Telescoping Composite Mechanics for Composite Behavior Simulation

C. C. Chamis[1], P. L. N. Murthy[1], P. K. Gotsis[1] , and S. K. Mital[2]

[1] *NASA Lewis Research Center, Cleveland, Ohio 44135, U.S.A.*
[2]*University of Toledo, Toledo, Ohio 43606, U.S.A.*

SUMMARY: Telescoping composite mechanics are described and implemented in terms of recursive laminate theory. The initial elemental scale is defined where simple equations are derived. Subsequently these mechanics are applied to homogeneous composites, composite structural components and hybrid composites. Results from those applications are presented in terms of tables/figures to illustrate the versatility and generality of telescoping composite mechanics. Comparisons with methods such as approximate, single cell, and 2-D and 3-D finite element demonstrate the predictive accuracy and computational effectiveness of composite telescoping mechanics.

KEYWORDS: Fiber composites, micromechanics, minimechanics, laminate theory, elemental scale, unit cell, hybrid composite, finite element, structural analysis, scale telescoping, scale substructuring, scale definition, computational simulation.

INTRODUCTION

Composites inherently contain several scales, [1]). Formulations to simulate composite behavior are usually based on some intermediate observation scale between the most elemental level (for example, micromechanics) and the laminate. The contention for any other than the most elemental scale is that element scale formulations may be either too difficult or credible testing cannot be performed to justify the relevant assumptions. On the other hand, formulations based on the observation scale, say for example laminate, usually are material and/or structural configuration dependent which must be repeated for every material/laminate. This latter approach is obviously time consuming and costly. For example, if the laminate response may be viewed as the observation scale, then the laminate theory based on known unidirectional composite (ply) properties may be viewed as meso-mechanics scale while micromechanics will be the micro scale where the formulation starts from constituent properties and incorporates composite processing variables as well.

Another example, formulations of structural response based on simulated component information may be viewed as meso-structural mechanics; while those based on the fundamental variables that describe the simulated component response will be viewed as micro-structural mechanics. The two examples mentioned above are generic and apply to other situations as well as such as (1) coupon testing versus granular structure, (2) granular structure versus metallurgical formulations, (3) polymer chains versus physical chemistry formulations, (4) structural system response versus finite element formulations etc. An alternate recently emerging view, which is by far more generic, is that of telescoping scale mechanics formulations based on fundamental (elemental) level variables and on subsequent propagation of that information to any desired progressively higher observation scale [2]).

A representative example of telescoping scale mechanics is composite mechanics which is performed by repeated application of laminate theory from fiber substructuring (where environmental and fiber architecture effects are included), to ply substructuring and to laminate substructuring. The objectives of this article are: (1) To describe telescoping scale mechanics and, (2) to demonstrate their generic features by using them to predict information that may be observed at different hierarchical scales. Three different sample cases are used to demonstrate the generic features of telescoping composite mechanics: (1) homogeneous composites, (2) composite structures and, (3) hybrid composites. These are described briefly by using one or two governing equations by presenting one or two typical results and by citing respective references for further details. It is important to note that the focus is on <u>what</u> can be done rather than on <u>step</u>-<u>by</u>-<u>step</u> <u>details</u> <u>on</u> <u>how</u> it is done. It is important to note that telescoping composite mechanics is only viable by computational simulation which readily applies recursive laminate theory.

TELESCOPING COMPOSITE MECHANICS CONCEPT

To describe telescoping composite mechanics it is convenient and helpful to define the multiple scales inherent in composites. Composite scales as defined herein (author's rational), refer to both substructuring and telescoping. Composite scale substructuring refers to substructuring a laminate progressively to lower consistent scales through-the-thickness: multiple-fiber ply, single fiber ply through-the-thickness of a ply, unit cell, and single fiber slicing (fiber substructuring). Scale telescoping reverses the process of substructuring. Both of these scales can be readily simulated by recursive application of laminate theory. The concept of substructuring/telescoping mechanics by recursive application of laminate theory is illustrated schematically in Figure 1.

Space for figure insertion

When we start with laminates we can progressively substructure (decompose) to lower scales of laminate behavior in terms of stresses and strains. On the other hand it is not easy to substructure laminate behavior in terms of mechanical, thermal or any other properties. We can also substructure any behavior from the highest scale to the lowest by using consistent formal composite mechanics methods. Obviously then formal scale substructuring methods are the inverse of those for scale telescoping.

We see in Figure 1 that: (1) our lowest scale is the slice with a scale a fraction of the fiber diameter; (2) the next scale in the telescoping sequence is a single fiber embedded in a matrix (typical cell) with a scale equal to fiber diameter plus some matrix; (3) the single fiber typical cell telescopes into a multi-fiber ply with a scale of ply thickness; and (4) the multi fiber ply telescopes into multi-ply laminate with a scale of laminate thickness. Other scales in the telescoping sequence up to the structural system scale (not shown in the Figure 1) include: (5) multi-laminate finite element; (6) multi-finite-element subcomponent; (7) multi-subcomponent component and (8) multi-component composite structural system. Scales (1), (2), (3) and (4) are readily simulated by recursive laminate application, while scales (5), (6) and (7) are structural scales and are not simulated by recursive laminate application (telescoping composite mechanics). The concept of scale telescoping by progressive laminate theory application is natural and has several advantages as follows: (1) the elemental equations remain very simple, (2) the computer keeps track of the information needed to propagate that scales information to the next higher scale, (3) laminate theory is widely used and extensively discussed in textbooks, (4) environmental effects are easily incorporated at the scale they occur, (5) fabrication processes are accounted, (6) nonlinear geometry and material behavior are readily simulated by incrementation and (7) time and related effects are also simulated incrementally or stated differently by updated Lagrangian Methods.

HOMOGENEOUS COMPOSITES

Homogeneous composites are defined herein to be made from one type of fibers in one type of matrix. Telescoping composite mechanics will be described in terms of elemental slice, single fiber cell, multi-fiber cell (single ply) and multi-ply laminate.

Elemental Scale – A significant part in telescoping composite mechanics is selection of elemental scale. This selection very much depends on the investigator’s experience. The authors consider fiber substructuring (Fig. 1, right) slice as the elemental scale to derive the governing equations. If we assume that the slice consists of matrix, interphase and fiber the equation for the transverse modulus is:

$$\mathrm{E}_{22s} = \frac{}{k_{ms}}\,\frac{}{\mathrm{E}_{is}}\,\frac{\mathrm{E}_{ms}}{\mathrm{E}_{fx} +}\,\frac{\mathrm{E}_{is}}{k_{is}}\,\frac{\mathrm{E}_{fs}}{\mathrm{E}_{fs}}\,\frac{}{\mathrm{E}_{ms} + k_{fs}\mathrm{E}_{ms}\mathrm{E}_{is}} \qquad (1)$$

The notation in equation (1) is as follows: E is the modulus of the constituent, k is the volume ratio of that constituent in the slice, subscripts *m, I* and *f* denote matrix interphase and fiber respectively, subscript *s* denotes slice and subscript *22* denote slice modulus on plane 2 in the 2 direction. The slice volume ratios are determined from specified: fiber diameter, average composite volume ratio type of fiber distribution array size (thickness or volume ratio) of the interphase and number of slices in substructuring the fiber.

Equation 1 is very simple. However, it is most inclusive because matrix voids interfacial disbonds, or partial bonds, and environmental effects are readily included. Comparable equations can be written for other mechanical properties as well as for thermal and hygral properties [3]. The first application or recursive laminate theory consists of stacking the slices up and predicting the composite unit cell properties (schematic above slice, Fig. 1, right). The major advantage is that all unit cell properties are predicted by the same assumed local uniformaties or non-uniformities.

Single Fiber Cell – It is instructive to compare the properties predicted by the slicing approach to those predicted by methods based or unit cell micromechanics equation, [4],

$$\mathrm{E}_{l22} = \frac{\mathrm{E}_m}{1 - \sqrt{k_f}\left(1 - \frac{\mathrm{E}_{f22}}{\mathrm{E}_{m22}}\right)} \tag{2}$$

and 3-D finite element methods [5] in Table 1. The comparisons are from very good to excellent. They demonstrate the effectiveness of the application of recursive laminate theory at that fiber subscale level as well as being inclusive and very, very simple. The alert reader would have recognized that what was done is application of elements in series at the slice level, application of elements in parallel at the slice stack and a form of trapezoidal numerical integration of the stacking process to represent the single fiber cell assumed in the derivation of Equation (2).

Space for figure insertion

Multi-fiber Ply – Commercially available tapes have normally a partially cured thickness of one ply (lamina). This includes about 15 fibers through-the-thickness of a ply (Glass, Graphite, Kevlar). Through-the-thickness non-uniformities are readily represented by substructuring each of the fifteen fibers into respective slices and then stack them by applying laminate theory again. Comparitive results with two other micromechanics methods (Fig. 2) are shown in Figure 3 for mechanical properties and in Figure 4 for thermal properties. Note the comparisons are from three different computer codes: ICAN [6]), METCAN [7]) and CEMCAN [3]). ICAN is based on square area unit-cell micromechanics, METCAN same as ICAN but includes interphase while CEMCAN is based on fiber substructuring slice. Again the comparisons are very good especially between CEMCAN and METCAN. Further demonstrating the effectiveness of telescoping composite mechanics.

Space for figure insertion

Space for figure insertion

Multi-ply Laminate/Composite - The governing equations for plate-type laminate behavior simulation in array form are:

$$\left\{\frac{\varepsilon_{co}}{\kappa_{c}}\right\} = \left[\begin{array}{cc} A_c & B_c \\ \hline B_c & D_c \end{array}\right]^{-1} \left\langle \left\{\frac{N_{ca}}{M_{ca}}\right\} - \left\{\frac{N_{ct}}{Mct}\right\} - \left\{\frac{N_{cm}}{M_{cm}}\right\} \right\rangle \qquad (3)$$

ε are reference plane strains; κ are the curvatures; A, B and D are the axial, coupling and bending stiffnesses, respectively; N, and M are the inplane forces and bending moments, respectively. The subscripts *o* refers to reference plane; *c* denotes laminate property; T and M temperature and moisture, respectfully. The elements of the arrays and vectors on the right side of equation (3) are evaluated by applying conventional laminate theory [6] with ply properties. Clearly, then, the laminate properties were obtained by three (slice, multi-fiber-ply, multi-ply) recursive applications of conventional laminate theory.

Equation (3) is used as input to finite element structural analysis of general composite plate type structures [6]. A typical result obtained from that type of composite structural analysis is shown in Figure 5 along with measured data comparisons [8]. The agreement is very good. It is important to note that recursive application of laminate theory is so generic that it simplifies simulation of several other composite architectures one of which is described subsequently.

Space for figure insertion

HYBRID COMPOSITES

There are two types of hybrid composites - interply (ply by ply) and intraply - tows of different fibers are intermingled in the same ply. Interply hybrid composites are comparable to homogeneous composites but each ply is made from different fiber and matrix. Recursive application of laminate theory is identical to that already described. Intraply hybrid composites are different as shown in the schematic in Figure 6, [9]. This simulation requires an additional recursive laminate application - two for the simulation of each composite as individual plies instead of one for the homogeneous case prior to laminate simulation. Another demonstration of the versatility of telescoping composite mechanics.

Space for figure insertion

The micromechanics equation for predicting transverse modules of intraply hybrid composite is [10].

$$E_{HC2} = \frac{E_{mp}}{\left[1 - k_{fp}\left(1 - \frac{E_{mp}}{E_{fp}}\right) + V_{sc}\left\{\frac{E_{mp}}{E_{ms}}\left[1 - \sqrt{k_{fs}}\left(1 - \frac{E_{ms}}{E_{f2s}}\right)\right] - \left[1 - k_{fp}\left(1 - \frac{E_{mp}}{E_{f2p}}\right)\right]\right\}\right]}$$

(4)

The notation in equation (4) is as follows: E is ply modulus; *k* is fiber volume ratio: Subscripts: H is hybrid; *c* is composite, *m* is matrix, *f* is fiber, *p* is primary composite, *s* is secondary composite. Predictions obtained from equation (4) are compared with more approximate equations, with recursive laminate theory, with 2-D finite element, and with measured data in Table 2 [10]. The comparisons are considered to be very good in view of the computational expediency of the telescoping composite mechanics through the application of recursive laminate theory.

Space for figure insertion

Space for figure insertion

CONCLUSION

An investigation is described which uses telescoping composite mechanics concepts and implements those concepts by recursive application of laminate theory. In these concepts formulations are based in the lowest possible scale-elemental equations for all composite properties. The composite behavior is then synthesized by recursive application of laminate theory up to the laminate. The laminate response is decomposed to lower scales by progressive laminate substructuring through laminate theory. Typical results predicted are compared with those obtained from other methods, single cell micromechanics, and 2-D and 3-D finite element. Those comparisons demonstrate the inclusiveness, accuracy, and computational effectiveness of telescoping composite mechanics. Additional results from homogeneous and hybrid, composites, and from composite structures illustrate the generality, and versatility of telescoping composite mechanics. Collectively these results demonstrate that laminate theory is a very efficient computational algorithm for composite mechanics and even homogeneous material experiencing layered-type nonlinear material behavior.

REFERENCES

1. Chamis, C. C., “Mechanics of Composite Materials: Past, Present, and Future”, *Journal of Composites Technology & Research*, l989, Vol. 11, No. 2, pp. 3-14.

2. Mital, S. K., Murthy, P.L.N. and Chamis, C. C., “ Concrete Thermomechanical Behavior Via Telescoping Scale Mechanics”, invited paper for the Prager Symposium to honor Professor Z.P. Bazant 33rd Annual Technical Meeting of the Society of Engineering Science, Tempe, Arizona, October, 1996.
3. Murthy, P.L.N. and Chamis, C. C., “Towards the Development of Micromechanics Equations for Ceramic Matrix Composites via Fiber Substructuring,” NASA TM 105246, February 1992.

4. Chamis, C. C., “Simplified Composite Micromechanical Equations for Hygral, Thermal, and Mechanical Properties.” *Sampe Quarterly*, l984, pp. 14-23.

5. Caruso, J. J. and Chamis, C. C., “Assessment of Simplified Composite Micromechanics Using Three-Dimensional Finite-Element Analysis”, *Journal of Composites Technology & Research,* l986, Vol. 8, No. 3, pp. 77-83.

6. Murthy, P. L. N and Chamis, C. C., “Integrated Composite Analyzer (ICAN), Users and Programmers Manual”, NASA TP 2515, March 1986.

7. Hopkins, D. A., and Chamis, C. C. “A Unique Set of Micromechanics Equations for High Temperature Metal Matrix Composites”,. NASA TM 87154, November 1985.

8. Chamis, C. C., “Vibration Characteristics of Composite Fan Blades and Comparison with Measured Data”, *Journal of Aircraft*, l977, Vol. 14, No. 7, pp. 644-647.

9. Chamis, C. C., Lark, R. F. and Sinclair, J. H., “Mechanical Property Characterization of Intraply Hybrid Composites”, NASA TM 79306, October 1979.

10. Chamis C. C. and Sinclair, J. H., “Micromechanics of Intraply Hybrid Composites: Elastic and Thermal Properties”, NASA TM 79253, December 1979.