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COMPUTING THE NUMBER OF FUZZY SUBGROUPS BY EXPANSION METHOD

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Abstract: The purpose of this paper is to construct the method to find the number of fuzzy subgroups for rectangle groups. The method is formulated by using the pattern of their lattice diagram. We construct the expansion methods, raw and column expansion.

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1. Introduction

In 1965, Zadeh [1] introduced the concept of fuzzy set. Rosenfeld in 1971 [2] was started study of fuzzy algebraic structures. He introduced the concept of fuzzy subgroups. One the most important problem of fuzzy theory is to classify the fuzzy subgroups of a finite group [3]. There are many papers have written. Lazlo (see [4]) constructed of fuzzy subgroups of groups of order one to six. Murali and Makamba (see [5], [6]) found the number of fuzzy subgroups of abelian groups of order $p^m q^n$ where p and q are different primes. Sulaiman (see [7] and [8]) constructed the fuzzy subgroups of symmetric group S_4 and alternating group A_4.

The definition of equivalence relation on fuzzy subgroups introduced in [9]. Some

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definitions and theorems in [9] is used in this paper. Sulaiman [9] developed a method to count the number of fuzzy subgroups of finite group. The method use the subgroup lattice of group. This method can be used not only for abelian group, but also for non-abelian as well. By using the method, we [10] constructed some formulas to count the number of fuzzy subgroups of "rectangle groups". In this paper, we develop the results to find some formulas. The formulas are constructed by expansion methods, raw and column expansion.

2. Preliminaries

In this paper, a group G is assumed to be a finite group. First of all, we present some basic notions and results that will be used later. Definition of equivalence relation on fuzzy subgroups use The Definition 6 in [9].

Definition 1. Let X be a nonempty set. A fuzzy subset of X is a function from X into [0, 1].

Definition 2. (Rosenfeld, see [2]). Let G be a group. A fuzzy subset μ of G is called a fuzzy subgroup of G if:

(1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in G$,

(2) $\mu(x^{-1}) \geq \mu(x), \forall x \in G$.

Theorem 3. *(Sulaiman, see Theorem 11 in [9]) Let H be a subgroup of G, and let the set of all subgroups of G which contain H (but are not equal to H) be $\{H_1, H_2, H_3, ..., H_k\}$. Then $n(F_{P_1=H}) = \Sigma_{i=1}^{k} n(F_{P_1=H_i})$.*

Definition 4. (Sulaiman and Budi Priyo, see [10]). Let G be a group and the number of it's subgroups is finite. The diagram of lattice subgroups of G is called "rectangle" and group G is called "rectangle group" if satisfies these conditions: The subgroups of G can be labeled by K_j^i where $1 \leq i \leq m, 1 \leq j \leq s$ for some $m, s \in N$ with $K_1^1 = G, K_s^m = \{e\}$ such that: (i) for fixed $i, 1 \leq i \leq m, K_k^i < K_{k-1}^i, \forall k, 2 \leq k \leq s$, (ii) for fixed $j, 1 \leq j \leq m, K_j^t < K_j^{t-1}, \forall t, 2 \leq t \leq m$. In this case the size of the diagram is $m \times s$. The diagram of "rectangle lattice" is shown in Figure 1.

3. Expansion Method

In this section we find the number of fuzzy subgroups of "rectangle group" by raw and column expansion.

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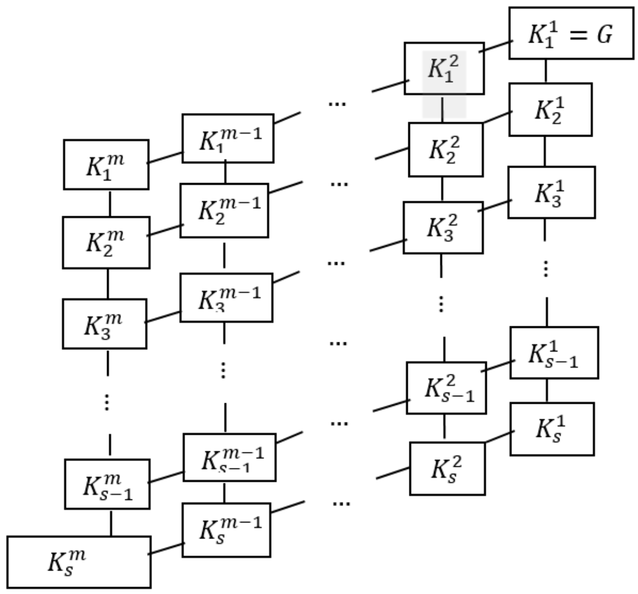

Figure 1: Rectangle Lattice

Theorem 5. *Let G be a rectangle group with $m, s \in N$. We have*

$$n(F_{P_1=K_s^m=\{e\}}) = 2.n(F_{P_1=K_{s-1}^m}) + \sum_{i=1}^{m-1} n(F_{P_1=K_s^i}).$$

Proof. Let $m, s \in N$. By analyzing Figure 1 and applying Theorem 3, we have,

$$n(F_{P_1=K_s^m=\{e\}}) = \sum_{i=1}^{s-1} n(F_{P_1=K_i^m}) + \sum_{i=1}^{s} n(F_{P_1=K_i^{m-1}}) + \sum_{i=1}^{s} n(F_{P_1=K_i^{m-2}}) + \dots + \sum_{i=1}^{s} n(F_{P_1=K_i^2}) + \sum_{i=1}^{s} n(F_{P_1=K_i^1}).$$

We can write

$$n(F_{P_1=K_s^m=\{e\}}) = \sum_{i=1}^{s-1} [n(F_{P_1=K_i^m}) + n(F_{P_1=K_i^{m-1}}) + n(F_{P_1=K_i^{m-2}}) + \dots + n(F_{P_1=K_i^2}) + n(F_{P_1=K_i^1})] + n(F_{P_1=K_s^{m-1}}) + n(F_{P_1=K_s^{m-2}}) + \dots + n(F_{P_1=K_s^2}) + n(F_{P_1=K_s^1}). \quad (1)$$

Analogue with this way we have

$$n(F_{P_1=K^m_{s-1}}) = \sum_{i=1}^{s-1}[n(F_{P_1=K^m_i}) + n(F_{P_1=K^{m-1}_i}) + n(F_{P_1=K^{m-2}_i}) + ... + n(F_{P_1=K^2_i}) + n(F_{P_1=K^1_i})] + n(F_{P_1=K^{m-1}_{s-1}}) + n(F_{P_1=K^{m-2}_{s-1}}) + ... + n(F_{P_1=K^2_{s-1}}) + n(F_{P_1=K^1_{s-1}}). \quad (2)$$

Eq. (1) can be written as

$$n(F_{P_1=K^m_s}) = \sum_{i=1}^{s-2}[n(F_{P_1=K^m_i}) + n(F_{P_1=K^{m-1}_i}) + n(F_{P_1=K^{m-2}_i}) + ...+ n(F_{P_1=K^2_i}) + n(F_{P_1=K^1_i})] + n(F_{P_1=K^m_{s-1}}) + n(F_{P_1=K^{m-1}_{s-1}}) + n(F_{P_1=K^{m-2}_{s-1}}) + ...+ n(F_{P_1=K^2_{s-1}}) + n(F_{P_1=K^1_{s-1}}) + n(F_{P_1=K^{m-1}_s}) + n(F_{P_1=K^{m-2}_s}) + ...+ n(F_{P_1=K^2_s}) + n(F_{P_1=K^1_s}). \quad (3)$$

Substitute (2) to (3) to get

$$n(F_{P_1=K^m_s}) = 2.n(F_{P_1=K^m_{s-1}}) + n(F_{P_1=K^{m-1}_s}) + n(F_{P_1=K^{m-2}_s}) + ...+ n(F_{P_1=K^2_s}) + n(F_{P_1=K^1_s}).$$

Finally we have

$$n(F_{P_1=K^m_s}) = 2.n(F_{P_1=K^m_{s-1}}) + \sum_{i=1}^{m-1}[n(F_{P_1=K^i_s}). \qquad \square$$

Theorem 6. (Raw Expansion) *Let G be a rectangle group with $m, s \in N$ and $s \geq 3$. We have*

$$n(F_{P_1=K^m_s}) = 2.n(F_{P_1=K^m_{s-1}}) + 2.n(F_{P_1=K^{m-1}_{s-1}}) + 2^2.n(F_{P_1=K^{m-2}_{s-1}}) + 2^3.n(F_{P_1=K^{m-3}_{s-1}}) + ...+ 2^{m-3}.n(F_{P_1=K^3_{s-1}}) + 2^{m-2}.n(F_{P_1=K^2_{s-1}}) + 2^{m-1}.n(F_{P_1=K^1_{s-1}}).$$

Proof. According to the Theorem 5 we have:

$$n(F_{P_1=K^2_s}) = 2.n(F_{P_1=K^2_{s-1}}) + n(F_{P_1=K^1_s}), \quad (4)$$

$$n(F_{P_1=K^3_s}) = 2.n(F_{P_1=K^3_{s-1}}) + n(F_{P_1=K^1_s}) + n(F_{P_1=K^2_s}), \quad (5)$$

$$n(F_{P_1=K^4_s}) = 2.n(F_{P_1=K^4_{s-1}}) + n(F_{P_1=K^1_s}) + n(F_{P_1=K^2_s}) + n(F_{P_1=K^3_s}). \quad (6)$$

Substitute (4) to (5) to get

$$n(F_{P_1=K^3_s}) = 2.n(F_{P_1=K^3_{s-1}}) + 2.n(F_{P_1=K^2_{s-1}}) + 2.n(F_{P_1=K^1_s}). \quad (7)$$

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Substitute (7) to (6) to get

$$n(F_{P_1=K_s^4}) = 2.n(F_{P_1=K_{s-1}^4}) + 2.n(F_{P_1=K_{s-1}^3}) + 2^2.n(F_{P_1=K_{s-1}^2}) + 2^2.n(F_{P_1=K_{s-1}^1}).$$

By recursive process we have

$$n(F_{P_1=K_s^{m-2}}) = 2.n(F_{P_1=K_{s-1}^{m-2}})+2.n(F_{P_1=K_{s-1}^{m-3}})+2^2.n(F_{P_1=K_{s-1}^{m-4}})+2^3.n(F_{P_1=K_{s-1}^{m-5}}) + ... + 2^{m-4}.n(F_{P_1=K_{s-1}^2}) + 2^{m-4}.n(F_{P_1=K_s^1}),$$

and

$$n(F_{P_1=K_s^{m-1}}) = 2.n(F_{P_1=K_{s-1}^{m-1}})+2.n(F_{P_1=K_{s-1}^{m-2}})+2^2.n(F_{P_1=K_{s-1}^{m-3}})+2^3.n(F_{P_1=K_{s-1}^{m-4}}) + ... + 2^{m-3}.n(F_{P_1=K_{s-1}^2}) + 2^{m-3}.n(F_{P_1=K_s^1}),$$

$$n(F_{P_1=K_s^m}) = 2.n(F_{P_1=K_{s-1}^m})+2.n(F_{P_1=K_{s-1}^{m-1}})+2^2.n(F_{P_1=K_{s-1}^{m-2}})+2^3.n(F_{P_1=K_{s-1}^{m-3}}) + ... + 2^{m-2}.n(F_{P_1=K_{s-1}^2}) + 2^{m-2}.n(F_{P_1=K_s^1}).$$

Therefore, we have

$$n(F_{P_1=K_s^m}) = 2.n(F_{P_1=K_{s-1}^m}) + (1+1+2+2^2+2^3+...+2^{m-4}+2^{m-3}).n(F_{P_1=K_s^1}) +2(1+1+2+2^2+2^3+...+2^{m-5}+2^{m-4}).n(F_{P_1=K_{s-1}^2})+2(1+1+2+2^2+2^3+...+2^{m-6} + 2^{m-5}).n(F_{P_1=K_{s-1}^3}) + ... + 2(1+1+2).n(F_{P_1=K_{s-3}^{m-3}}) + 2(1+1).n(F_{P_1=K_{s-1}^{m-2}}) + 2.n(F_{P_1=K_{s-1}^{m-1}}).$$

By manipulation algebraic, use the fact that

$$(1+1+2+2^2+2^3+...+2^{k-1}+2^k) = 2^{k+1},$$

and Corollary 10 in [9] we have

$$n(F_{P_1=K_s^m}) = 2.n(F_{P_1=K_{s-1}^m})+2.n(F_{P_1=K_{s-1}^{m-1}})+2^2.n(F_{P_1=K_{s-1}^{m-2}})+2^3.n(F_{P_1=K_{s-1}^{m-3}}) + ... + 2^{m-3}.n(F_{P_1=K_{s-1}^3}) + 2^{m-2}.n(F_{P_1=K_{s-1}^2}) + 2^{m-1}.n(F_{P_1=K_{s-1}^1}). \quad \square$$

Theorem 7. *Let G be a rectangle group with $m \times n$ in size. Then*

$$n(F_{P_1=K_s^m=\{e\}}) = 2.n(F_{P_1=K_s^{m-1}}) + \sum_{i=1}^{s-1} n(F_{P_1=K_i^m}).$$

Proof. Analogue with the proof of Theorem 5. □

Theorem 8. *(Column Expansion) Let G be a rectangle group with $m, s \in N$ and $m \geq 3$. We have*

$$n(F_{P_1=K_s^m}) = 2.n(F_{P_1=K_s^{m-1}}) + 2.n(F_{P_1=K_{s-1}^{m-1}}) + 2^2.n(F_{P_1=K_{s-2}^{m-1}}) + 2^3.n(F_{P_1=K_{s-3}^{m-1}}) + ... + 2^{s-3}.n(F_{P_1=K_3^{m-1}}) + 2^{s-2}.n(F_{P_1=K_2^{m-1}}) + 2^{s-1}.n(F_{P_1=K_1^{m-1}}).$$

Proof. Analogue with the proof of Theorem 6. □

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