Time and Frequency domain design of Functional Filters

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Abstract— This paper proposes both time and frequency domain design of functional filters for linear time-invariant multivariable systems where all measurements are affected by disturbances. The order of this filter is equal to the dimension of the vector to be estimated. The time procedure design is based on the unbiasedness of the filter using a Sylvester equation; then the problem is expressed in a singular system one and is solved via Linear Matrix Inequalities (LMI) to find the optimal gain implemented in the observer design. The frequency procedure design is derived from time domain results by defining some useful Matrix Fractions Descriptions (MFDs) and mainly, establishing the useful and equivalent form of the connecting relationship that parameterizes the dynamic behavior between time and frequency domain, given by Hippe in the reduced-order case. A numerical example is given to illustrate our approach.

I. INTRODUCTION

The problem of functional filter design is equivalent to find a filter, that estimates a linear combination of the states of a system using the input and output measurements. The filter has the same order as the functional to be estimated. It has been the object of numerous studies in time domain since the original work of Luenberger [17], [18], first appeared. In most case it is related to the constrained or unconstrained Sylvester equations, see [20] and [19]. In addition [1], [10], [11] and [12] give different procedures for designing time domain functional observers, where one can view how the Sylvester equation was solved.

In frequency domain, there is less literature although it is the basis for most analysis performed on control systems [4], [5], [9]. It is well known that both linear optimal state feedback and optimal linear filtering problems can be formulated and solved in the frequency domain. A first solution for the filter transfer matrix was presented by MacFarlane [21], and it was demonstrated in [22] that optimal state feedback low and the optimal filter can be characterized by polynomial matrix that directly parameterize the state feedback control and the observer in the frequency domain.

In this framework, a time and frequency domain design procedure of functional filters is proposed. In fact, after using the unbiasedness condition, we propose a new method

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to treat the error dynamics where in order to avoid the time derivative of the disturbance w in this error (see [2]), we transform the problem into a singular one problem. Then a LMI approach is used to find the optimum gain implemented in the observer design. Then, based on time domain results, we propose a new functional filter with the aid of polynomial approach. The main reason of formulating the results of the time domain in the frequency one is the advantages that it presents for the observer-based control [23]. In fact, in this case, the compensator is driven by the input and the output of the system. So only the input-output behavior of the compensator (characterized by its transfer function) influences the properties of the closed-loop system. The additional degrees of freedom given by the frequency approach can then be used for robustness purpose for example [23].

The organization of this paper is as follows. Section II gives assumptions used through this paper and states the functional filtering problem that we propose to solve. Section III, presents the first contribution of the paper by giving the design procedure of a functional filter in the time domain. Using the unbiasedness condition, the problem is transformed into a singular problem one in order to avoid to have the derivative of the disturbance in the error dynamics. A LMI approach is then applied to optimize the gain implemented in the observer . Then section IV presents the second result of the paper by giving a frequency domain description of the time domain functional filter designed in section III using useful polynomial MFDs. We mainly establish the equivalent connecting relationship between time and frequency domain approach that will be useful for functional filter design. This can be viewed as a generalization of that of [7] given for the reduced order case . Section V gives a numerical example to illustrate our approach and finally section VI concludes the paper.

II. PROBLEM STATEMENT

Consider the following linear time-invariant multivariable system

$$\dot{x}(t) = Ax(t) + Bu(t) + D_1 w(t) \quad \text{(1a)}$$
$$z(t) = Kx(t) \quad \text{(1b)}$$
$$y(t) = Cx(t) + D_2 w(t) \quad \text{(1c)}$$

where $x(t) \in \Re^n$ and $y(t) \in \Re^m$ are the state vector and the measured output vector of the system, $w(t) \in \Re^q$ represents the disturbance vector, $u(t) \in \Re^p$ is the control input vector of the system and $z(t) \in \Re^{m_z}$ is the unmeasurable outputs to be estimated. A, B, D_1, K, C and D_2 are known constant matrices of appropriate dimensions.

Further, it is assumed through the paper that :

Assumption 1: $rank\ K = m_z$ and $rank\ C = m$

Problem :

We intend in this paper, to give a time and a frequency domain design of functional filters, that generates an estimate $\hat{z}(t)$ and $\hat{z}(s)$ for $z(t)$ and $z(s)$, using the input and output measurements $u(t)$ and $y(t)$. The order of these filters (m_z) is equal to the dimension of the vector to be estimated.

III. Time domain design

Our aim is to design a functional filter of order (m_z) for system (1) , of the form :

$$\dot{\varphi}(t) \ = \ N\varphi(t) + Jy(t) + Hu(t) \tag{2a}$$
$$\hat{z}(t) \ = \ \varphi(t) + Ey(t) \tag{2b}$$

where $\hat{z}(t) \in \Re^{m_z}$ is the estimate of $z(t)$.
The estimation error is given by

$$e(t) \ = \ z(t) - \hat{z}(t) \tag{3a}$$
$$= \ \psi x(t) - \varphi(t) - ED_2 w(t) \tag{3b}$$

with

$$\psi = K - EC \tag{4}$$

So, its dynamics can be written as

$$\dot{e}(t) \ = \ \psi\dot{x}(t) - \dot{\varphi}(t) - ED_2\dot{w}(t) \tag{5a}$$
$$= \ Ne(t) + (\psi A - JC - N\psi)x(t) + (\psi B - H)u(t)$$
$$+ \ (\psi D_1 - JD_2 + NED_2)w(t) - ED_2\dot{w}(t) \tag{5b}$$

The problem of the filter design is to determine N, J, H and E such that [1], [2]

$i)$ the filter (2) is unbiased if $w(t) = 0$

$ii)$ the filter (2) is stable, i.e N is Hurwitz.

The unbiasedness of the filter is achieved if and only if the following Sylvester equation holds

$$\psi A - JC - N\psi = 0 \tag{6}$$

with

$$H = \psi B \tag{7}$$

The Sylvester equation (6) can be written by taking into account (4) as

$$-NK + (NE - J)C + KA - ECA \ = \ 0 \tag{8}$$

Let

$$L = J - NE \tag{9}$$

Then, equation (8) can be transformed as

$$[N \ \ L \ \ E] \begin{bmatrix} K \\ C \\ CA \end{bmatrix} = KA \tag{10}$$

For the resolution of (10), let set

$$[N \ \ L \ \ E] = X \tag{11}$$

$$\begin{bmatrix} K \\ C \\ CA \end{bmatrix} = \Sigma \tag{12}$$

$$KA = \Theta \tag{13}$$

therefore (10) becomes

$$X\Sigma = \Theta \tag{14}$$

This equation has a solution X if and only if

$$rank \begin{pmatrix} \Sigma \\ \Theta \end{pmatrix} = rank\ \Sigma \tag{15}$$

and a general solution for (14), if it exists, is given by

$$X = \Theta\Sigma^+ - Z(I - \Sigma\Sigma^+) \tag{16}$$

where Σ^+ is a generalized inverse of matrix Σ given by (12) and Z is an arbitrary matrix of appropriate dimensions, that will be determined in the sequel using LMI approach.

Once matrix X is determined, it is easy to give the expressions of matrices N, L and E. In fact,

$$N \ = \ X \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix} = A_{11} - ZB_{11} \text{ where} \tag{17}$$

$$A_{11} \ = \ \Theta\Sigma^+ \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix} \tag{18}$$

$$B_{11} \ = \ (I - \Sigma\Sigma^+) \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix}, \tag{19}$$

$$L \ = \ X \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix} = A_{22} - ZB_{22} \text{ where} \tag{20}$$

$$A_{22} \ = \ \Theta\Sigma^+ \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix} \tag{21}$$

$$B_{22} \ = \ (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix}, \text{ and} \tag{22}$$

$$E \ = \ X \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix} = A_{33} - ZB_{33} \tag{23}$$

where

$$A_{33} \ = \ \Theta\Sigma^+ \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix} \tag{24}$$

$$B_{33} \ = \ (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix}. \tag{25}$$

Hence all filter matrices are determined if and only if the matrix Z is known.

So, the dynamics of the estimation error (5b) reads

$$\dot{e}(t) = Ne(t) + \alpha w(t) + \beta \dot{w}(t) \tag{26}$$

with

$$\alpha = \psi D_1 - JD_2 + NED_2 = \alpha_1 - Z\alpha_2 \tag{27}$$

where

$$\alpha_1 = KD_1 - \Theta\Sigma^+ \begin{pmatrix} 0 \\ D_2 \\ CD_1 \end{pmatrix} \tag{28}$$

$$\alpha_2 = (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ D_2 \\ CD_1 \end{pmatrix} \tag{29}$$

and

$$\beta = -ED_2 \tag{30}$$
$$= \beta_1 - Z\beta_2 \tag{31}$$

where

$$\beta_1 = -A_{33}D_2 \tag{32}$$
$$\beta_2 = -B_{33}D_2 \tag{33}$$

Notice that the error dynamics (26) contains the derivative of $w(t)$. This problem is generally solved by adding an additional constraint on the filter matrices (see [2]) or by choosing a new type of norm for the system. Here we propose another method which consists in rewriting the error system into a descriptor (singular) form. Then the following descriptor system is given from equation (26):

$$\begin{pmatrix} I & -\beta \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{e}(t) \\ \dot{\rho}_1(t) \end{pmatrix} = \begin{pmatrix} N & \alpha \\ 0 & -I \end{pmatrix} \begin{pmatrix} e(t) \\ \rho_1(t) \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ I \end{pmatrix} w(t) \tag{34a}$$

$$e(t) = (I \quad 0) \begin{pmatrix} e(t) \\ \rho_1(t) \end{pmatrix} \tag{34b}$$

where $e(t)$ is the estimation error and $\rho_1(t)$ is such that $\rho_1(t) = w(t)$.

Before continuing, let us recall the following result on descriptor systems:

Lemma 1: [13]

Consider a singular system of the form

$$F\dot{x} = Ax + Bw \tag{35}$$
$$z = Cx \tag{36}$$

where x is the state, w is the exogenous input and z is a controlled output; matrices F, A, B and C are known. The pair (F, A) is admissible and $\|G\|_\infty < \gamma$ ($G = C(sF - A)^{-1}B$) if and only if there exists $X \in \Re^{n \times n}$ such that
1)

$$F^T X = X^T F \geq 0 \tag{37}$$

2)

$$A^T X + X^T A + C^T C + \frac{1}{\gamma^2} X^T B B^T X < 0 \tag{38}$$

Before applying this result on the singular system (34), we consider that the gain matrix Z satisfies the following relation

$$Z\beta_2 = 0 \tag{39}$$

in order to avoid an unknown (to be designed) gain matrix Z in the singular matrix $\begin{pmatrix} I & -\beta \\ 0 & 0 \end{pmatrix}$ of system (34).

So it exists a matrix Z_1 that satisfies

$$Z = Z_1(I - \beta_2\beta_2^+) \tag{40}$$

Setting

$$\rho_{20} = \begin{pmatrix} I & -\beta \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} I & -\beta_1 \\ 0 & 0 \end{pmatrix} \tag{41}$$

$$A_{20} = \begin{pmatrix} N & \alpha \\ 0 & -I \end{pmatrix} = \begin{pmatrix} A_{11} - ZB_{11} & \alpha_1 - Z\alpha_2 \\ 0 & -I \end{pmatrix} \tag{42}$$

$$\beta_{20} = \begin{pmatrix} 0 \\ I \end{pmatrix} \tag{43}$$

and

$$\chi(t) = \begin{pmatrix} e(t) \\ \rho_1(t) \end{pmatrix} \tag{44}$$

the singular system (34) reads

$$\rho_{20} \dot{\chi}(t) = A_{20} \chi(t) + \beta_{20} w(t) \tag{45a}$$
$$e(t) = (I \quad 0) \chi(t) \tag{45b}$$

Now, a LMI approach is used in the following theorem to get the gain matrix Z which parameterizes the filter matrices

Theorem 1: The pair (ρ_{20}, A_{20}) is admissible and

$$\|W(s) = (I \quad 0)(s\rho_{20} - A_{20})^{-1}\beta_{20}\|_\infty < \gamma$$

if and only if there exist $X_1 \in \Re^{m_z \times m_z}$, $X_2 \in \Re^{q \times q}$ and $Y \in \Re^{(m_z + 2m) \times m_z}$ such that the following LMIs hold
1)

$$\begin{pmatrix} X_1 & 0 \\ -\beta_1^T X_1 & 0 \end{pmatrix} = \begin{pmatrix} X_1^T & -X_1^T\beta_1 \\ 0 & 0 \end{pmatrix} \geq 0 \tag{46}$$

2)

$$\begin{bmatrix} P_{11} & P_{12} & 0 & I \\ P_{13} & P_{14} & X_2^T & 0 \\ 0 & X_2 & -\gamma^2 I & 0 \\ I & 0 & 0 & -I \end{bmatrix} < 0 \tag{47}$$

with $\theta_0 = (I - \beta_2\beta_2^+)^T$ and

$$P_{11} = A_{11}^T X_1 + X_1^T A_{11} - B_{11}^T \theta_0^T Y - Y^T \theta_0 B_{11}$$
$$P_{12} = X_1^T \alpha_1 - Y^T \theta_0 \alpha_2$$
$$P_{13} = \alpha_1^T X_1 - \alpha_2^T \theta_0^T Y$$
$$P_{14} = -X_2 - X_2^T$$

Then the gain Z_1 is given by $Z_1 = (YX_1^{-1})^T$.

Proof 1: Using lemma 1 and the Schur lemma [14] on the singular system (45) yields to

1)

$$\Phi_1 = \Phi_1^T \geq 0 \quad \text{with } \Phi_1 = \rho_{20}^T \mathcal{X} \qquad (48)$$

2)

$$\begin{bmatrix} \Phi_2 + \Phi_2^T & \Phi_3^T & (I \quad 0)^T \\ \Phi_3 & -\gamma^2 I & 0 \\ (I \quad 0) & 0 & -I \end{bmatrix} < 0 \qquad (49)$$

with

$$\Phi_2 = A_{20}^T \mathcal{X} \qquad (50)$$

$$\Phi_3 = \beta_{20}^T \mathcal{X} \qquad (51)$$

By taking $\mathcal{X} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}$ and using (41), (48) is equivalent to (46). Then, replacing A_{20} and β_{20}, from (49), by their expressions (42), (43) and using (40) , the LMI (47) holds, with $\theta_0 = (I - \beta_2\beta_2^+)^T$ and $Y = Z_1^T X_1$.

Design of the time domain functional filter
The different steps of the filter computation in the time domain are summarized in the following design method:

1) Compute Σ and Θ from (12), (13).

2) All known matrices implemented in LMIs i.e. A_{11}, B_{11}, α_1, α_2, B_{33}, β_2 , A_{33} and β_1 are computed using respectively (18) , (19), (28), (29), (25), (40), (24), (32). So, θ_0 is known.

3) Resolution of the LMIs ((46), (47)) gives the gain Z_1, so from (40), Z is known.

4) Compute the filter matrix N from (17).

5) Matrix E is also obtained by (23). Therefore ψ is known (4) and H is determined as (7)

6) Finally, the time domain representation of the functional filter (2) is known, indeed after computing A_{22} and B_{22} from (21), (22), L can be easily calculated from (20) and therefore matrix J implemented in time design is computed using (9).

IV. FREQUENCY DOMAIN DESIGN

The next theorem presents the main result of this section by giving a frequency domain description of the functional filter designed in section III:

Theorem 2: : Consider the following Matrix Fraction Descriptions (MFDs)

i)

$$\begin{bmatrix} M_0 \\ M_1 \end{bmatrix} (sI - N)^{-1} = \bar{D}^{-1}(s)\bar{N}_x(s) \qquad (52)$$

where

a) M_0 and M_1 are arbitrary matrix of dimension $(m_z \times m_z)$ and $(m \times m_z)$ such that matrix $\begin{bmatrix} M_0 \\ M_1 \end{bmatrix}$ is of full column rank.

b) N is the filter (2) matrix of order m_z , and the two polynomial matrix $\bar{D}(s)$ and $\bar{N}_x(s)$ are of dimensions $((m_z + m) \times (m_z + m))$ and $((m_z + m) \times m_z)$ respectively. They have the specification to be left coprime . These transfer functions can be calculated from the factorization approach presented by Vidyasager [15].

ii)

$$\begin{bmatrix} M_0 \\ M_1 \end{bmatrix} (sI - N)^{-1}H = \bar{D}^{-1}(s)\bar{N}_u(s) \qquad (53)$$

where

$$\bar{N}_u(s) = \bar{N}_x(s)H \qquad (54)$$

iii)

$$\bar{D}^{-1}(s)\tilde{D}(s) = I_{m_z+m} + \begin{bmatrix} M_0 \\ M_1 \end{bmatrix} (sI - N)^{-1}[0 \quad J] \qquad (55)$$

with $\bar{D}(s)$ is given by (52) and the polynomial matrix $\tilde{D}(s)$ parameterizes the dynamics of the functional filter (2) in the frequency domain. This left coprime MFD is a generalization of the connecting relationship that parameterizes the dynamics behavior between time and frequency domain given by Hippe [7] for the reduced order case.

Then a frequency domain representation of the functional filter (2) of order m_z , $m_z \leq n$, related to system (1) is given by,

$$\begin{aligned} \hat{z}(s) &= \tilde{D}^{-1}(s) [\tilde{D}(s) \begin{bmatrix} M_0 \\ M_1 \end{bmatrix}^+ ((\bar{D}^{-1}(s) \tilde{D}(s) - I) \\ &\times \begin{bmatrix} 0 \\ y(s) \end{bmatrix} + \bar{D}^{-1}(s)\bar{N}_u(s) u(s)) + \\ &\tilde{D}(s) E \times y(s)] \end{aligned} \qquad (56)$$

The filter denominator matrix $\tilde{D}(s)$ satisfies

$$\tilde{D}(s) = \bar{D}(s) + \bar{N}_x(s) [0_{m_z \times m_z} \quad J] \qquad (57)$$

Proof 2: : The Laplace transform of (2a) reads as

$$\begin{aligned} \varphi(s) &= (sI - N)^{-1}Jy(s) + (sI - N)^{-1}Hu(s) \quad (58) \\ &= (sI - N)^{-1}[0_{m_z \times m_z} \quad J] \begin{bmatrix} 0_{m_z \times 1} \\ y(s) \end{bmatrix} \\ &+ (sI - N)^{-1}Hu(s) \end{aligned} \qquad (59)$$

So, the Laplace transform of the estimated vector $\hat{z}(t)$ (2b), can be written by taking into account(59) as

$$\begin{aligned} \hat{z}(s) &= (sI - N)^{-1}[0_{m_z \times m_z} \quad J] \begin{bmatrix} 0_{m_z \times 1} \\ y(s) \end{bmatrix} \\ &+ (sI - N)^{-1}Hu(s) + Ey(s) \end{aligned} \qquad (60)$$

Or, following the proposed left coprime MFDs (55), we have

$$(sI - N)^{-1}[0_{m_z \times m_z} \quad J] = \begin{bmatrix} M_0 \\ M_1 \end{bmatrix}^+ \times$$
$$(\bar{D}^{-1}(s)\tilde{\bar{D}}(s) - I_{m_z+m}) \quad (61)$$

and from (53)

$$(sI - N)^{-1}H = \begin{bmatrix} M_0 \\ M_1 \end{bmatrix}^+ \times \bar{D}^{-1}(s)\bar{N}_u(s) \quad (62)$$

the desired estimate of the functional filter reads

$$\hat{z}(s) = \begin{bmatrix} M_0 \\ M_1 \end{bmatrix}^+ \times (\bar{D}^{-1}(s)\tilde{\bar{D}}(s) - I_{m_z+m})$$
$$\times \begin{bmatrix} 0_{m_z \times 1} \\ y(s)_{m \times 1} \end{bmatrix} + \begin{bmatrix} M_0 \\ M_1 \end{bmatrix}^+ \bar{D}^{-1}(s)\bar{N}_u(s)\ u(s)$$
$$+ \quad E\ y(s) \quad (63)$$

Therefore and in view of,

$$\tilde{\bar{D}}^{-1}(s)\tilde{\bar{D}}(s) = I_{m_z+m} \quad (64)$$

the frequency domain description (56) holds. The polynomial matrix equation of the filter denominator (57) directly follows from equation (55) using (52).

V. NUMERICAL EXAMPLE

Consider the system presented in section II, where [5]

$$A = \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -4 \\ -7 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 0 & 0.067 \end{bmatrix}$$

1 - Time domain functional filter design

By applying the proposed algorithm of section III, we obtain the following results:

1)

$$\Sigma = \begin{bmatrix} 0 & 0.0670 \\ 1 & 1 \\ -2 & -4 \end{bmatrix}, \quad \Theta = \begin{bmatrix} 0 & -0.1340 \end{bmatrix} \quad (65)$$

2)

$$A_{11} = -0.0112, B_{11} = \begin{bmatrix} 0.9944 \\ 0.0666 \\ 0.0333 \end{bmatrix} \quad (66)$$

$$\alpha_1 = 0.1306, \alpha_2 = \begin{bmatrix} -0.2998 \\ -0.0201 \\ -0.0100 \end{bmatrix} \quad (67)$$

$$B_{33} = \begin{bmatrix} 0.0333 \\ 0.0022 \\ 0.0011 \end{bmatrix}, \beta_2 = \begin{bmatrix} -0.0333 \\ -0.0022 \\ -0.0011 \end{bmatrix} \quad (68)$$

$$A_{33} = 0.0666\ \beta_1 = -0.0666 \quad (69)$$

3) For $\gamma = 12$

$$Z_1 = \begin{bmatrix} 1.0698 & -80.6126 & 158.7342 \end{bmatrix} \quad (70)$$

and therefore

$$Z = \begin{bmatrix} 0.0890 & -80.6783 & 158.7013 \end{bmatrix} \quad (71)$$

Consequently, the filter matrices are given by
$N = -0.0112$, $E = 0.0666$, $H = -0.0663$, $L = 0.1333$
and $J = 0.1325$.

So, the time domain description of the functional filter of order 1, reads

$$\dot{\varphi}(t) = -0.0112\ \varphi(t) + 0.1325\ y(t) -$$
$$0.0663\ u(t) \quad (72a)$$
$$\hat{z}(t) = \varphi(t) + 0.0666\ y(t) \quad (72b)$$

2 - Frequency domain functional filter design

For that, let us choose $M_0 = 1$ and $M_1 = 0$, so $\begin{bmatrix} M_0 \\ M_1 \end{bmatrix}$ is of full column rank. By using the factorization approach given in [15] and the algorithms of [16] about realization of RH_∞ matrices satisfying the Bezout identity, we obtain

$$\bar{D}(s) = \begin{bmatrix} \frac{s+0.0112}{s+2.0112} & \frac{-1}{s+2.0112} \\ 0 & 1 \end{bmatrix} \quad (73)$$

and

$$\bar{N}_x(s) = \begin{bmatrix} (s+2.0112)^{-1} \\ 0 \end{bmatrix} \quad (74)$$

one can verify that (52) is satisfied. From (54), we have

$$\bar{N}_u(s) = \begin{bmatrix} -0.0663(s+2.0112)^{-1} \\ 0 \end{bmatrix} \quad (75)$$

consequently, from (57) the denominator matrix of the functional filter reads as

$$\tilde{D}(s) = \begin{bmatrix} \frac{s+0.0112}{s+2.0112} & \frac{-0.8675}{s+2.0112} \\ 0 & 1 \end{bmatrix} \quad (76)$$

Therefore, following (63) , and by taking into account (73), (75) and (76), $\hat{z}(s)$ can be rewritten as

$$\hat{z}(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & \frac{0.1325}{s+0.0112} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0_{1 \times 1} \\ y(s)_{1 \times 1} \end{bmatrix}$$
$$- \frac{0.0663}{s+0.0112}\ u(s) + 0.0666\ y(s) \quad (77)$$

so, its readily follows that the desired frequency description (56) of the functional filter is determined .

In other hand, we propose to draw the frequency estimation error. For that, (77) reads

$$\hat{z}(s) = [\frac{(0.0666s+0.1332)(2s+2)}{(s+0.0112)(s+2)^2} - \frac{0.0663}{s+0.0112}]\ u(s)$$
$$+ [\frac{(0.0666s+0.1332)(s^2-7s-4)}{(s+0.0112)(s+2)^2}]\ w(s) \quad (78)$$

therefore supposing that $u = 0$, and using Laplace transform of $(1b)$, the frequency estimation error satisfies the following equation

$$e(s) = z(s) - \hat{z}(s) \quad (79)$$
$$= \frac{-0.0666s^3 + 0.802s^2 + 2.142s + 0.5433}{s^3 + 4.011s^2 + 4.045s + 0.0448}w(s)$$

The simulations are done with an input $u = 0$. For the time domain design, the disturbance $w(t)$ is taken to be of bounded energy and is given by figure 1. We take $(e(0) = 2.5)$ as initial condition. Finally, the figures 2 and 3 show the time and frequency domain behavior of the filter and so, the effectiveness of our approach.

VI. CONCLUSION

A new time and frequency domain design of functional filter for linear multivariable systems is proposed in this paper. The proposed filter has the same order as the functional to be estimated. The time domain procedure is based on the resolution of Sylvester equation and the use of a singular system approach in order to avoid time derivative of the disturbance. LMI approach is then used to find the optimum gain implemented in the observer design. An algorithm that summarizes the different step of resolution is given. Then, based on time domain results, the frequency domain description of the functional filter is derived. In fact, we define some useful matrix fraction descriptions and mainly propose a connecting relation between time and frequency domain parameterizations of functional filter, which is equivalent to that proposed by Hippe [7] in the reduced order case. The proposed design procedure has been applied on numerical example and it shows its effectiveness.
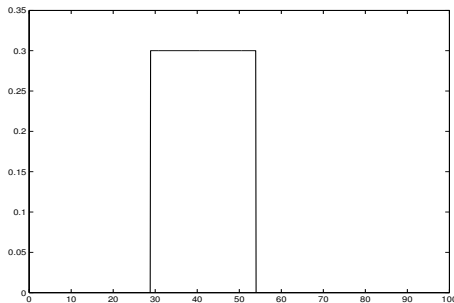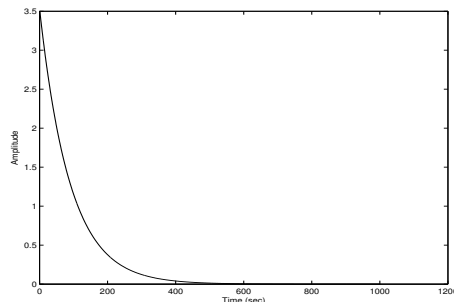

Fig. 1.   The behavior of the used disturbance w(t)

Fig. 2.   Evolution of the estimation error e(t) designed in time domain

Fig. 3.   The Bode diagram of the transfert function from w(s) to e(s)

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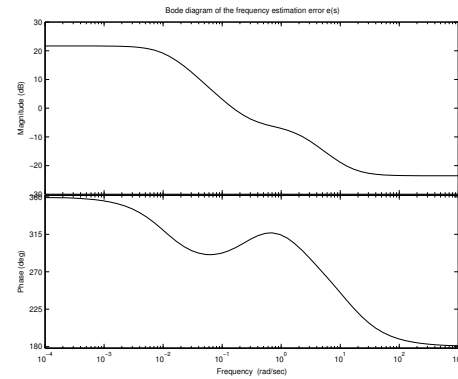
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