Graphical Technique to Locate the Center of Curvature of a Coupler Point Trajectory

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This paper presents an original technique to locate the center of curvature of the path traced by an arbitrary point fixed in the coupler link of a planar four-bar linkage. The method is purely graphical and the center of curvature; i.e., the center of the osculating circle, can be located in a direct manner with few geometric constructions. The advantage of this technique, compared to the classical approach using the Euler-Savary equation, is that measurements of angles and distances between points are not required. Also, it is not necessary to locate inflection points or draw the inflection circle for the instantaneous motion of the coupler link. The technique is based on the concept of a virtual link which is valid up to, and including, the second-order properties of motion of the coupler link. The virtual link is coincident with the path normal to the coupler curve; i.e., the line connecting the coupler point to the velocity pole of the coupler link. The absolute instant center of the virtual link defines the ground pivot for the link and is, therefore, coincident with the center of the osculating circle. The authors believe that the graphical approach presented in this paper represents an important contribution to the kinematics literature on the curvature of a point trajectory. [DOI: 10.1115/1.1798091]

Keywords: *Coupler Point Trajectory, Center of Curvature, Osculating Circle, Virtual Link, Graphical Techniques*

1 Introduction

The planar four-bar linkage is the simplest lower pair mechanism with a mobility of one. Therefore, it is natural that the trajectory of a point fixed in the coupler link, referred to as a coupler curve, has received considerable attention in the kinematics literature [1,2]. It is well known that an arbitrary coupler curve of the four-bar linkage is a tricircular sextic [3,4]. It is also well known that for a study of the instantaneous, second-order properties of motion of the coupler link, the coupler curve can be replaced by a circle, referred to in the literature as the osculating circle [5–7], osculation circle [3,8], or osculatory circle [9]. The osculating circle and the coupler curve share three infinitesimally separated points and, therefore, cannot be used for a study of the properties of motion higher than second-order. For the coupler point under investigation, the osculating circle is a unique property of the path traced by this point (i.e., the center is coincident with the center of curvature of the path and the radius is the radius of curvature of the path). In other words, for the purpose of analyzing the instantaneous velocity and acceleration of a coupler point, the point trajectory can be regarded as a circle [6,10]. Replacing a point trajectory by the osculating circle has proved useful in the kinematic analysis and synthesis of complex mechanisms with more than four links since the special properties of a circle make it relatively easy to investigate the path curvature of such mechanisms.

This paper will show that the concept of a virtual link, which connects the coupler point with the center of the osculating circle, provides geometric insight into the study of path curvature. A virtual link must be clearly distinguished from a physical link because it is not a rigid body (i.e., the length of a virtual link is a function of the position of the linkage). However, the virtual link is valid for an investigation of the first- and second-order properties of motion of the coupler link. This concept was utilized by Waldron and Kinzel [10] in a study of the path curvature of a mechanism which contains higher pairs. Pennock and Sankaranarayanan [11] also used virtual links to investigate the curvature of a coupler point trajectory of a geared seven-bar mechanism with kinematic indeterminancy. More recently, virtual links were used to study the path curvature of coupler curves of the single flier and the double flier eight-bar linkages [12,13].

The focus of this paper is to use a virtual link to locate the center of the osculating circle of a coupler curve of the planar four-bar linkage. The virtual link will connect the coupler point to the ground link and the first- and second-order properties of this link define the center of the osculating circle. The virtual link is coincident with the path normal to the coupler curve at a specified instant in time or a specified position of the linkage (henceforth referred to as the design position). In other words, the virtual link is collinear with the line that connects the coupler point to the instantaneous center of zero velocity of the coupler link. An advantage of this technique over other techniques, such as the Euler-Savary equation, is that the method is purely graphical; i.e., analytical equations and measurements of angles and distances are not required. Other advantages are the simplicity of the geometric constructions and it is not necessary to find inflection points of the coupler link. Sandor et al. [14] presented a graphical method to solve the Euler-Savary equation, however, their method requires the construction of the inflection circle for the coupler link.

The important problem, in the proposed approach, is to determine the length of the virtual link; i.e., the distance between the coupler point and the center of curvature of the path traced by this point. The solution is based on the result that the virtual link creates two additional four-bar linkages which are kinematically equivalent to the original four-bar linkage. Two four-bar linkages are said to be equivalent to the nth-order ($n \geqslant 1$) if the coupler point of one linkage traces a curve which has the same geometric properties as the coupler curve of the other linkage up to and including the nth-order [3,7,10]. In other words, the two linkages are said to be equivalent to the first-order if the coupler point of one four-bar has the same velocity as the coupler point of the other four-bar. Similarly, the two linkages are said to be equivalent to the second-order if the two coupler points have the same

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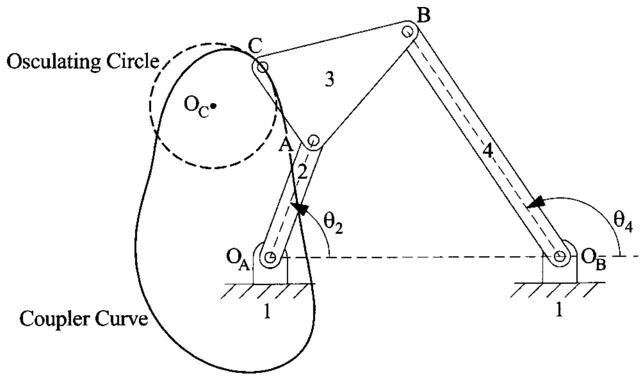

Fig. 1 Coupler curve and the osculating circle

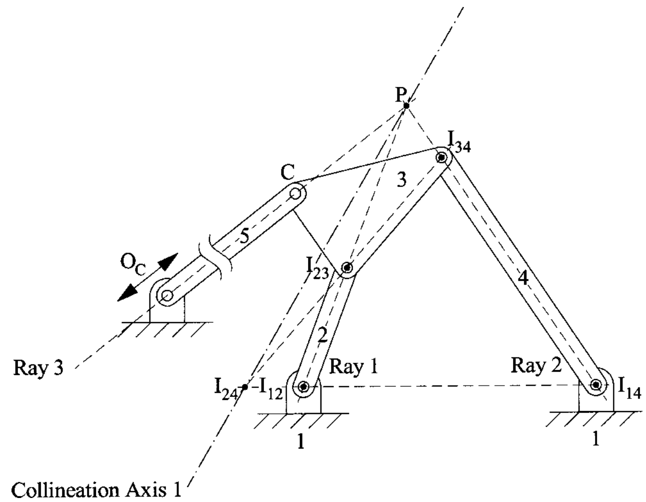

Fig. 2 Instant centers, collineation axis 1 and ray 3

acceleration. Note that if the two linkages are acceleration equivalent then they are also velocity equivalent. The concept of an equivalent four-bar linkage has proved useful in the kinematic synthesis of planar mechanisms. For example, kinematically equivalent linkages are commonly used to duplicate the instantaneous position, velocity, and acceleration of a cam-follower mechanism [5,8,9]. There are an infinite number of acceleration equivalent four-bar linkages for the cam mechanism and the relative motions permitted between connected links are the same even though the joints are physically quite different.

The paper is arranged as follows. Section 2 presents the graphical technique to locate the center of curvature of a coupler point trajectory of a four-bar linkage. The procedure takes advantage of the concept of a virtual link to create two four-bar linkages which are equivalent to the original linkage and locate the center of the osculating circle. This circle can then be used to study the second-order properties of the path traced by the coupler point. Section 3 presents a numerical example to compare this original graphical technique with a traditional approach which utilizes the Euler-Savary equation and the inflection circle for the coupler link. Finally, Sec. 4 presents some observations, conclusions and suggestions for future research.

2 Graphical Technique to Locate the Center of Curvature

Consider the planar four-bar linkage O_AABO_B in the design position shown in Fig. 1. The ground link O_AO_B is denoted as link 1, the side links O_AA and O_BB are denoted as links 2 and 4, respectively, and the coupler link AB is denoted as link 3. The orientation of links 2 and 4 relative to the ground link are denoted as θ_2 and θ_4, respectively. An arbitrary point fixed in the coupler link is denoted as coupler point C. The figure also shows the trajectory of point C; i.e., the coupler curve, and the osculating circle. The purely graphical procedure to locate the center of curvature of the coupler curve for the specified design position (i.e., the center of the osculating circle) will be presented here in three steps.

The first step is to locate secondary instant centers I_{13} and I_{24} using the Aronhold-Kennedy theorem. Instant center I_{13} will henceforth be referred to as the pole of the coupler link and denoted as point P, see Fig. 2. The line that connects the pole to instant center I_{12} is referred to as the first ray (and denoted as ray 1) and the line that connects the pole to instant center I_{14} is referred to as the second ray (and denoted as ray 2). Note that the first and second rays are coincident with the normals to the paths of coupler points A and B, respectively. The line that connects the pole to instant center I_{24} will be referred to as the first collineation axis and denoted as collineation axis 1. The line that connects the pole to coupler point C will be denoted as ray 3 and is normal to the coupler curve in the design position. Therefore, the virtual link (denoted as link 5), which is pinned to the coupler link at C and pinned to the ground link at O_C, must be coincident with ray 3, as shown in Fig. 2.

The second step is to find the collineation axis for the four-bar linkage formed by links 1, 2, 3, and virtual link 5; i.e., the line connecting the pole to instant center I_{25}. This line will be referred to as the second collineation axis and denoted as collineation axis 2, see Fig. 3(*a*). Note that the instant center I_{35} is coincident with coupler point C, and instant center I_{15} will be coincident with the pin connecting the virtual link to the ground link; i.e., O_C. Also note that the coupler link for the four-bar linkage formed by links 1, 2, 3, and virtual link 5 has the same first- and second-order properties of motion as the coupler link of the original four-bar linkage formed by links 1, 2, 3, and 4. Therefore, the two four-bar linkages must have the same pole and the same pole tangent. Also note that the two linkages have the same ray 1.

The graphical technique to find collineation axis 2 is to draw a circle of arbitrary radius and whose center is coincident with the pole. This circle will henceforth be referred to as the construction circle, as shown in Fig. 3(*a*). Denote the points of intersection of this circle with ray 3 as point D; with the collineation axis 1 as point E, with ray 1 as point F, and with ray 2 as point G. Now draw a line, denoted as L_1, connecting points D and E and then draw a line parallel to L_1, denoted as L_2, through point G. The point of intersection of line L_2 with the construction circle is denoted as point H. Finally, draw a line connecting point H and the pole P. The result of this geometrical construction is that line PH is collineation axis 2, and this can be proved as follows.

According to Bobillier's theorem [6,9] the angle between collineation axis 1 and ray 1, denoted as α in Fig. 3(*b*), is equal to the angle between ray 2 and the pole tangent. Similarly, the angle between the pole tangent and ray 1, denoted as β in the figure, is equal to the angle between ray 3 and collineation axis 2. If the angle between ray 3 and line PH is denoted as γ; i.e., $\angle DPH=\gamma$, then line PG is collineation axis 2 if it can be shown that the angle $\gamma=\beta$. Recall the theorem that the angle at the center of a circle is twice the angle at the circumference of the circle when subtended by the same arc. Therefore, observe from Fig. 3(*a*) that

$$\angle EPG=2\angle EHG \quad \text{and} \quad \angle DPH=2\angle DEH \tag{1a}$$

Since lines L_1 and L_2 are parallel then

$$\angle EHG=\angle DEH \tag{1b}$$

The conclusion from Eqs. (1*a*) and (1*b*) is that

$$\angle EPG=\angle DPH=\gamma \tag{2}$$

Also observe from Figs. 3(*a*) and 3(*b*) that

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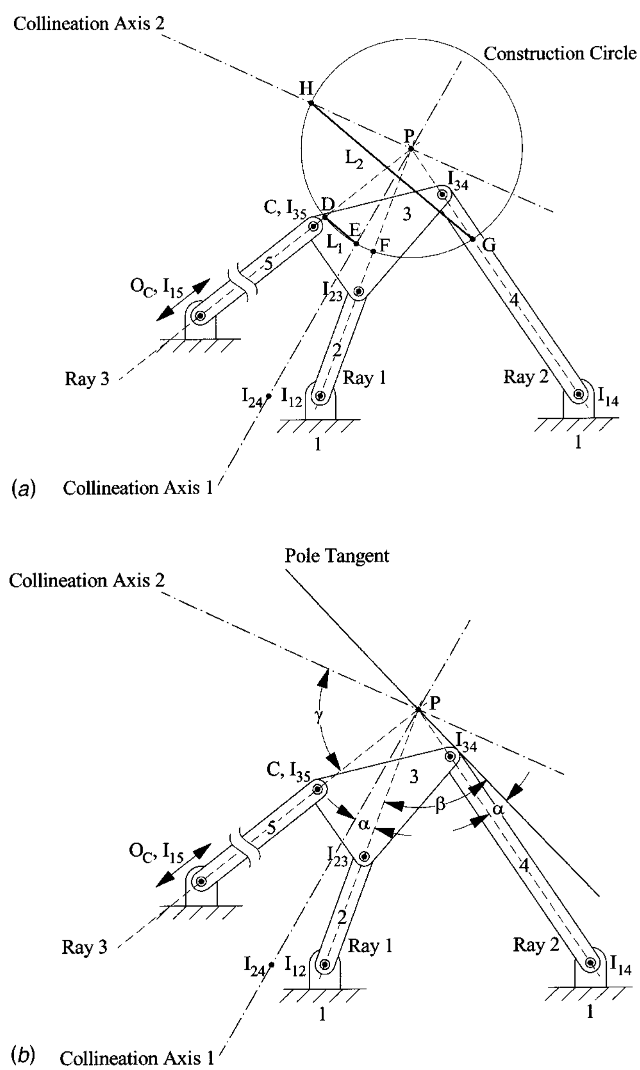

Fig. 3 (*a*) The construction circle and collineation axis 2, (*b*) a proof of the geometric construction

$$\angle \mathrm{EPG} = \alpha + \angle \mathrm{APB} = \beta \tag{3}$$

Equating Eqs. (2) and (3) gives the result that the angle $\gamma = \beta$. Therefore, the line PG is indeed collineation axis 2.

The third, and final, step is to locate the absolute instant center of the virtual link; i.e., I_{15}, using the equivalent four-bar-linkage formed by the ground link O_CO_A, link 2 (O_AA), link 3 (AC), and virtual link 5 (CO_C). From the Aronhold-Kennedy theorem, the instant center I_{25} is the point of intersection of the line connecting instant centers I_{23} and I_{35} with collineation axis 2, see Fig. 4. Therefore, instant center I_{15} is the point of intersection of the line connecting I_{12} and I_{25} with ray 3 as shown in the figure. Recall that the center of curvature of the coupler curve, in the design position, is coincident with the absolute instant center of the virtual link. Also recall that the center of curvature O_C is the center of the osculating circle, as shown in Fig. 5. The distance from the center of curvature to coupler point C defines the length of virtual link 5; i.e., the radius of curvature of the trajectory of point C.

Note that the equivalent four-bar linkage formed by ground link O_CO_B, link 4 (O_BB), link 3 (BC), and virtual link 5 (CO_C) can also be used to locate instant center I_{15}. The graphical construction is the same as before and, in this case, the collineation axis is referred to as the third collineation axis and denoted as collineation axis 3, as shown in Fig. 6. It is interesting to note that instant centers I_{24}, I_{45}, and I_{25} lie on the same straight line. This is in

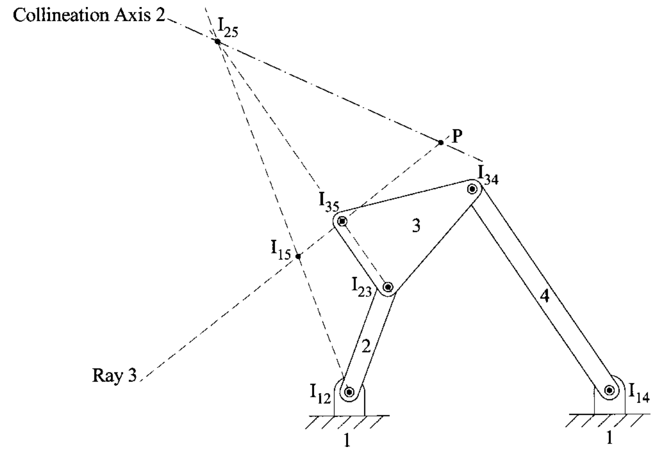

Fig. 4 Instant centers I_{25} and I_{15}

agreement with the Aronhold-Kennedy theorem for the five-bar linkage formed by links 1, 2, 3, 4, and the virtual link 5 which has an instantaneous single degree-of-freedom. Also note that due to Bobillier's theorem, the angle between collineation axis 1 and ray 1 is equal to the angle between collineation axis 3 and ray 3. Therefore, it can be shown that the angle between collineation axes 2 and 3 is equal to the angle between rays 1 and 2 (which is the difference in the orientation of side links 2 and 4 relative to

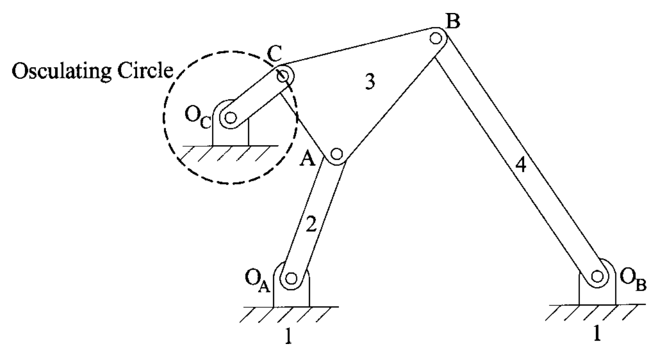

Fig. 5 Center of curvature of the trajectory of point C

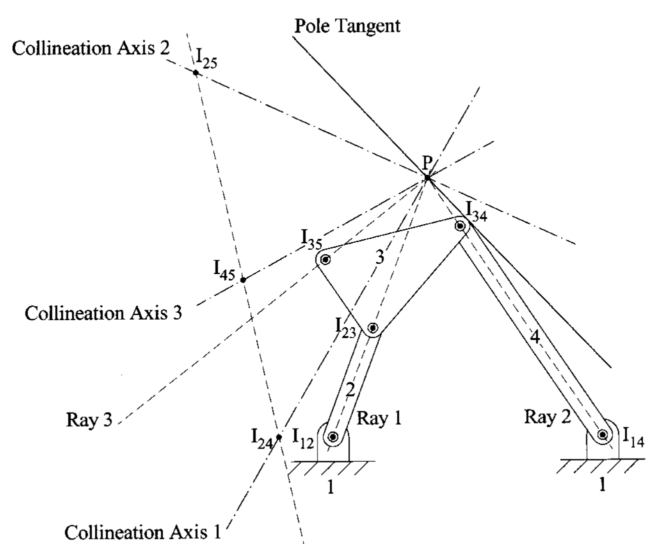

Fig. 6 Collineation axis 3

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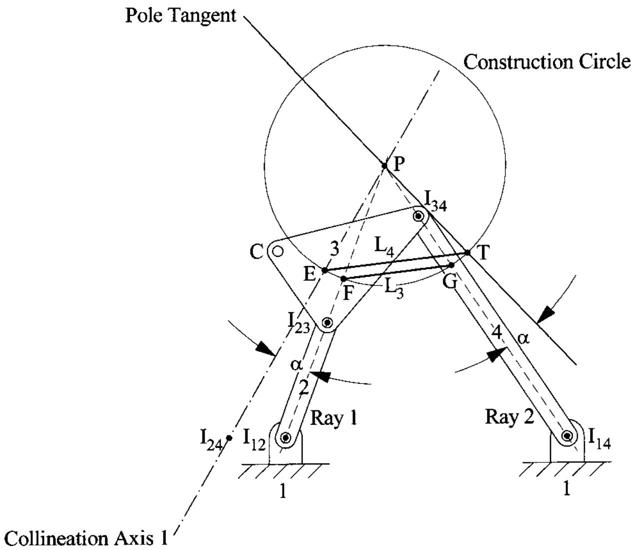

Fig. 7 Purely graphical technique to locate the pole tangent

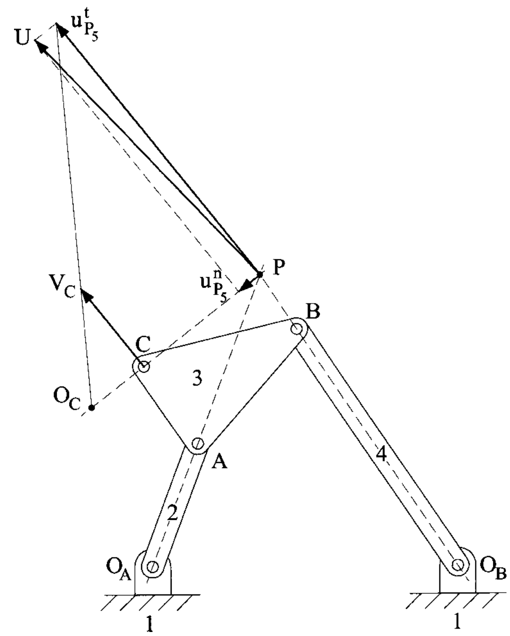

Fig. 8 Center of curvature of the path trajectory point C

ground link O_AO_B; i.e., $\theta_4 - \theta_2$). Similarly, it can be shown that the angle between collineation axes 1 and 2 is equal to the angle between rays 2 and 3.

Summary of the Graphical Technique

1. Locate pole P and instant center I_{24} and draw collineation axis 1 and ray 3; i.e., the line through P and the coupler point C. (See Fig. 2).
2. Draw the construction circle. Then draw the lines L_1 and L_2 and collineation axis 2. (See Fig. 3(*a*)).
3. Locate the instant center I_{25} and then locate the instant center I_{15}. (See Fig. 4). The center of curvature of the path traced by the coupler point C is coincident with instant center I_{15}.

Pole Tangent. Note that the graphical procedure presented in this paper, to obtain the center of curvature of the coupler point trajectory, does not require the pole tangent. The pole tangent defines the direction of the velocity of the pole. For the sake of completeness, a purely graphical technique to locate the pole tangent will be presented here. Draw a construction circle; i.e., a circle of arbitrary radius and center coincident with the pole. For convenience, the construction circle shown in Fig. 3(*a*) will be used again here, see Fig. 7. Then draw a line, denoted as L_3, connecting points F and G and draw a line, denoted as L_4, parallel to L_3 through point E. The point of intersection of line L_4 with the construction circle is denoted as point T. Finally, draw a line connecting point T and the pole P. The result of this geometrical construction is that line PT is the pole tangent, and this can be proved as follows.

Since the angle at the center of a circle is twice the angle at the circumference of the circle, when subtended by the same arc, then

$$\angle EPF = 2\angle ETF \quad \text{and} \quad \angle GPT = 2\angle GFT \tag{4a}$$

Also, since lines L_3 and L_4 are parallel then

$$\angle ETF = \angle GFT \tag{4b}$$

The conclusion from Eqs. (4*a*) and (4*b*) is that

$$\angle EPF = \angle GPT \tag{5}$$

which states that the angle between collineation axis 1 and ray 1 is equal to the angle between ray 2 and line PT. Recall Bobillier's theorem states that the angle between collineation axis 1 and first ray 1 is equal to the angle between ray 2 and the pole tangent. Therefore, the line that connects pole P and point T is indeed coincident with the pole tangent. Recall that the pole, the pole tangent, and the pole normal (which indicates the direction of the pole acceleration), define the canonical system for the coupler link [15].

It is interesting to note that the center of curvature of the trajectory traced by point C can be obtained directly from the pole velocity, denoted as U, see Fig. 8. The procedure is to project the pole velocity unto the line that is drawn through the pole and parallel to the path tangent of point C. This vector, denoted as $u^t_{P_5}$, represents the velocity of a point fixed in the virtual link 5 and coincident with pole P. Next draw a line through the tip of this vector and the tip of the velocity vector of point C, denoted as V_C. The intersection of this line with the line passing through the pole and point C (i.e., the path normal) is the center of curvature of the trajectory traced by the coupler point.

The following section will show that the results of the purely graphical concepts, presented in this section, are in complete agreement with the traditional approach which requires the inflection circle for the coupler link and the Euler-Savary equation [3,8,10]. The reader can compare the two approaches and note that neither inflection points of the coupler link nor the pole tangent are required in the graphical technique proposed in this paper.

3 Numerical Example

Consider the four-bar linkage O_AABO_B, shown in Fig. 9, with the link dimensions $O_AO_B = 40$ cm, $O_AA = 17.5$ cm, $AB = 20$ cm, and $O_BB = 38$ cm (consistent with the dimensions of Figs. 1 through 7). Note that linear dimensions can be presented without units because they can be scaled uniformly to any convenient scale [12]. For convenience, the metric length of centimeters will be used in this section. The location of coupler point C is specified by the distance $AC = 12.5$ cm and the angle $\angle BAC = 75$ deg. For convenience, the design position is specified by the angle $\theta_2 = 70$ deg.

Following the purely graphical technique presented in Sec. II, the location of the center of curvature of the path traced by point C (in the design position) is as shown in Fig. 9. The important observation is that no measurements are required in finding this result. If the radius of curvature of the path traced by point C (i.e., the radius of the osculating circle) is required then the distance

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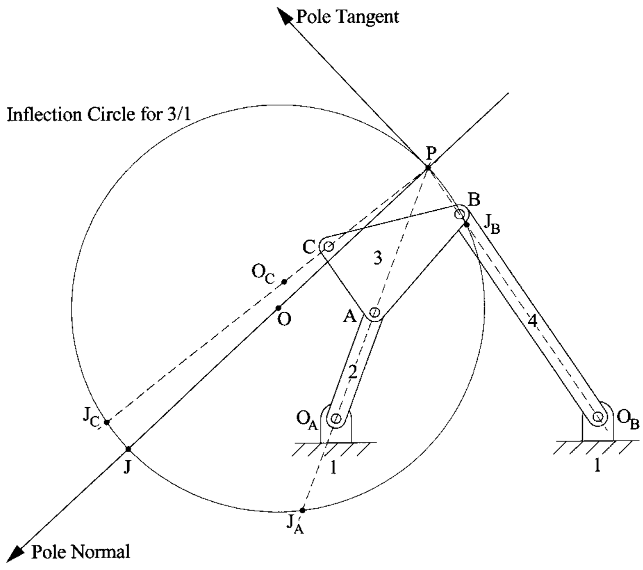

Fig. 9 Inflection circle and center of curvature of trajectory of point C

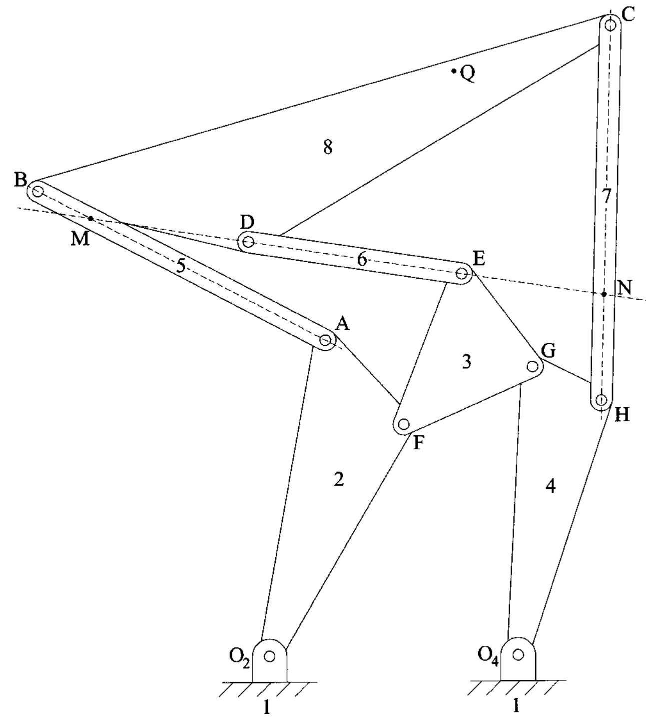

Fig. 10 Single flier eight-bar linkage

O_CC can be measured from the scaled drawing. For this example, the radius of curvature of the point trajectory is measured as $O_CC=8.74$ cm.

The Euler-Savary equation states that the radius of curvature of the path traced by the coupler point C can be written as

$$O_CC=\frac{(PC)^2}{J_CC} \tag{6}$$

where J_C is an inflection point of the coupler link, see Fig. 9. To locate this inflection point it is standard practice to first draw the inflection circle for the coupler link. Recall that the inflection circle is defined as the locus of points in the coupler link that are instantaneously traveling on straight lines or the locus of points in the coupler link that have no normal acceleration; i.e., inflection points. The inflection circle can be drawn by locating the inflection points J_A and J_B that correspond to points A and B, respectively, in the coupler link. These two inflection points are obtained from the Euler-Savary equation; i.e.,

$$J_AA=\frac{(PA)^2}{O_AA} \quad \text{and} \quad J_BB=\frac{(PB)^2}{O_BB} \tag{7}$$

From a scaled drawing of the linkage, the measured distances are PA=23.90 cm and PB=8.70 cm. Substituting these values, and the specified link dimensions, into Eqs. (7) gives $J_AA=32.64$ cm and $J_BB=1.99$ cm. Since pole P and the inflection points J_A and J_B lie on the circumference of the inflection circle then the circle can be drawn, see Fig. 9. The center of the inflection circle is denoted as O and the inflection pole is denoted as J. The diameter of the inflection circle is measured as PJ=63.27 cm. The pole tangent and pole normal are also shown in the figure. The pole tangent is inclined at 133.33 deg to the line passing through the ground pins O_AO_B.

The intersection of the line connecting pole P to coupler point C with the inflection circle gives the inflection point J_C. The distances from P to C and from J_C to C are measured and the results are PC=19.51 cm and $J_CC=43.54$ cm, respectively. These measurements are then substituted into Eq. (6) to give the location of the center of curvature of the path traced by point C on the path normal; i.e., $O_CC=8.74$ cm. The distance from the pole to the center of curvature is $PO_C=PC+CO_C=28.25$ cm. Alternatively, the Euler-Savary equation can be written as

$$\frac{1}{PO_C}=\frac{1}{PC}-\frac{1}{PJ_C} \tag{8}$$

Substituting the measured values into this equation gives the same result; namely, $PO_C=28.25$ cm. Therefore, the analytical results obtained in this section are in complete agreement with the purely graphical technique that is presented in Sec. 2.

4 Concluding Remarks

This paper presents a graphical technique to locate the center of curvature of the path traced by an arbitrary point fixed in the coupler link of a planar four-bar linkage. The feature of this technique is that a virtual link is used to connect the coupler point to the ground link. The virtual link is valid up to, and including, the second-order properties of motion of the coupler link. The virtual link is coincident with the path normal of the coupler point and the length of the link is the distance between the coupler point and the center of curvature of the path traced by this point. The technique presented in this paper is purely graphical; i.e., no analytical equations or measurements are required. Another advantage is that the technique requires few geometric constructions and it is not necessary to draw the inflection circle for the coupler link.

A previous paper [12] investigated the curvature of the path traced by coupler point Q of the single flier eight-bar linkage shown in Fig. 10. The graphical technique used the equivalent four-bar linkage defined by ground link 1 (O_NO_M), virtual link 15 (O_MM), coupler link 8 (MQN), and virtual link 18 (NO_N) as shown in Fig. 11(*a*). The coupler pins M and N are defined as the points of intersection of link 6 with links 5 and 7 (or the links extended), respectively. The paper showed that the centers of curvature O_M and O_N can be obtained in a straightforward manner by using the concept of virtual links and equivalent four-bar linkages. Finally, the location of the center of curvature of the trajectory of point Q was obtained from the pole P_{18}, the inflection circle (specified by inflection points J_M and J_N) and the Euler-Savary equation, see Fig. 11(*b*). This paper, however, now makes it possible to perform this final step without finding inflection points, drawing the inflection circle, using analytical equations, or mak-

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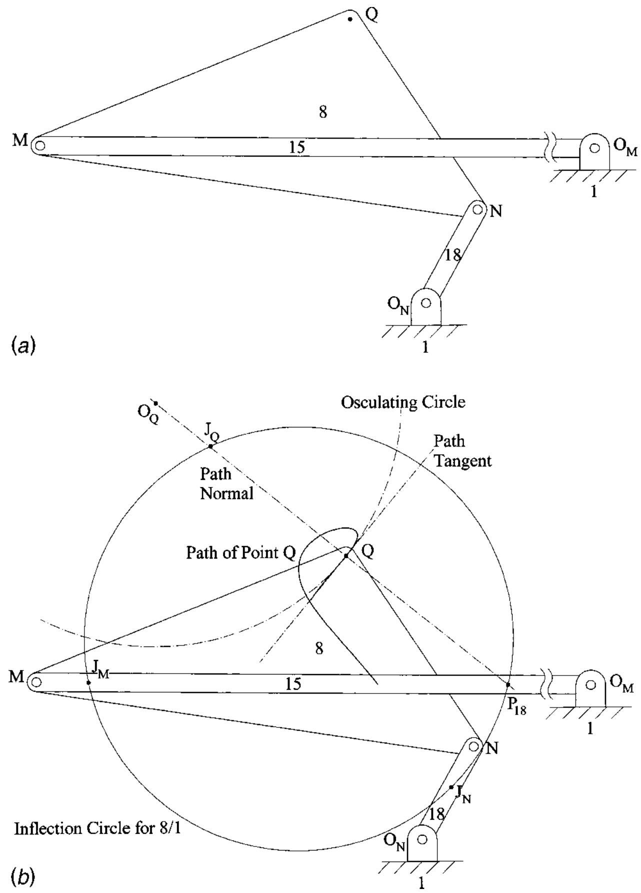

Fig. 11 (*a*) Equivalent four-bar linkage for the coupler link, (*b*) inflection circle and osculating circle for the coupler link

ing measurements of lengths and angles. For purposes of illustration, the figure shows the path of point Q over a considerable distance away from the design position.

The authors are of the opinion that the graphical ideas proposed in this paper will prove especially helpful when used in conjunction with parametric computer aided design software such as PRO/ENGINEER, SOLIDWORKS, and SOLID EDGE. These commercially available packages have powerful integrated numeric solvers that function behind easy to use and well-developed interfaces. The engineer can quickly and easily create kinematic figures and determine the properties of a linkage (for example, the locations of instant centers and the radius of curvature of a point trajectory) using geometric constructions. These figures can be created to make use of the parametric solvers so that they will be updated as the design is iterated as part of the design process. This technique is an alternative to writing analytical code and will provide the engineer with geometric insight without having to formulate nonlinear equations. The proliferation of the computer aided design packages makes this a viable educational and industrial tool.

The graphical techniques, developed in this paper, will be used in a future research activity to create a graphical program that uses parametric constraints. The goal is to make the program be easily adapted to investigate the path curvature of a coupler point trajectory of any planar, single-degree-of-freedom linkage. The research will extend the graphical kinematic computer programming of path curvature to the kinematic synthesis of the planar four-bar linkage and the Stephenson and the Watt six-bar linkages. The authors also believe that the work presented in this paper can be extended to include the higher-order properties of motion of the coupler link. For example, the concept of a virtual link should provide important insight into the third-order properties of motion of a coupler link and afford a purely graphical technique to draw the well-known cubic of stationary curvature [6,9].

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