The algorithm satisfies that

$$f(\boldsymbol{\lambda}^{(k)}) \geq f(\boldsymbol{\lambda}^{(0)}) \tag{27}$$

and hence

$$f(\boldsymbol{\lambda}^{(k)}) \geq \delta. \tag{28}$$

We denote

$$g_j(\boldsymbol{\lambda}) = \beta_j \log(\mathbf{a}_j^T\boldsymbol{\lambda}) - \frac{\mathbf{a}_j^T\boldsymbol{\lambda}}{c_j}. \tag{29}$$

From (27) and (29), we obtain

$$\sum_{j=1}^{Bf} g_j(\boldsymbol{\lambda}) \geq \delta. \tag{30}$$

The maximum value of $g_j(\boldsymbol{\lambda})$ is $(g_j(\boldsymbol{\lambda}))_{\max} = \beta_j \log(\beta_j c_j) - \beta_j = C_j$. Hence, $g_j(\boldsymbol{\lambda}) \leq C$ where $C = \max_j\{C_j\}$. Using this observation, we can write (30) as

$$g_j(\boldsymbol{\lambda}) \geq \delta - (Bf-1)C = \delta' \tag{31}$$

implying that for all j

$$\mathbf{a}_j^T\boldsymbol{\lambda} \geq e^{\delta'/\beta} = \epsilon \tag{32}$$

and $\beta = \max_j\{\beta_j\}$.

From (24) and (32), we obtain M to be

$$M = e^{-2\delta'/\beta}\sum_{j=1}^{Bf}\|\mathbf{a}_j\|^2. \tag{33}$$

The gradient projection method in each step calculates the projection on the set given by (21a), i.e., $\mathcal{C}_{\boldsymbol{\lambda}} = \{\boldsymbol{\lambda} \in \mathcal{R}^n : \boldsymbol{\lambda} \geq 0\}$. It follows that $\mathcal{P}_{\mathcal{C}}(x) = x_+$. The algorithm is shown in the table below.

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ALG 1: Solving dual (21)
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Choose a staring point: $\boldsymbol{\lambda}^{(0)} = \mathbf{1}$

Repeat:

1) Choose step size $t^{(k)} = 1/M$
2) $\boldsymbol{\lambda}^{(k+1)} = \left(\boldsymbol{\lambda}^{(k)} + t^{(k)} \bigtriangledown f(\lambda^{(k)})\right)_+$

until $|\bigtriangledown f(\lambda)| \leq \eta$.

In the algorithm η is chosen to satisfy the desired accuracy. The output of the algorithm will be the optimum solution to (21), denoted $\boldsymbol{\lambda}^*$.

The convergence of the algorithm to the optimum value follows from [14]. Furthermore, given the value of M (33), we use [14, Thm.3.1.] to obtain the rate of convergence as:

$$f(\boldsymbol{\lambda}^{(k)}) - f(\boldsymbol{\lambda}^*) \leq \frac{M\|\boldsymbol{\lambda}^{(k)} - \boldsymbol{\lambda}^*\|^2}{2k}. \tag{34}$$

By forming the Lagrangian for (19) and determining the KKT conditions, we obtain the optimum powers for each CN b transmitting at frequency i to picocell $j \in b$ as

$$P_b^*(i) = \left[\frac{w_j/f_j}{h_b(\boldsymbol{\lambda}^*)} - \frac{1+\Delta_b}{\gamma_{jb}(i)}\right]_+ \tag{35}$$

where

$$h_b(\boldsymbol{\lambda}) = \frac{\lambda_b}{f} + \sum_{v=1, v\neq b}^{B} \lambda_{B+(b-1)f+i}\gamma_{kb}(i) \tag{36}$$

and $k \in v$ is the picocell k receiving from v on frequency i. Given the solution to the dual problem (21), $\boldsymbol{\lambda}^*$, we now obtain the optimal power allocation from (35).

We next consider the special case in which only power constraints (19a) are active. We have $\lambda_b = 0$ for $b > B$. The optimum powers (35) for $b = 1, \ldots, B$ become

$$P_b^*(i) = \left[\frac{w_j/f_j}{\lambda_b^*} - \frac{1+\Delta_b}{\gamma_{jb}(i)}\right]_+. \tag{37}$$

where λ_b is obtained from binding constraints (19a). The optimum solution is waterfilling over frequencies, as expected.

V. Joint Power Allocation and Scheduling

A solution to the optimization problem (19) performs optimal power allocation for all picocells and over all frequencies. It determines the point on the achievable rate region for a given set of weights $\mathbf{w}$. Values of the weights are updated based on the long-term average demands, via the scheduling algorithm (4) or (13). The joint scheduling and power allocation algorithm is given in the table below.

Initialize $\mathbf{w}^{(0)} = \mathbf{1}$

Repeat:

1) Determine $\boldsymbol{\lambda}^{*(k)}$ from ALG 1
2) Determine $\mathbf{P}^{*(k)}$ from (35)
3) Determine $\mathbf{R}^{*(k)}$ from (19)
4) Update $\mathbf{w}^{(k)}$ via scheduling algorithm.

VI. Conclusion and Future Work

We considered scheduling and power allocation of wireless backhaul. For scheduling transmissions, we applied the gradient scheduling algorithm to our setting, and then proposed an alternative, simpler, scheduling algorithm. Our approach assigns rates based on traffic demands in order to maximize picocell utility. This metric can further incorporate network performance objectives as desired by the network operator, such as total throughput, fairness, priorities, and system revenue. As the load in the access network changes, the backhaul scheduling adapts to the time-varying load. For OFDM transmissions, we further determined an optimal power allocation under power and interference constraints. In the special case of interference-free regimes, the optimum power allocation over frequencies reduces to water-filling. Finally, we presented an algorithm that performs joint scheduling and power allocation for OFDMA transmissions in the backhaul channel to maximize picocell utility. To apply it, a server in the backhaul network needs knowledge of the following: 1) channel state information at the picocells, and 2) allowed interference levels in the backhaul network.

In the paper, the *downlink backhaul channel*, i.e., transmissions from CNs to pico base stations, was analyzed. In our future work, the considered approach will be applied to uplink transmissions. This approach will also be generalized to a system with multiple antennas at both ends. We will also extend this result to arbitrary signal constellations by following the approach in [4]. The considered scheduling problem can also be generalized to consider backhaul allocation that assigns the backhaul capacity jointly to a group of neighboring picocells. This avoids scenarios in which neighboring cells cannot meet their traffic demands and cannot handoff to their neighbors, and thus have to drop calls.

The wireless backhaul problem has similarities with the wireless cellular network setting, allowing many existing solutions to be readily deployed for the backhaul application. The differences, such as stationary users, a smaller number of users per cell and different channel dynamics, can alleviate some of the challenges facing cellular networks (e.g., extensive CSI, extensive coordination between base stations, etc.) and can result in simpler solutions (e.g., no need for handoff, simpler scheduling, etc.). Another difference is the hierarchal structure of the backhaul allocation problem, as the optimization of the backhaul wireless network depends on the optimization of rates and performance of the access network dictating the backhaul demand.

VII. Acknowledgment

The authors would like to thank Andrea Montanari for an insightful discussion regarding the problem considered in the paper.

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