INTERNATIONAL JOURNAL OF ADVANCED RESEARCH IN ENGINEERING AND TECHNOLOGY (IJARET)

ISSN 0976 - 6480 (Print)
ISSN 0976 - 6499 (Online)
Volume 5, Issue 1, January (2014), pp. 177-181
© IAEME: www.iaeme.com/ijaret.asp
Journal Impact Factor (2013): 5.8376 (Calculated by GISI)
www.jifactor.com

IJARET
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CANONICAL-LAPLACE TRANSFORM AND ITS VARIOUS TESTING FUNCTION SPACES

N.V. Kalyankar,   S.B.Chavhan

Yeshwant Mahavidyalaya Nanded-431602

ABSTRACT

The linear canonical transform is a useful tool for optical analysis and signal processing. In this paper we have defined canonical-Laplace transform and have also established some testing functions spaces using Gelfand-shilov technique.

Index Terms: Canonical Transform, Fourier Transform, Fractional Fourier Transform**,** Laplace Transform, Testing Function Space.

1. INTRODUCTION

The Fourier analysis is undoubtedly the one of the most valuable and powerful tools in signal processing, image processing and many other branches of engineering sciences [6],[7].the fractional Fourier transform, a special case of linear canonical transform is studied through different analysis .Almeida[1],[2].had introduced it and proved many of its properties . The fractional Fourier transform is a generalization of classical Fourier transform, which is introduce from the mathematical aspect by Namias at first and has many applications in optics quickly[5]. The definition of Laplace transform with parameter p of $f(x)$ denoted by $L[f(x)] = F(p)$

$$L[f(x)] = \int_0^\infty e^{-px} f(x)$$

And definition of canonical transform with parameter s of $f(t)$ denoted by

$$\{CT\,f(t)\}(s) = \frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{-i\left(\frac{s}{b}\right)t} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)$$

The definition of canonical-Laplace transform is given in section 2. S-type spaces using Gelfand-shilov technique are developed in section 3.Section 4 is devoted for the results on countable union s-type space. The notation and terminology as per Zemanian [8],[9]. Gelfand-Shilov [3],[4].

2. DEFINITION CANONICAL CANONICAL-LAPLACE TRANSFORMS

The definition of Laplace transform with parameter p of $f(x)$ denoted by $L\left[f(x)\right]=F(p)$

$$L\left[f(x)\right]=\int_0^{\infty} e^{-px} f(x)$$

The definition of Laplace transform with parameter s of $f(t)$ denoted by

$$\{CT\, f(t)\}(s)=\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{-i\left(\frac{s}{b}\right)t} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t)$$

The definition of conversional canonical -Laplace transform is defined as

$$CL\{f(t,x)\}(s,p)=\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty}\int_0^{\infty} e^{-i\left(\frac{s}{b}\right)t} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{-px} f(t,x)\,dxdt$$

3. VARIOUS TESTING FUNCTION SPACES

3.1 The space $CL_{a,b,\gamma}$

It is given by

$$CL_{a,b,\gamma}=\left\{\phi:\phi\in E_+ / \sigma_{l,k,p}\phi(t,x)=\sup\left|t^l D_t^k D_x^p \phi(t,x)\right| \le C_{kp} A^l . l^{l\gamma}\right\}$$

The constant $C_{k,p}$ and A depend on ϕ.

3.2 The space $CL_{ab,\beta}$

$CL_{ab,\beta}$ this space is given by

$$CL_{a,b,\beta}=\left\{\phi.\phi\in E_+ / \rho_{l,k,p}\phi(t,x)=\sup\left|t^l D_t^k D_x^p \phi(t,x)\right| \le C_{l,p} B^k k^{k\beta}\right\}$$

The constants $C_{l,p}$ and B depend on ϕ.

3.3 The space $CL_{a,b,\beta}^{\gamma}$

This space is formed by combining the condition (3.1) and (3.2)

$$CL_{a,b,\beta}^{\gamma}=\left\{\phi:\phi\in E_+ / \xi_{l,k,p}\phi(t,x)=\sup_{I_1}\left|t^l D_t^k D_x^p \phi(t,x)\right| \le C\, A^l l^{l\gamma} B^k k^{k,\beta}\right\}$$

$l,k,p=0,1,2............$ Where A,B,C depend on ϕ.

3.4 The space: $CL_{a,b,\beta}^{\gamma,m}$

It is defined as,

$$CL_{a,b,\beta}^{\gamma,m}=\left\{\phi:\phi\in E_+ / \sigma_{l,k,p}\phi(t,x)=\sup_{I_1}\left|t^l D_t^k D_x^p \phi(t,x)\right| \le C_{k,p,\mu}(m+\mu)^l l^{l\gamma}\right\}$$

For any $\mu>0$ where m is the constant, depending on the function ϕ.

3.5 The space $CL_{a,b,\beta,n}$:

This space is given by

$$CL_{a,b,\beta,n} = \left\{\phi : \phi \in E_+ \,/\, \rho_{l,k,p}\phi(t,x) = \sup_{I_1} \left|t^l D_t^k D_x^p \phi(t,x)\right| \le C_{l,p,\delta}(n+\delta)^k k^{k\beta}\right\}$$

For any $\delta > 0$ where n the constant is depends on the function ϕ.

3.6 The space $CL_{a,b,\beta,n}^{\gamma,m}$:

This space is defined by combining the conditions in (3.4) and (3.5).

$$CL_{a,b,\beta,n}^{\gamma,m} = \left\{\phi : \phi \in E_+ \,/\, \xi_{l,k,p}\phi(t,x) = \sup_{I_1} \left|t^l D_t^k D_x^p \phi(t,x)\right| \right.$$
$$\left. \le C_{\mu\delta}(m+\mu)^l (n+\delta)^k . l^{l\gamma} k^{k\beta}\right\} \qquad (3.6)$$

4. RESULTS ON COUNTABLE UNION S-TYPE SPACE

Proposition 4.1: If $m_1 < m_2$ then $CL_{\gamma,m_1}^{a,b} \subset CL_{\gamma,m_2}^{a,b}$. The topology of $CL_{\gamma,m_1}^{a,b}$ is equivalent to the topology induced on $CL_{\gamma,m_1}^{a,b}$ by $CL_{\gamma,m_2}^{a,b}$

$i.e\ T_{\gamma,m_1}{}^{a,b} \sim T_{\gamma,m_2}{}^{a,b} / CL_{\gamma,m_1}^{a,b}$

Proof: For $\phi \in CL_{\gamma,m_1}^{a,b}$ and $\quad \delta_{l,k,p}(\phi) \le C_{k,\mu}(m_1+\mu)^l l^{l\gamma}$

$\le C_{k,\mu,p}(m_2+\mu)^l l^{l\gamma} \qquad$ Thus, $CL_{\gamma m_1}^{a,b} \subset CL_{\gamma m_2}^{a,b}$

The space $CL_{\gamma,}^{a,b}$ can be expressed as union of countable normed spaces.

Proposition 4.2: $CL_{\gamma}^{a,b} = \overset{\infty}{\underset{i=1}{U}} CL_{\gamma,m_1}^{a,b}$ and if the space $CL_{\gamma}^{a,b}$ is equipped with strict inductive limit topology $S_{a,b,m}$ defined by injective map from $CL_{\gamma,m_1}^{a,b}$ to $CL_{\gamma}^{a,b}$ then the sequence $\{\phi_n\}$ in $CL_{\gamma}^{a,b}$ converges to zero.

Proof: we show that $\quad CL_{\gamma}^{a,b} = \overset{\infty}{\underset{i=1}{U}} CL_{\gamma.m_1}^{a,b}$

Clearly $\overset{\infty}{\underset{i=1}{U}} CL_{\gamma.m_1}^{a,b} \subset CL_{k}^{a,b}$ for proving the other inclusion, let $\phi \in CL_{\gamma}^{a,b}$ then

$$\delta_{l,k,p}(\phi(t,x)) = \sup_{I_1} \left|t^l D_t^k D_x^p \ \phi_n(t,x)\right|$$
$$\le C_{k,p} A^l l^{l\gamma}, \qquad (4.1)$$

where A is some positive constant, choose an integer $m = m_A$ and $\mu = 0$ such that $C_{k,p}A^l \le C_{k,p}(m+\mu)^l$.

.

Then (4.1) we get $\phi \in CL_{\gamma,m_1}^{a,b}$ implying that $CL_{\gamma}^{a,b} = \overset{\infty}{\underset{i=1}{U}} CL_{\gamma,m_1}^{a,b}$

Proposition 4.3: If $\gamma_1 < \gamma_2$ and $\beta_1 < \beta_2$ then $CL_{\gamma_1}^{a,b,\beta_1} \subset CL_{\gamma_2}^{a,b,\beta_2}$ and the topology of $CL_{\gamma_i}^{a,b,\beta_i}$ is equivalent to the topology induced on $CL_{\gamma_1}^{a,b,\beta_1}$ by $CL_{\gamma_2}^{a,b,\beta_2}$.

Proof: Let $\phi \in CL_{\gamma_1}^{a,b,\beta_1}$

$$\xi_{l,k,p}(\phi) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right|$$
$$\leq C A^l l^{l\gamma_1} B^k k^{k\beta_1}$$
$$\leq C A^l l^{l\gamma_2} . B^p k^{p,\beta_2} \qquad \text{where} l, k, p = 0,1,2,3$$

Hence $\phi \in CL_{\gamma_2}^{a,b,\beta_2}$. Consequently, $CL_{\gamma_1}^{a,b,\beta_1} \subset CL_{\gamma_2}^{a,b,\beta_2}$. The topology of $CL_{\gamma_1}^{a,b,\beta_1}$

Is equivalent to the topology $T_{\gamma_2}^{a,b,\beta_2} / CL_{\gamma_2}^{a,b,\beta_2}$

It is clear from the definition of topologies of these spaces.

Proposition 4.4: $CL^{a,b} = \underset{\gamma_i \beta_i = 1}{\overset{\infty}{U}} CL_{\gamma_i}^{a,b,\beta_i}$ and if the space $CL^{a,b}$ is equipped with the strict $CL^{a,b}$ inductive limit topology defined by the injective maps from $CL_{\gamma_i}^{a,b,\beta_i}$ to $CL^{a,b}$ then the sequence $\{\phi_n\}$ in $CL^{a,b}$ converges to zero iff $\{\phi_n\}$ is contained in some $CL_{\gamma_i}^{a,b,\beta_i}$ and converges to zero.

Proof: $CL^{a,b} = \underset{\gamma_i \beta_i = 1}{\overset{\infty}{U}} CL_{\gamma_i}^{a,b,,\beta_i}$

Clearly $\underset{\gamma_i \beta_i = 1}{\overset{\infty}{U}} CL_{\gamma_i}^{a,b,\beta_i} \subset CL^a$

For proving other inclusion, let $\phi(t,x) \in CL^{a,b}$ then

$$\eta_{l,k,p}(\phi) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right|,$$

is bounded by some number. We can choose integers γ_m and β_m such that

$$\eta_{l,k,p}(\phi) \leq C\, A^l l^{l,\gamma} B^{k,m}, k^{k,\beta,m}$$

$\therefore \phi \in CL_{\gamma_i}^{a,b,\beta_i}$ for some integer γ_i and β_i

Hence $CL^{a,b} \subset \underset{\gamma_i \beta_i = 1}{\overset{\infty}{U}} CL_{\gamma_i}^{a,b,\beta_i}$ Thus $CL^{a,b} = \underset{\gamma_i \beta_i = 1}{\overset{\infty}{U}} CL_{\gamma_i}^{a,b,\beta_i}$

5. CONCLUSION

In this paper canonical-Laplace is generalized in the form the distributional sense, and proved results on countable union s-type space. Also discussed the topological structure of some testing function spaces.

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