

Kaluza-Klein FRW cosmological models in Lyra manifold

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Abstract

We have constructed five dimensional FRW cosmological models for $k = -1, 1, 0$ in Lyra manifold with time dependent displacement field. The matter field is considered in the form of a perfect fluid with isotropic matter pressure. It is found that the model for $k = -1$ is inflationary. For $k = 1$, the model is inflationary for set of values of arbitrary constant n and decelerates in the standard way for another set of values of n . Moreover the concept of Lyra manifold does not exist at infinite time.

Keywords: five dimensions, FRW, Lyra manifold.

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1 Introduction

In view of the Kaluza-Klein theories [1, 2, 3, 4] the study of higher dimensional cosmological models has acquired much significance. Various authors [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] constructed higher dimensional cosmological models in general theory of relativity.

The geometrization of gravitation by Einstein in his general theory of relativity inspired several authors to geometrize other physical fields. Weyl [17] proposed a unified theory to geometrize gravitation and electromagnetism. This theory inspired Gehard Lyra [18] to develop Lyra's geometry. Subsequently Sen [19] and Sen and Dunn [20] suggested a new scalar tensor theory of gravitation. They modified the Einstein's field equations based on Lyra's manifold in normal gauge as

$$R_{ik} - \frac{1}{2}g_{ik}R + \frac{3}{2}\phi_i\phi_k - \frac{3}{4}g_{ik}\phi_m\phi^m = -\chi T_{ik} \quad (1)$$

where ϕ_i is the displacement vector and other symbols have their usual meanings in the Riemannian geometry. Jeavons et al.[21] pointed out that the field equations proposed by Sen and Dunn are heuristically useful even though they are not derived from the usual variational principle. A brief note on Lyra's geometry is given by Singh and Singh [22].

Halford [23] showed that the energy conservation law does not hold in the cosmological theory based on Lyra's geometry. Halford [24] showed that the scalar tensor theory of gravitation in Lyra manifold gives same effects, within observational limits, as in the Einstein theory. Soleng [25] pointed out that the constant gauge vector ϕ in Lyra's geometry together with a creation field becomes Hoyle's [26] creation field cosmology or contains a special vacuum field, which together with the gauge vector may be considered as a cosmological term. Further Soleng [27] showed that for matter with zero spin the field equations of his scalar tensor theory reduce to those of Brans-Dicke theory. Beesham [28] constructed four dimensional FRW cosmological model in Lyra geometry. Assuming the energy density of the universe equal to its critical value he showed that the models have $k = -1$ geometry. Singh and Desikan [29] obtained the exact solutions for four dimensional FRW cosmological model in Lyra geometry with constant deceleration parameter. They examined the behavior of the displacement field β and the energy density ρ for perfect

fluid distribution. However they found that the expressions for β^2 and ρ are not valid for empty universe and the stiff matter distribution. Mohanty et al. [30, 31] showed the non existence of five dimensional perfect fluid cosmological model in Lyra manifold. Further they obtained the exact solutions of the field equations for empty universe. Mohanty et al.[32] showed that in a five dimensional space-time the general perfect fluid distribution does not survive but degenerates stiff fluid distribution in Lyra manifold. Various higher dimensional cosmological models [33, 34, 35, 36, 37] are constructed in Lyra manifold.

In this paper we constructed various five dimensional FRW cosmological models in Lyra manifold when the source of gravitation is a perfect fluid. The isotropy of pressure is assumed in all dimensions including the extra ones [38, 39, 40, 41, 30, 31, 32]. In section 2 we obtained the field equations for the 5D FRW line element. In section 3 the solutions of the field equations are obtained and some physical and kinematical properties of the models are discussed. In section 4 concluding remarks are given.

2 Field equations

Here we consider the five dimensional FRW metric in the form

$$ds^2 = -dt^2 + R^2(t) \left[\frac{dr^2}{1 - k r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] + A^2(t) d\mu^2 \quad (2)$$

where R and A are functions of cosmic time “ t ” only and k characterizes the spatial curvature. The fifth co-ordinate μ is assumed to be space like.

The energy momentum tensor for perfect fluid distribution is taken as

$$T_{ij} = (p + \rho)u_i u_j + p g_{ij} \quad (3)$$

together with the co-moving co-ordinates

$$g^{ij}u_i u_j = -1, \quad (4)$$

where p and ρ are isotropic pressure and energy density of the cosmic fluid distribution respectively and u_i is the five velocity vector of the fluid which has components $(1, 0, 0, 0, 0)$.

The displacement vector ϕ_h is defined as

$$\phi_h = (\beta(t), 0, 0, 0, 0). \quad (5)$$

The field equations (1) together with the equations (3), (4) and (5) for the metric (2) yield the following equations

$$2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2 + 2\frac{R'A'}{RA} + \frac{A''}{A} + \frac{3}{4}\beta^2 + \frac{k}{R^2} = -\chi p \quad (6)$$

$$-3\left(\frac{R'}{R}\right)^2 - 3\frac{R'A'}{RA} + \frac{3}{4}\beta^2 - 3\frac{k}{R^2} = -\chi\rho \quad (7)$$

and

$$3\frac{R''}{R} + 3\left(\frac{R'}{R}\right)^2 + \frac{3}{4}\beta^2 + 3\frac{k}{R^2} = -\chi p \quad (8)$$

where prime denotes differentiation with respect to time 't'.

3 Solution of the field equations

In equations (6)-(8) there are five unknowns viz. R , A , β , P and ρ involved in three independent field equations. In order to obtain explicit exact solutions, we consider

$$A = R^n, \quad n(\neq 0) \quad \text{is a parameter} \quad (9)$$

and the equation of state i.e.

$$p = m\rho, \quad 0 \leq m \leq 1 \quad (10)$$

Now the set of equations (6)-(10) admit an exact solution given by

$$R = at + b, \quad a(\neq 0), b \quad \text{are constants} \quad (11)$$

$$A = (at + b)^n \quad (12)$$

$$\chi\rho(= \frac{\chi p}{m}) = \frac{6a^2 + 3na^2 + 6k}{(1-m)(at+b)^2} \quad (13)$$

$$\frac{3}{4}\beta^2 = \frac{a^2(1+n+n^2+5m+2mn-mn^2) + k(1+5m)}{(m-1)(at+b)^2} \quad (14)$$

where

$$a^2 = \frac{2k}{(n+2)(n-1)} \quad \text{and} \quad m \neq 1 \quad (15)$$

In the following subsections we intend to construct different cosmological models for different values of k i.e. $k = 0, -1, +1$

3.1 Case I. *Flat Model* ($k = 0$)

In this case we obtained a five dimensional flat vacuum model with zero curvature in general theory of relativity.

3.2 Case II. *Open Model* ($k = -1$)

In this case the metric (2) takes the form

$$ds^2 = -dt^2 + (at + b)^2 \left(\frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) + (at + b)^{2n} d\mu^2 \quad (16)$$

The pressure (p), energy density (ρ) and gauge function (β) become

$$\chi\rho (= \frac{\chi p}{m}) = \frac{6a^2 + 3na^2 - 6}{(1-m)(at+b)^2} \quad (17)$$

$$\frac{3}{4}\beta^2 = \frac{a^2(1+n+n^2+5m+2mn-mn^2) - 5m - 1}{(m-1)(at+b)^2} \quad (18)$$

where

$$a^2 = \frac{-2}{(n+2)(n-1)} \quad \text{and} \quad m \neq 1 \quad (19)$$

Here $\chi\rho > 0$ for $n \in (0, 1)$ and $a^2 > 0$ for $n \in (-2, 1)$. Thus equ.(16) together with (17)-(19) represent the FRW perfect fluid open cosmological model in Lyra geometry for $n \in (0, 1)$.

At $t = 0$ the model (16) becomes flat and the pressure, energy density and gauge function β have finite values. As time increases the scale factors R and A increase indefinitely. So in the above open model there is no compactification of extra dimension. Further the energy density (ρ) of the universe is positive throughout the evolution and $\rho \rightarrow \infty$ as $t \rightarrow \frac{-b}{a}$. Hence the model is free from initial singularity but possesses line singularity at $t = \frac{-b}{a}$. The energy density of the universe decreases with increase of cosmic time t . The gauge function $\beta \rightarrow \infty$ as $t \rightarrow \frac{-b}{a}$ and as $t \rightarrow \infty$. Hence the concept of Lyra manifold does not remain for a very large time.

The scalar expansion (θ), shear scalar (σ^2), spatial volume (V) and deceleration parameter (q) for the model (16) are obtained as

$$\theta = u_{;i}^i = \frac{a(n+3)}{at+b} \quad (20)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2} \left[\frac{4}{9} + 3 \left(\frac{a}{at+b} + \frac{1}{3} \right)^2 + \left(\frac{an}{at+b} + \frac{1}{3} \right)^2 \right] \quad (21)$$

$$V = \sqrt{-g} = (at+b)^{n+3} \quad (22)$$

and

$$q = -\frac{V\ddot{V}}{\dot{V}^2} = -\frac{n+2}{n+3} \quad (23)$$

Here we observe that the deceleration parameter (q) is negative when $n \in (0, 1)$, which confirms that the open model (16) is inflationary.

3.3 Case III. *Closed Model* ($k = +1$)

In this case the values of R and A are same as obtained earlier in equations (11) and (12) respectively and the metric (2) takes the form

$$ds^2 = -dt^2 + (at+b)^2 \left(\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) + (at+b)^{2n} d\mu^2 \quad (24)$$

The values of energy density (ρ) and the gauge function (β) reduce to

$$\chi\rho (= \frac{\chi p}{m}) = \frac{6a^2 + 3na^2 + 6}{(1-m)(at+b)^2} \quad (25)$$

$$\frac{3}{4}\beta^2 = \frac{a^2(1+n+n^2+5m+2mn-mn^2)+5m+1}{(m-1)(at+b)^2} \quad (26)$$

where

$$a^2 = \frac{2}{(n+2)(n-1)} \quad \text{and} \quad m \neq 1 \quad (27)$$

Here $\chi\rho > 0$ for $n \in (-\infty, 0) \cup (1, \infty)$ and $a^2 > 0$ for $n \in (-\infty, -2) \cup (1, \infty)$. Thus in this case equ.(24) together with (25) and (26) represent the FRW perfect fluid closed cosmological model in Lyra geometry for $n \in (-\infty, -2) \cup (1, \infty)$.

In this model the compactification of the extra dimension takes place when $n \in (-\infty, -2)$ where as the scale factor A of the three spatial directions increases with the increase of cosmic time t . In this case it is observed that

- (i) the behavior of energy density and the gauge function are similar to that of open model discussed earlier in section 3.2.
- (ii) The spatial volume V given in equ.(22) decreases with increase of cosmic time t when $n \in (-\infty, -3)$ and as $t \rightarrow \infty$, $V \rightarrow 0$.
- (iii) The behavior of spatial volume V is similar to that of the open model discussed earlier in section 3.2 when $n \in (-3, -2) \cup (1, \infty)$.
- (iv) The deceleration parameter (q) given in equation (23) is positive when $n \in (-3, -2)$ and negative when $n \in (-\infty, -3) \cup (1, \infty)$

4 Conclusion

In the preceding section we have constructed five dimensional FRW cosmological models when the source of gravitation is generated by a perfect fluid.

In case of flat model with zero curvature i.e. $k = 0$, we found that the model reduces to five dimensional flat vacuum model of the universe in Einstein's general theory of relativity and the volume is finite whereas the volume is infinite in four dimensional case.

In case of open model of the universe with negative curvature i.e. $k = -1$, we observed that the model is inflationary and compactification of the extra dimension does not occur. The energy density of the universe remains positive throughout the evolution. The model admits singularity at $t = \frac{-b}{a}$. The energy density and pressure of the universe decrease with increase of cosmic time. The volume of the space is infinite which is similar to that of four dimensional case.

In case of closed model of the universe with positive curvature i.e. for $k = 1$, we examined that the model is inflationary for $n \in (-\infty, -3) \cup (1, \infty)$ and the model decelerates in the standard way for $n \in (-3, -2)$. Moreover the compactification of the extra dimension occurs in this model for $n \in (-\infty, -2)$. The spatial volume of the universe of this model decreases as cosmic time t increases and becomes zero as t tends to infinity when $n \in (-\infty, -3)$. This behavior is similar to that of four dimensional case.

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Kaluza-Klein FRW kosmološki modeli u Lyra mnogostrukosti

Konstruisani su petodimenzioni FRW kosmološki modeli za $k = -1, 1, 0$ u Lyra mnogostrukosti sa vremenski promenljivim poljem pomeranja. Polje materije se razmatra u obliku idealnog fluida sa izotropnim materijalnim pritiskom. Nadjeno je da je model pri $k = -1$ inflatoran. Za $k = 1$ model je inflatoran za skup vrednosti proizvoljne konstante n i usporava se na standardan način za neki drugi skup vrednosti n . Štaviše koncept Lyra mnogostrukosti ne postoji pri beskonačnom vremenu.