

DECOMPOSED PYRAMID VECTOR QUANTIZATION BY USING BARNES-WALL LATTICE

Ján MIHALÍK – Iveta GLADIŠOVÁ – Viktor MICHALČIN
Dept. of Electronics and Multimedia Telecommunication FEI TU Košice
Park Komenského 13, 041 20 Košice, Slovak Republic
tel.:+42195 6022854; fax:+42195 6323989
mihalik@ccsun.tuke.sk, gladis@ccsun.tuke.sk, viki07@pobox.sk

Abstract

The decomposed pyramid vector quantization (DPVQ) by using 16-dimensional Barnes-Wall lattice is presented and also its fast algorithm. DPVQ is motivated by a geometric structure of the memoryless Laplacian source of vectors. Its quantization vectors are the points that lie on the surface of 16-dimensional pyramid and also are points of Barnes-Wall lattice, which is an optimum one for this dimension according to minimization of the quantization noise.

1. Introduction

The pyramid vector quantizer (PVQ) [1], [2] is technical unpretentious solution with fast and effective processing of input data. The PVQ is one member of a class of similarly defined vector quantizers that select quantization vectors from a suitable lattice, subject to the geometry induced by the probability density function of the source. The region of high probability will have a geometry that is dependent on the source - for Laplacian source it is a pyramid. Thus good codebooks of PVQ can also be constructed as an intersection of the points of suitable lattice and the geometric region of high probability for this source.

2. Barnes-Wall lattice

PVQ with Barnes-Wall lattice includes quantization vectors lying on the surface of its pyramid and also are points of the lattice, which is an optimum one in 16-dimensional space from viewpoint of minimum quantization noise. Barnes-Wall lattice [6] was described as

$$\Lambda_{16} = \bigcup_{i=0}^{31} (r_i + 2D_{16}) = \bigcup_{i=0}^{31} \Lambda_{16}^{(i)}, \quad (1)$$

where r_i are offset vectors. Λ_{16} has the sublattice $2D_{16}$ with 32 coset representatives which are codewords of [16, 5, 8] first-order Reed-Muller code (vector r_i). The lattice D_{16} is 16-dimensional one with points $(b_1, b_2, \dots, b_{16})$ having integer coordinates giving an even sum

$$D_{16} = \left\{ \mathbf{b} = (b_1, b_2, \dots, b_{16}); \sum_{i=1}^{16} b_i = \text{even number} \right\}. \quad (2)$$

The surface of 16-dimensional integer pyramid [5] is defined as follows

$$S(16, K) = \left\{ \mathbf{x} : \sum_{i=1}^{16} |x_i| = K \right\}, \quad (3)$$

where the scalar parameter K is an integer radius of the pyramid, $\mathbf{x}=(x_1, x_2, \dots, x_{16})$ is 16-dimensional point lying on the surface of pyramid $S(16, K)$. With regard to the possible intersection of the lattice Λ_{16} and the pyramid with $S(16, K)$ can be modified definition of the Barnes-Wall lattice in dependence on the radius K of the integer pyramid such a way

$$0 \leq K \leq 7; \Lambda_{16} = \Lambda_{16}^{(0)} = 2D_{16} + r_0 = 2D_{16} \quad (4)$$

$$8 \leq K \leq 15; \Lambda_{16} = 2D_{16} \bigcup \Lambda_{16}^{(2)} \bigcup \Lambda_{16}^{(3)} \dots \bigcup \Lambda_{16}^{(31)} = 2D_{16} \bigcup \text{complement} \quad (5)$$

$$K \geq 16; \Lambda_{16} = 2D_{16} \bigcup \Lambda_{16}^{(1)} \bigcup \Lambda_{16}^{(2)} \bigcup \Lambda_{16}^{(3)} \dots \bigcup \Lambda_{16}^{(31)} = 2D_{16} \bigcup \text{complement}, \quad (6)$$

where \mathbf{U} is an operator of unification and the complement is unification all lattices $\Lambda_{16}^{(i)}$.

3. Decomposition of pyramid with Barnes-Wall lattice

DPVQ uses for looking up of quantization vectors the idea of decomposition of 16-dimensional pyramid and thus decomposition of Barnes-Wall lattice on this pyramid. Every 16-dimensional quantization vector $(b_1, b_2, \dots, b_{16})$ can be decomposed on eight 2-dimensional subvectors $(b_1, b_2), (b_3, b_4), \dots, (b_{15}, b_{16})$. These subvectors are the points lying on eight 2-dimensional pyramids with radius

$$K_j = |b_{2j-1}| + |b_{2j}|, \quad j=1,2,\dots,8. \quad (7)$$

If we order individual radiuses K_j to 8-dimensional vector, then we obtain the vector of global scelet (GS) (K_1, K_2, \dots, K_8) . GS represents a part of codebook, which contains the determinate number of quantization vectors (Fig.1a). GS consists of a finite number of subscelets, which are represented by their vectors. Coordinates of the vectors of subscelets correspond to the values of patterns of 2-dimensional pyramids with a given radius by which are generated the quantization vectors of DPVQ, as it is seen in Fig.1b for $K=3$. The lattice $2D_{16}$ has the vector of GS the same as that one of subscelet. Total codebook generated on the pyramid with radius

$$K = \sum_{j=1}^{16} |2d_j + r_{ij}|, \quad (8)$$

where d_j are the coordinates of points of the lattice D_{16} and r_{ij} are j -th coordinates of the offset vector \mathbf{r}_i , can be decomposed on a finite number of partial codebooks corresponding to the separate GS which numbers for some values of radius together with numbers of quantization vectors are in Tab.1.

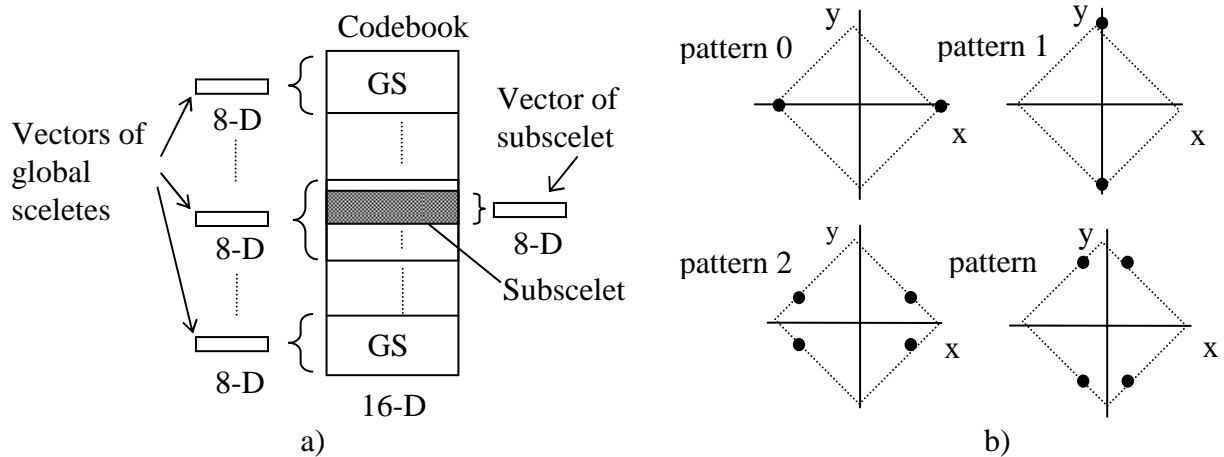


Fig.1a) Decomposition of the codebook under GS; b) patterns of 2-dimensional pyramids for radius $K=3$.

Tab. 1

	Number of quantization vectors			Number of global sceletes		
	$2D_{16}$	complement	sum	$2D_{16}$	complement	sum
$K=4$	512	0	512	4	0	4
$K=8$	44 032	3 840	47 872	18	30	48
$K=10$	0	92 160	92 160	0	236	236

Generation of quantization vectors i.e. points of Barnes-Wall lattice on the 16-dimensional pyramid on the basis of GS [A, B, C, D, E, F, G, H] of lattice $2D_{16}$ is carried out under Fig.2a, in which the 4-dimensional vectors are created by combination of all 2-dimensional vectors from two belonging 2-dimensional pyramids which radiuses are combined as follows AB, CD, EF, GH. By analogues way all 8-dimensional vectors are created by combination of 4-dimensional vectors and then 16-dimensional vectors by combination of 8-dimensional vectors from belonging 4 or 8-dimensional pyramids, respectively. The principle of generation of quantization vectors on the basis of GS [A B C D E F G H] of the complement is described in Fig.2b. Bellow the every radius of the GS of complement are points lying on 2-dimensional pyramids of the patterns, which values create the coordinates of the GS of complement. Lines in Fig.2b show the manner of grouping of the points successively to 16-dimensional vectors. Then full lines crossing from the left to the right represent the quantization vectors, which must lie on the surface of 16-dimensional pyramid with radius K and are also the points of Barnes-Wall lattice.

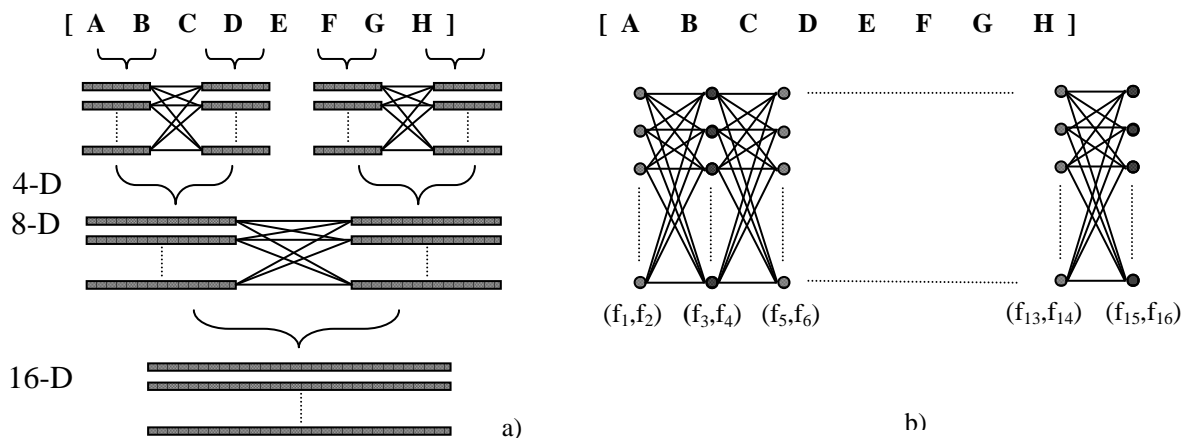


Fig.2 The principle of generation of 16-dimensional quantization vectors represented by a global scele from a) lattice $2D_{16}$; b) complement.

4. Algorithm of decomposed pyramid vector quantization

On the basis of decomposition of 16-dimensional pyramid with Barnes-Wall lattice the fast algorithm of DPVQ search for only a part instead of full search for the codebook by using calculation of Euclidean distances thereby reduce a number of matematic operations. The fast algorithm of DPVQ is as follows:

1. Scale input vector $\mathbf{x}=(x_1, x_2, \dots, x_{16})$ by the coefficient c , i.e. $\mathbf{x}_n=c\mathbf{x}$. By scaling we can increase signal to noise ratio (SNR). Maximum of SNR we get for the optimum coefficient c_{opt} , which is determined experimentally in dependency on the radius K and the energy of input vectors..
2. Scaled 16-dimensional vector \mathbf{x}_n is arranged to 8-dimensional vector $\mathbf{d}=(d_1, d_2, \dots, d_8)$, where d_1 is a sum of absolute values of the first two components of the vector \mathbf{x}_n , d_2 – next two and so on.

3. Finding such a 8-dimensional vector $\mathbf{p}=(A, B, C, D, E, F, G, H)$ from the set of vectors of GS and their modification, which has the smallest Euclidean distance δ_{min} to the vector \mathbf{d}

$$\delta_{min} = (A - p_1)^2 + (B - p_2)^2 + \dots + (H - p_8)^2 \quad (9)$$

4. Generation all 16-dimensional quantization vectors represented by the vector \mathbf{p} of GS.
5. Input vector \mathbf{x}_n is compared to the generated 16-dimensional quantization vectors and the vector \mathbf{b} is selected with the smallest Euclidean distance from the vector \mathbf{x}_n .
6. Inverse scaling of the vector \mathbf{b} to the output quantized vector $\tilde{\mathbf{x}} = \mathbf{b}/c$.

5. Simulation results

The proposed algorithm of the DPVQ was verified on the PC Pentium 166 MHz and was simulated for the sequence of 22000 input vectors and 16-dimensional pyramid with radius $K=4, 8, 10$. The input training sequence of vectors was generated by independent and identically distributed Laplacian random variables with zero mean values and unitary variances. From the values K of 16-dimensional pyramid with Barnes-Wall lattice follow out number of quantization vectors N in Tab.2, that determines a bit rate n bit/sample. Results of simulation are also in this table, where efficiency of DPVQ is measured by SNR in decibels, while σ_q^2 is mean square value of quantization noise.

Tab. 2

K	N	n(bit/sample.)	c_{opt}	SNR(dB)	σ_q^2
4	512	0.5625	1.02	3.1187	0.4874
8	47 872	0.9717	1.25	5.1487	0.3059
10	92 160	1.0307	1.42	5.6354	0.2718

The time of the vector quantization in the PC by full searching of the codebook of PVQ was 2ours and 27 minutes and for the fast algorithm the time was only 11 minutes for 16-dimensional pyramid with radius $K=10$. On the other side the number of operations [multiplications / divisions] for the full search algorithm is 183 312 per sample and for the fast algorithm only 336 at the same parameters.

6. Conclusion

DPVQ is a suboptimum vector quantization [3], which can reach higher efficiency then that one of optimum vector quantization [4] for assumption the same complexity of implementation, because it can has larger dimension at equal number of bits per sample. The proposed algorithm of DPVQ is very fast, because it does not look up the full codebook of PVQ, but only its part represented by a vector of GS. It results in lower requirements for memory and also the computational complexity, what gives possibility of fast vector quantization of images with high efficiency.

Acknowledgement

The work presented in this paper was supported by the Grant Agency of Ministry of Education and Academy of Science of Slovak Republic under Grant No. 1/7387/20.

References

- [1] Fischer,T.R.: A Pyramid Vector Quantizers. IEEE Trans. on Inform. Theory, IT-32, No.4, (1986), pp.568-583.
- [2] Mihalík,J.- Gladišová,I.: Weighted Pyramid Vector Quantizer. Electrical Engineering Journal, 46, No.2, (1995), pp.46-50.
- [3] Gersho,A.-Gray,R.M.: Vector Quantization and Signal Compression. Kluwer Academic Publishers, Dordrecht, Notherlands, 1992.
- [4] Chmúrny, J.- Mihalík,J.: Algorithms for Optimization of Vector Quantizers. Slaboproudý obzor, 47, No.9, (1986), pp.435-440. (In Slovak)
- [5] Gladišová,I.- Mihalík,J.: An Algorithm of Pyramid Vector Quantization. Proc. Intl. Conf. “Digital Signal Processing”, Košice, 1993, pp.236-239. (In Slovak)
- [6] Gibson,J.D.: Lattice Quantization. Advances in Electronics and Electron Physics, Vol.72, (1988), pp.259-330.