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Singularity Analysis for a 5-DoF Fully-Symmetrical Parallel Manipulator 5-RRR(RR)

Si-Jun Zhu, Zhen Huang, Ming-Yang Zhao

***Abstract*—A 5-DoF *3R2T* (three dimensional rotation and two dimensional translation degrees of freedom) fully-symmetrical parallel manipulator can be adopted in many applications such as simulating the motion of spinal column. However, kinematics of this type parallel manipulator has not been studied enough because of short history. The study of kinematics of the manipulators leads inevitably to the problem of singular configuration. Singularity of a 5-DoF *3R2T* fully-symmetrical parallel manipulator, 5-RRR(RR), is illustrated in this study. According to the singularity classification by Fang and Tsai, both limb singularity and actuation singularity are illustrated by screw theory and Grassmann geometry. The result of this study will be helpful for singularity analysis of 5-DoF *3R2T* fully-symmetrical parallel manipulators because of their similar constraint property.**

Keywords: singularity analysis, 5-DoF, fully-symmetrical, parallel manipulator

I. Introduction

THE history for 5-DoF fully-symmetrical[1] parallel manipulator is not very long[2], thereby kinematics of these manipulators have not been studied enough. Most of existent fully-symmetrical parallel manipulators are *3R2T* type[3;4], namely the end-effector has three dimensional rotation and two dimensional independent translation motions. Such a mechanism can be used to simulate the motion of a spinal column. The translation along the axis of spinal column for the top relative to the bottom is very small and consequently can be ignored. So the top of the spinal column has three rotational and two translation freedoms relative to the bottom.

The study of the kinematics of the mechanisms leads inevitably to the problem of singular configuration. The singular configuration for a parallel manipulator is more complex than a serial one. Sometimes, the end-effector of parallel manipulator may lose one or more DoF similar to the case of a serial manipulator. In the other cases, one or more DoF of end-effector of a parallel manipulator may be uncontrollable.

In recent twenty years, many researchers had studied the singularity of parallel manipulators. Different methods for singularity classification are proposed:

With Grassmann geometry, Merlet[5] studied singularity configuration of Gough-Stewart manipulator and defined three types of singularities based on the constrained motions.
Huang *et al.*[6] proposed a geometry sufficiency and necessary condition to analyze singularity configurations of Stewart manipulator.

Gosselin and Angeles[7] studied the general singularity for closed-loop kinematic chains. Three types of singularities are defined based on the property of Jacobian matrices of the chain. Ma and Angeles[8] pointed out that the third type singularity in ref. [7] is a subset of architecture singularity. Tsai named the singularity classification in ref. [7] as inverse kinematic, direct kinematic, and combined singularity respectively[9].

Zlatanov *et al.* pointed out that the approach in ref. [7] may fail to detect certain singularities in the general closed-loop case[10]. They also proposed six fundamental singularity types (RI, RO, II, IO, RPM, IIM) and 21 combination cases of six ones above. After that, Bonev *et al.*[11] and Merlet[12] discussed the puzzling singularity of translational parallel robot prototype built at Seoul National University, respectively.

Park and Kim[13] studied the singularity of parallel manipulators with differential geometry in a Euclidean space for the first time.

Fang and Tsai[14] identified the singularity configuration of parallel manipulator with three types: limb singularity, platform singularity and actuation singularity.

In this paper, singularity of a fully-symmetrical 5-DoF *3R2T* 5-RRR(RR) parallel manipulator is studied. Both screw theory[15,16], Grassmann Geometry and singularity classification defined by Fang and Tsai[14] are adopted in the singularity analysis since they are convenient for comprehension of singular configuration. The result of this study will be helpful for singularity analysis of 5-DoF *3R2T* fully-symmetrical parallel manipulators because of their similar constraint relationship.

The paper was arranged as follows, at first, screw theory are recalled briefly. Second, the mobility of this manipulator is analyzed with screw theory. Third, according to singularity classification by Fang and Tsai[14], both limb singularity and

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actuation singularity for the mechanism 5-RRR(RR) are illustrated. Moreover, methods to avoid actuation singularities are also presented.

II. BASIC SCREW THEORY

The singularity analysis in this study is mainly based on the screw theory. Hereby, some basic screw concepts are recalled here briefly.

In the screw theory[15;16], a unit screw is given by a dual vector.

$$\boldsymbol{\$}=[\boldsymbol{S}^T;\ \boldsymbol{S_0}^T]=[\boldsymbol{S}^T;\ (\boldsymbol{r}\times\boldsymbol{S}+h\boldsymbol{S})^T\ ]^T=[\ l\ m\ n\ ;\ p\ q\ r\ ]^T \quad (1)$$

where $\boldsymbol{\$}$ denotes a unit screw; $\boldsymbol{S}=[l, m, n]$ the unit vector along the screw axis; $\boldsymbol{S_0}=\boldsymbol{r}\times\boldsymbol{S}+h\boldsymbol{S}=[p, q, r]$ the dual part of the screw; $\boldsymbol{r}$ the position vector of any point on the screw axis; h the pitch of the screw.

Kinematical screw for a revolute joint is a linear vector whose h=0,

$$\boldsymbol{\$}_{revolute}=[\boldsymbol{S}^T;\ \boldsymbol{S_0}^T]^T=[\boldsymbol{S}\ ;\ \boldsymbol{r}\times\boldsymbol{S}]^T \quad (2)$$

where $\boldsymbol{\$}_{revolute}$ denotes a kinematic screw for a revolute joint.

Kinematic screw for a prismatic pair is a couple vector whose $h=\infty$,

$$\boldsymbol{\$}_{prismatic}=[\boldsymbol{0}^T;\ \boldsymbol{S}^T]^T \quad (3)$$

where $\boldsymbol{\$}_{prismatic}$ denotes a kinematic screw for a prismatic pair; $\boldsymbol{0}$ is a 3×1 zero-vector.

Two screws ($\boldsymbol{\$}_A$ and $\boldsymbol{\$}_B$) are reciprocal to each other if they satisfy[15;16]

$$\boldsymbol{S_A}\cdot\boldsymbol{S_{0B}}+\boldsymbol{S_{0A}}\cdot\boldsymbol{S_B}$$
$$=(l_A\cdot p_B+m_A\cdot q_B+n_A\cdot r_B)+(p_A\cdot l_B+q_A\cdot m_B+r_A\cdot n_B)=0 \quad (4)$$

where $\boldsymbol{\$}_A=[\boldsymbol{S_A}^T;\boldsymbol{S_{0A}}^T]=[l_A\ m_A\ n_A\ ;\ p_A\ q_A\ r_A]^T$, $\boldsymbol{\$}_B=[\boldsymbol{S_B}^T;\boldsymbol{S_{0B}}^T]^T=[l_B\ m_B\ n_B\ ;\ p_B\ q_B\ r_B\ ]^T$.

According to Eq. (4), two coplanar linear vectors $\boldsymbol{\$}_A$ and $\boldsymbol{\$}_B$ are reciprocal to each other.

III. MANIPULATOR DESCRIPTION AND MOTION ANALYSIS

As shown in Fig. 1, a movable platform (end-effector) and base are connected by five identical limbs each with five joints R_i, i=1,2,3,4,5. Axes of R_1, R_2, R_3 are perpendicular to the base plane and the parallel structure is denoted with underline RRR. The other two joints, R_4 and R_5, intersect at a common point called rotation center. The intersection structure is denoted with parentheses (RR). Five R_1 are chosen as actuators.

The kinematical screw system[15;16] for one RRR(RR) limb consists of five joint screws ($\boldsymbol{\$}_i$, i=1,2,3,4,5) corresponding to five joints, respectively. Since the rank of kinematical screw system is independent to the reference frame, we here assume Z-axis of the reference frame parallel the axis of R_1, origin O locate at the rotation center for convenience, as shown in Fig. 2. Then, the kinematical screw system for one limb is

$$\boldsymbol{\$}_1=[0, 0, 1; y_1, -x_1, 0]^T$$
$$\boldsymbol{\$}_2=[0, 0, 1; y_2, -x_2, 0]^T$$
$$\boldsymbol{\$}_3=[0, 0, 1; y_3, -x_3, 0]^T$$
$$\boldsymbol{\$}_4=[l_4, m_4, n_4; 0, 0, 0]^T$$
$$\boldsymbol{\$}_5=[l_5, m_5, n_5; 0, 0, 0]^T \quad (5)$$

where $[x_1, y_1, 0]$, $[x_2, y_2, 0]$ and $[x_3, y_3, 0]$ are the coordinate of intersection points of axes of R_1, R_2, R_3 with O-XY plane, respectively; $[l_4, m_4, n_4]$, $[l_5, m_5, n_5]$ denotes the direction cosine for axes of R_4, R_5.

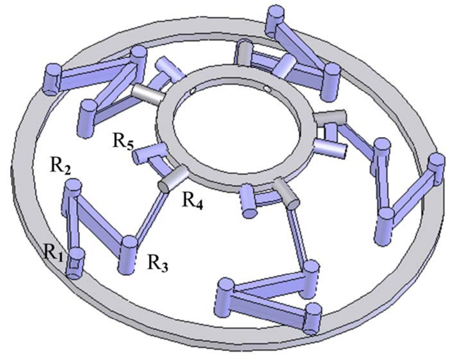

Fig. 1. Sketch of 5-RRR(RR)

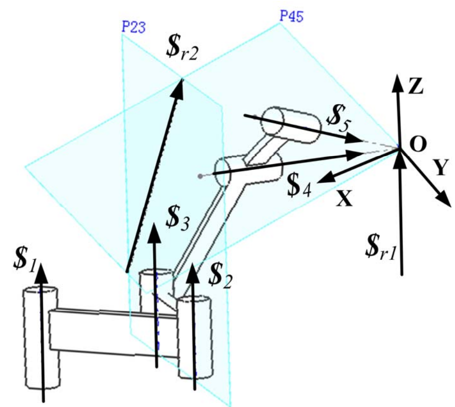

Fig. 2. Twists and wrenches of a RRR(RR) limb

According to the screw theory[15;16], the constraint wrench of the limb is a screw which is reciprocal to the kinematical screw system in Eq.(5), namely

$$\boldsymbol{\$}_{r1}=[0, 0, 1; 0, 0, 0]^T \quad (6)$$

where $\boldsymbol{\$}_{r1}$ denotes a constraint force along Z-axis.

Similarly, $\boldsymbol{\$}_{r1}$ of five limbs are the same, and they form a common constraint force which constrains the translation of the end along Z-axis. As a result, the movable platform has three dimensional rotation freedoms and two dimensional translational freedoms which parallel to O-XY plane.

In another way, according to the modified Kutzbach-Grüble formula[17]

$$M=d(n-g-1)+\sum_{i=1}^{g} f_i+v-\varsigma \quad (7)$$

where M denotes the number of DoF; $d=6-\lambda$, λ denotes the number of the common constraint; n the number of links; g the number of pairs; f_i the freedoms of the i^{th} pair; v the number of redundant constraints not included in common constraint and ς denotes the local freedom. For the manipulator 5-RRR(RR), λ=1, n=22, g=25, $\sum_{i=1}^{g} f_i=25$, v=0, ς=0, so M=5, namely the movable platform of this manipulator has five DoF.

IV. SINGULARITY ANALYSIS

In this study, the singularity of the manipulator will be discussed according to the classification defined by Fang and Tsai[14]. They classified the singularities for lower-mobility parallel manipulators into three types: limb singularity, platform singularity and actuation singularity.

(a) Limb singularity is similar to the singularity of a serial manipulator. It occurs when the limb kinematical screw system degenerates. In such a condition, the joint screws in the kinematical screw system become linearly dependent and the number of independent wrenches reciprocal to the kinematical screw system increases accordingly. As a result, the movable platform loses one or more DoF.

(b) Platform singularity occurs when the overall wrench system of the movable platform (which consists of wrenches from all limbs) degenerates. As a result, one or more DoF of movable platform are uncontrollable. For a 5-DoF fully-symmetrical parallel manipulator, wrenches from all limbs are always the same. Therefore, platform singularity is unlike to occur for a 5-DoF fully-symmetrical parallel manipulator.

(c) Actuation singularity occurs if the movable platform still possesses certain DoF after locking all actuators. Its another well-known name is self-motion[18]. In this case, the rank of wrench system is less than six after locking actuators. As a result, actuators can not fully control the movable platform in this configuration.

Moreover, both limb singularity and platform singularity are independent of the input selection (the selection of actuated joints). They should be avoided in trajectory plan stage. However, different with them, actuation singularity depends on the input selection. It may be avoided by adjusting assembly configuration or changing actuated joints.

Since the platform singularity for 5-RRR(RR) is unlikely occur, only limb singularity and actuation singularity are discussed in the following sections.

A. Limb singularity

Assume the coordinate for the intersection of O-XY plane and axis of i^{th} kinematic pair is

$$\boldsymbol{C}_i = [x_i, y_i, 0] \tag{8}$$

Then the limb screw system is

$$\boldsymbol{\$}_{limb} = \begin{bmatrix} \boldsymbol{\$}_1 \\ \boldsymbol{\$}_2 \\ \boldsymbol{\$}_3 \\ \boldsymbol{\$}_4 \\ \boldsymbol{\$}_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & y_1 & -x_1 & 0 \\ 0 & 0 & 1 & y_2 & -x_2 & 0 \\ 0 & 0 & 1 & y_3 & -x_3 & 0 \\ l_4 & m_4 & n_4 & 0 & 0 & 0 \\ l_5 & m_5 & n_5 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

where $\boldsymbol{\$}_{limb}$ denotes kinematical screw system for a limb; l, m, n is the direction cosines of the screw. Since the last column is a zero column, the rank of the screw system depends on the former five columns. Determination of the former five columns, $\left|\boldsymbol{\$}_{limb(:,1:5)}\right|$, is

$$\begin{aligned} &\left|\boldsymbol{\$}_{limb(:,1:5)}\right| \\ &= (l_4 m_5 - l_5 m_4)[(x_1 y_2 - x_2 y_1) + \\ &(x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)] \\ &= (l_4 m_5 - l_5 m_4)[(x_1 y_2 - x_2 y_1) + \\ &(x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3) + (x_2 y_2 - x_2 y_2)] \\ &= (l_4 m_5 - l_5 m_4)[(x_2 - x_1)(y_3 - y_2) - (x_3 - x_2)(y_2 - y_1)] \end{aligned} \tag{10}$$

Assume

$$\boldsymbol{N}_{45} = \boldsymbol{S}_4 \times \boldsymbol{S}_5 = [N_{45x}, N_{45y}, N_{45z}] \tag{11}$$

Then Eq.(10) can be rewritten as

$$\begin{aligned} &\left|\boldsymbol{\$}_{limb(:,1:5)}\right| \\ &= N_{45z}[(x_2 - x_1)(y_3 - y_2) - (x_3 - x_2)(y_2 - y_1)] \end{aligned} \tag{12}$$

The sufficient and necessary condition for the limb singularity of RRR(RR) is Eq.(12) equals zero. According to Eq.(12), there are two special singular cases for a RRR(RR) limb listed in Table I.

TABLE I
GEOMETRY CONDITIONS FOR LIMB SINGULARITY OF 5-RRR(RR)

Singular Cases	Geometry Conditions
$\frac{(y_3 - y_2)}{(x_3 - x_2)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$	Axes of R_1, R_2 and R_3 are coplanar
$N_{45z} = 0$	Axis of R1 parallels to $\mathbf{P}_{45}$

where $\mathbf{P}_{ij}$ denote the plane determined by axes of R_i and R_j,

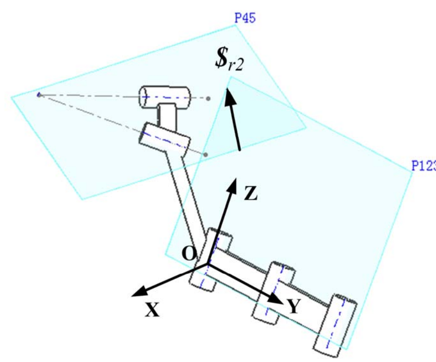

Fig. 3. The first case of limb singularity

1) First case: As shown in Fig.3, axes of R_1, R_2, R_3 are coplanar, where P_{123} is the plane determined by the axes of R_1, R_2, R_3; P_{45} is the plane determined by axes of R_4, R_5.

Assume O-YZ plane of the reference frame be the same with plane P_{123}, Z-axis be along the axis of R_3 for convenience. Thus, the kinematical screw system for a limb is

$$\begin{aligned} &\boldsymbol{\$}_1 = [0, 0, 1; y_1, 0, 0]^T \\ &\boldsymbol{\$}_2 = [0, 0, 1; y_2, 0, 0]^T \\ &\boldsymbol{\$}_3 = [0, 0, 1; 0, 0, 0]^T \\ &\boldsymbol{\$}_4 = [l_4, m_4, n_4; (x_0, y_0, z_0) \times (l_4, m_4, n_4)]^T \\ &\boldsymbol{\$}_5 = [l_5, m_5, n_5; (x_0, y_0, z_0) \times (l_5, m_5, n_5)]^T \end{aligned} \tag{13}$$

where $[x_0, y_0, z_0]$ is the coordinate of the rotation center.

In this case, $\$_1$, $\$_2$, $\$_3$ are dependent and the rank of kinematical screw system is four. There are two independent wrenches reciprocal to the kinematical screw system,

$$\$_{r1}=[0, 0, 1; y_0, -x_0, 0]^T \tag{14}$$

$$\$_{r2}=\frac{1}{\sqrt{m_{r2}^2+1}}[0, m_{r2}, 1; p_{r2}, 0, 0]^T \tag{15}$$

where $p_{r2}=(m_4l_5-l_4m_5)/(l_5n_4-l_4n_5)$;
$m_{r2}=(-z_0l_4m_5+l_5m_4z_0+y_0l_4n_5-l_5n_4y_0+m_4l_5-l_4m_5)/(n_5m_4-m_5n_4)$.

$\$_{r2}$ in Eq. (15) is a constraint force along the intersection line of planes P_{123} and P_{45}. Under the two constraint forces, $\$_1$ and $\$_2$, the end of the limb can not translate along their axes or any axis intersects with them, except the common vertical line of two axes. Furthermore, rotation of the movable platform is also limited[19]. It can only rotate around the axis intersects with both axes.

By the way, a special case may occur if the lengths of links connected R_1 with R_2 and R_2 with R_3 are equal, R_1, R_3 are coaxial. Then, the limb will achieve a local rotational freedom, and infinite input will lead zero end-output. This case is the third type singularity in Ref. [7].

For the manipulator 5-<u>RRR</u>(RR), the first case of limb singularity may occurs simultaneously in three limbs. Then, four constraint force acts on the movable platform. The movable platform consequently has two independent DoF instead of five.

2) Second case: As shown in Fig.4, the plane P_{45} is perpendicular to the base plane.

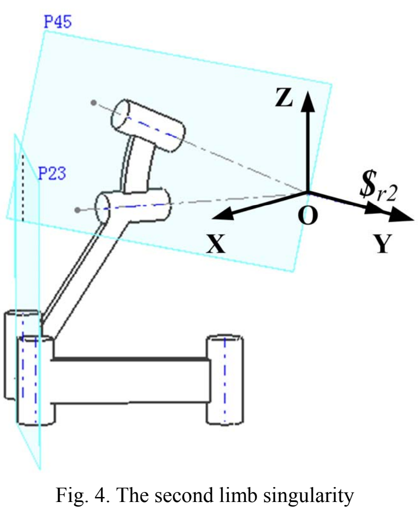

Fig. 4. The second limb singularity

Assume O-XZ plane be the same with plane P_{45}, Z-axis parallel to the axis of R_1, origin O locate at the rotation center. Thus, the kinematical screw system will be

$$\$_1=[0, 0, 1; y_1, -x_1, 0]^T$$
$$\$_2=[0, 0, 1; y_2, -x_2, 0]^T$$
$$\$_3=[0, 0, 1; y_3, -x_3, 0]^T$$
$$\$_4=[l_4, 0, n_4; 0, 0, 0]^T$$
$$\$_5=[l_5, 0, n_5; 0, 0, 0]^T \tag{16}$$

where $[x_1, y_1, 0]$, $[x_2, y_2, 0]$ and $[x_3, y_3, 0]$ are the intersection points of axes of R_1, R_2, R_3 with O-XY plane, respectively; $[l_4, 0, n_4]$, $[l_5, 0, n_5]$ denotes the direction cosine of axes of R_4, R_5.

Then, there are two independent constraint wrenches

$$\$_{r1}=[0, 0, 1; 0, 0, 0]^T \tag{17}$$
$$\$_{r2}= [0, 0, 0; 0, 1, 0]^T \tag{18}$$

where $\$_{r2}$ in Eq. (18) is a constraint couple which constrains the rotation around Y-axis.

Such a singularity configuration can occurs only in one limb simultaneously. In this case, the end of the limb can not translate along Z-axis or rotate around any axis parallels to Y-axis.

B. Actuation singularity

When the actuator fixed on the joint R_1 is locked, the kinematical screw system under the frame in Fig. 2 is

$$\$_2=[0, 0, 1; y_2, -x_2, 0]^T$$
$$\$_3=[0, 0, 1; y_3, -x_3, 0]^T$$
$$\$_4=[l_4, 0, n_4; 0, 0, 0]^T$$
$$\$_5=[l_5, 0, n_5; 0, 0, 0]^T \tag{19}$$

There are two independent wrenches, $\$_{r1}$ and $\$_{r2}$, at general configuration for a limb $R_2R_3R_4R_5$. The axis of $\$_{r2}$ for a limb is the intersection line of planes P_{23} and P_{45}, shown in Fig. 2.

In another way, according to screw theory[15;16],

$$\$_{r2}=[l_{r2}, m_{r2}, n_{r2}, p_{r2}, q_{r2}, r_{r2}]^T \tag{20}$$

where $l_{r2}= (1/\lambda)\ (x_3-x_2)/(x_2y_3- x_3y_2)$;
$m_{r2}= (1/\lambda)\ (y_3-y_2)/(x_2y_3- x_3y_2)$;
$n_{r2}= 0$;
$p_{r2}= (1/\lambda)\ (m_4n_5-m_5n_4)/(l_4m_5-l_5m_4)$;
$q_{r2}= (1/\lambda)\ (l_5n_4-l_4n_5)/(l_4m_5-l_5m_4)$;
$r_{r2}= (1/\lambda)$;
$\lambda=\sqrt{l_{r2}^2+m_{r2}^2}$;

Assume the unit direction of normal vector for plane P_{45} is $\boldsymbol{n}_{45}=[n_{45x},n_{45y},n_{45z}] = [m_4n_5-m_5n_4,\ l_5n_4-l_4n_5,\ l_4m_5-l_5m_4]$, then

$$p_{r2}= (1/\lambda)\ n_{45x} / n_{45z} \tag{21}$$
$$q_{r2}= (1/\lambda)\ n_{45y} / n_{45z} \tag{22}$$
$$\$_{r2}=(1/(\lambda n_{45z}))[n_{45z}l_{r2},n_{45z}m_{r2},n_{45z}n_{r2},\ n_{45x},n_{45y},n_{45z}]^T \tag{23}$$

Thus, wrench system $\$_r$ of the movable platform consists of six wrenches, $\$_{r2}$ of five limbs and $\$_{r1}$,

$$\$_r=[\$_{r1},\$_{r2}^{(1)},\$_{r2}^{(2)},\$_{r2}^{(3)},\$_{r2}^{(4)},\$_{r2}^{(5)}]^T \tag{24}$$

where $\$_{r2}^{(i)}$ denotes the $\$_{r2}$ of the i^{th} limb, i=1,2,3,4,5.

At general configuration, the rank of the wrench system in Eq.(18) is usually six and hence the movable platform can be controlled by five actuators. However, if wrench system degenerates at some special configuration, the actuation singularity occurs. There are three cases of actuation singularities for the manipulator 5-<u>RRR</u>(RR).

1) First case: As shown in Fig.5, rotation center locates in P_{23} of five limbs. Let origin O of reference frame locate at the rotation center, and Z-axis be perpendicular to the base plane.

In this case, rotation center is on the intersection line of P_{23} and P_{45} since it is always in P_{45}, namely the axis of $\$_{r2}$. Moreover, $\$_{r1}$ also passes through the rotation center. Then six constraint wrenches shown in Eq. (24) intersect at the rotation center. According to the Grassmann geometry[5], there are only three linear independence vectors in the spatial intersection case.

Based on screw theory, from Eq.(19), $x_2y_3- x_3y_2=0$. So,

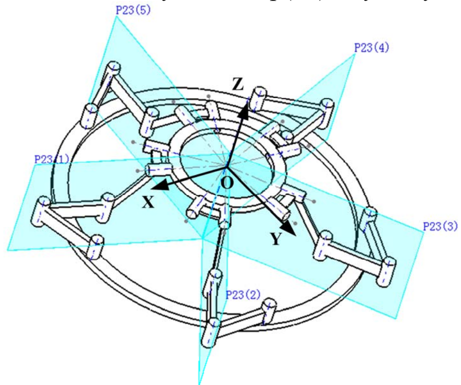

Fig. 5. The first case of actuation singularity

$$\$_{r2}^{(i)} = [l_{r2}^i, m_{r2}^i, n_{r2}^i, 0,0,0]^{\mathrm{T}} \tag{25}$$

namely,

$$\$_r = \begin{bmatrix} 0 & l_{r2}^1 & l_{r2}^2 & l_{r2}^3 & l_{r2}^4 & l_{r2}^5 \\ 0 & m_{r2}^1 & m_{r2}^2 & m_{r2}^3 & m_{r2}^4 & m_{r2}^5 \\ 1 & n_{r2}^1 & n_{r2}^2 & n_{r2}^3 & n_{r2}^4 & n_{r2}^5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

where $[l_{r2}^i, m_{r2}^i, n_{r2}^i]$ denotes the direction cosine of axis $\$_{r2}^{(i)}$, i=1,2,3,4,5. Obviously, rank of wrench system, $\$_r$ is three.

In this case, the movable platform can still rotate about the rotation center even after locking five actuators. In other words, there are three uncontrollable rotation DoF.

This actuation singularity can be passed by choosing different joints as actuators. For example, six wrenches in Eq.(24) are not dependent if choosing three R_1 and two R_2 as actuators.

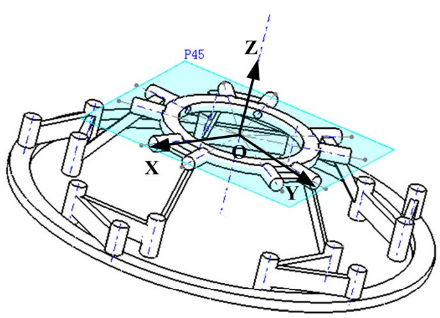

Fig. 6. The second case of actuation singularity

2) Second case: As shown in Fig.6, the plane P_{45} of five limbs parallel to the base plane. Let the reference frame be the same with Fig. 5.

In this case, P_{45} of five limbs will be the same plane. And $\$_{r2}$ of five limbs will be also in the plane. According to Grassmann geometry[5], the rank of coplanar linear vectors (five $\$_{r2}$) is three. Thus, the number of linear independent constraint wrenches (five $\$_{r2}$ and $\$_{r1}$) is four.

According to screw theory, from Eq.(23), the last three entries of the $\$_{r2}$ are the direction cosine of the normal vector for P_{45}. When the movable platform parallel to the base plane, the five P_{45} planes are the same. Then, their normal vectors parallel to each other, namely

$$\$_{r2}^{(i)} = (1/\lambda)[l_{r2}^i, m_{r2}^i, n_{r2}^i, 0,0,1]^{\mathrm{T}} \tag{27}$$

Then

$$\$_r = (1/\lambda)\begin{bmatrix} 0 & l_{r2}^1 & l_{r2}^2 & l_{r2}^3 & l_{r2}^4 & l_{r2}^5 \\ 0 & m_{r2}^1 & m_{r2}^2 & m_{r2}^3 & m_{r2}^4 & m_{r2}^5 \\ \lambda & n_{r2}^1 & n_{r2}^2 & n_{r2}^3 & n_{r2}^4 & n_{r2}^5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{28}$$

Obviously, rank of the wrench system, $\$_r$ is four.

In this case, there are two uncontrollable freedoms for the movable platform. The movable platform can instantaneously rotate around any axis in plane O-XY even after locking five actuators.

One of input selections to avoid this singular configuration is choosing three R_1 and two R_4 as actuators.

3) Third case: Configurations of five limbs are symmetrical about Z-axis. Let the reference frame be the same with Fig. 5.

In this case, five $\$_{r2}$ will also be symmetrical about Z-axis. One $\$_{r2}$ can be achieved by transforming another by a rotation around Z-axis.

Based on the screw theory, assume O-YZ plane of reference frame parallel $\$_{r2}^{(1)}$, and $\$_{r2}^{(1)}$ intersect with X-axis at $[x, 0, 0]$, namely

$$\$_{r2}^{(1)} = [0 \quad m \quad n \quad 0 \quad -xn \quad xm]^{\mathrm{T}} \tag{29}$$

Since any $\$_{r2}^{(i)}$ can be achieved by transforming $\$_{r2}^{(1)}$ after a rotation around Z-axis, then $\$_{r2}^{(i)}$ can be expressed as

$$\$_{r2}^{(i)} = [-ms_{\alpha i} \quad mc_{\alpha i} \quad n \quad xns_{\alpha i} \quad -xnc_{\alpha i} \quad xm]^{\mathrm{T}} \tag{30}$$

where αi is the angle between the first limb and i^{th} limb; $s_{\alpha i}$, $c_{\alpha i}$ denote $\sin(\alpha i)$ and $\cos(\alpha i)$, respectively.

Given $\$_{r2}^{(1)}$, $\$_{r2}^{(2)}$, $\$_{r2}^{(3)}$, any $\$_{r2}^{(i)}$ can be expressed as

$$\$_{r2}^{(i)} = k_1\$_{r2}^{(i)} + k_2\$_{r2}^{(i)} + k_3\$_{r2}^{(i)} \tag{31}$$

where

$k_1=(s_{\alpha3}c_{\alpha i}-s_{\alpha i}c_{\alpha3}+c_{\alpha2}s_{\alpha i}-c_{\alpha i}s_{\alpha2}-s_{\alpha3}c_{\alpha2}+c_{\alpha3}s_{\alpha2})/t$;

$k_2=(s_{\alpha3}-s_{\alpha3}c_{\alpha i}-s_{\alpha i}+s_{\alpha i}c_{\alpha3})/t$;

$k_3=(s_{\alpha i}-s_{\alpha2}-c_{\alpha2}s_{\alpha i}+c_{\alpha i}s_{\alpha2})/t$;

$t=s_{\alpha3}-s_{\alpha2}-s_{\alpha3}c_{\alpha2}+c_{\alpha3}s_{\alpha2}$.

Thus, the max linear independent number of five $\$_{r2}$ is three. Hence, only four of six wrenches work efficiently. In other words, there are two uncontrollable DoF.

Moreover, this type of actuation singularity can be avoided by arranging three limbs with clockwise and the other two with counter-clockwise at assembly configuration. As shown in Fig. 7, limbs 1, 3, 5 are assembled with one current and the other two with contrary current.

Furthermore, for all existent 5-DoF *3R2T* fully-symmetrical parallel manipulators, it is always a common constraint force is acted on the movable platform at general configurations. Singularity analysis for them is similar because of their similar constraint property. Hence, this study is helpful for singularity analysis of other 5-DoF

3R2T fully-symmetrical parallel manipulators. For example, actuation singularity of manipulator 5-PRR(RR)[3] is the same with 5-RRR(RR).

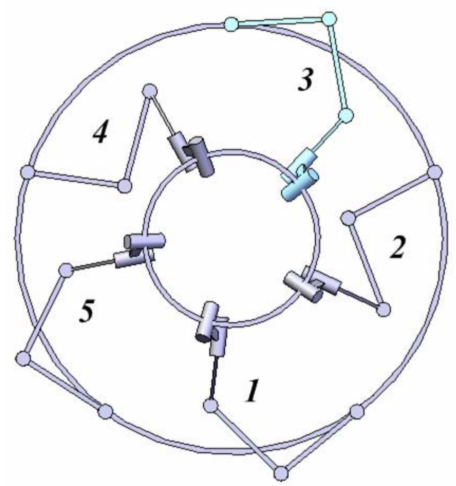

Fig. 7. Assembly configuration to avoid the third case of actuation singularity

V. CONCLUSIONS

Based on the singularity classification by Fang and Tsai, this paper studied the singularity configurations of a 5-DoF *3R2T* fully-symmetrical parallel manipulator, 5-RRR(RR). Both limb singularity and actuation singularity are illustrated with screw theory and Grassmann geometry. The influence of these singularities and methods to avoid them are also presented. The result of this study is helpful for singularity analysis of other 5-DoF *3R2T* fully-symmetrical parallel manipulators.

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