Pulse Propagation in Random Media

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Abstract: Pulse propagation in a random inhomogeneous medium is studied using a time-domain progressive wave equation and its associated path integral representation. From an eikonal-like solution one obtains the time-dependent statistics of pulses propagating through this random medium as well as a simple way of simulating the time-series of received signals.

INTRODUCTION

Pulse propagation in random inhomogeneous media is of much interest in applied sciences such as ocean acoustics and atmospheric optics. Theoretical work in this area (see review in Ref.[1]) consists mostly of analysis in the frequency domain. Recently (see Ref.[2]) an approach for treating wave propagation in random media in a space-time framework was proposed using a time-domain progressive wave equation derived a few years ago by McDonald and Kuperman [3] from the wave equation. This equation is the equivalent in the space-time formulation of the well known parabolic (or paraxial) wave equation in the space-frequency formulation [4,5] and mathematically similar to the time dependent Schrödinger's equation. The parabolic equation in the frequency domain has been extensively used to discuss wave propagation in random media, an important example of its use can be found in Ref.[6]. In the present article a previously developed (see Ref.[2]) path integral representation for the solution of the time-domain progressive wave equation is used to study pulse propagation in random media. With his approach one can use knowledge about time dependent wave propagation in homogeneous media to approximately evaluate the path integrals and obtain analytical expressions for the moments of the propagating pulse in a weakly and randomly inhomogeneous medium. Besides expressions for the various statistical moments, one also obtains a simple algorithm for producing simulations of pulse propagation in such media. Thus time-series at a point detector are computed using this method and their behavior can then be related to the properties of the medium.

PULSE PROPAGATION IN A WEAKLY RANDOM MEDIUM

For a pulse generated by a point source of the form $S(\mathbf{r},t) = \delta(\mathbf{r})F(t)$, where $F(t)$ is the pulse shape function, one has the following expression for the acoustic pressure field in a medium with sound speed $c(\mathbf{r},t) = c_0 + c_1(\mathbf{r},t)$ (c_1 is the weak random part):

$$P(\mathbf{r},t) = \frac{1}{4\pi r} f(t - \frac{r}{v_0})/|1 - \frac{\partial V}{\partial v_0}| \,. \tag{1}$$

The effective propagation speed $v_0(\mathbf{r},t)$ is the solution of the equation $v_0 = c_0 + V(\mathbf{r},t,v_0)$ with $V(\mathbf{r},t,v_0) = \int_0^1 d\mu\, c_1(\mu\mathbf{r}, t-(1-\mu)r/v_0)$. Assuming that $c_1(\mathbf{r},t)$ is a Gaussian random process with zero mean and $< c_1(\mathbf{r},t)c_1(\mathbf{r}',t') >= \rho(|\mathbf{r}-\mathbf{r}'|, |t-t'|)$ one can produce simulated time series of the acoustic pressure field at a point. Below, for the case $\rho(r,t) = \rho_0 exp(-(r + c_0 t)/L)$ two examples are shown with different values of the correlation length L. The values used were $r = 1500m$, $c_0 = 1500m/s$, $\rho_0 = 10^{-4}c_0^2$. The pulse shape function chosen was $F(t) = exp(-(t/\tau)^2)cos(2\pi f t)$, with $\tau = .005s$ and $f = 300Hz$.

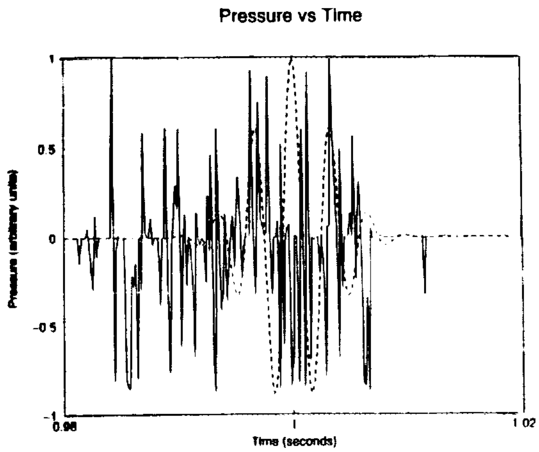

FIGURE 1. Correlation length $L = 1m$.

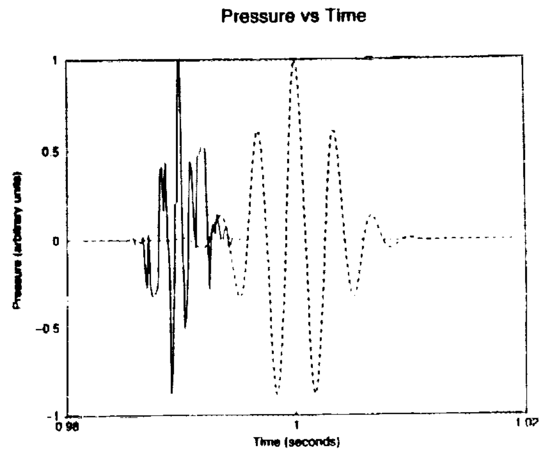

FIGURE 2. Correlation length $L = 100m$.

In figure 1 the continuous line is the pressure time series in the presence of fluctuations in the sound speed with a short correlation length. The dotted line is the corresponding time series in the absence of fluctuations. The short correlation length of the fluctuations causes the pulse to spread out and lose coherence. Figure 2 is similar, except with a much longer correlation length which allows the pulse to retain some similarity with the source shape function, the main effect is the change in the arrival time for the center of the pulse, in this case the arrival is delayed. In both cases the strength of the fluctuations is the same. The marked contrast in the propagation is due only to the difference in the correlation length of the sound speed fluctuations. This result is intuitively expected since with a shorter correlation length the sound speed field would be expected to be more randomized over the distance from the source to the detector than in the case of a longer correlation length.

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REFERENCES

1. Ishimaru, A., *Wave Propagation and Scattering in Random Media*, Academic Press, New York, 1978.

2. Dacol, D. K., Waves in Random Media **4**, 117-126 (1994).

3. McDonald, B. E. and Kuperman, W. A., J. Acoust. Soc. Am. **81**, 1406-1417 (1987).

4. Leontovich, M. A. and Fock, V. A., J. Phys. USSR **10**, 13-24 (1946).

5. Tappert, F. D.,"The Parabolic Approximation Method", *Wave Propagation and Underwater Acoustics*, edited by J. B. Keller ans J. S. Papadakis, Lecture Notes in Physics, Vol. 70, Springer-Verlag, New York, 1977, ch.5, pp. 224-286.

6. Dashen, R., J. Math. Phys. **20**, 894-920 (1979).