Review Paper

On Cosmological Magnetic Fields, Extended Navier-Stokes equation, and Newtonian Raychaudhury equation

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ABSTRACT

Traditionally it has been argued that due to the electric neutrality of the universe on large scales, the only relevant interaction in cosmology should be gravitation. However, the behavior of electromagnetic fields on astrophysical and cosmological scales is still far from clear, the most evident example being the unknown origin of magnetic fields observed in galaxies and galaxy clusters. Therefore, in the present paper I will explore how cosmological magnetic fields can play some roles in the form of extended electromagnetism, Newtonian version of Raychaudhury equation, and also extended covariant Navier-Stokes equation. Then this may be viewed as a way to generalize Navier-Stokes systems on Cantor Sets. Further investigation can be recommended.

Key Words: Navier-Stokes equations, Raychaudhury equation, Navier-Stokes cosmology, Extended Electromagnetism.

Introduction

Traditionally it has been argued that due to the electric neutrality of the universe on large scales, the only relevant interaction in cosmology should be gravitation. However, the behavior of electromagnetic fields on astrophysical and cosmological scales is still far from clear, the most evident example being the unknown origin of magnetic fields observed in galaxies and galaxy clusters. Therefore, in the present paper I will explore how cosmological magnetic fields can play some roles in the form of extended electromagnetism, Newtonian version of Raychaudhury equation, and also covariant Navier-Stokes equation. Then this may be viewed as a way to generalize Navier-Stokes systems on Cantor Sets.

It can be expected that the cosmological magnetic fields have played some roles to cause cosmic vorticity. But cosmic vorticity is another mysterious subject in cosmology, and it would need further research to clarify the connection between cosmological magnetic fields and cosmic vorticity [7]. The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.

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Extended Electromagnetism equation

According to Jimenez & Maroto [4], it is possible to consider that large-scale cosmic magnetic fields can be generated in an extended electromagnetic model. The corresponding modified Maxwell equations read [4]:

$$\nabla_{\nu} F^{\mu\nu} + \xi \nabla^{\mu} (\nabla_{\nu} A^{\nu}) = J^{\mu}. \tag{1}$$

The above equations imply that there exists small additional term in extended electromagnetism.

Extended Navier-Stokes equation

The 2D Navier-Stokes equation for a steady viscous flow can be written as follows [6]:

$$\rho(\vec{\upsilon} \cdot \nabla)\vec{\upsilon} = -\nabla p + \rho\vec{f} + \mu\Delta\vec{\upsilon} \tag{2}$$

Argentini obtained a general exact solution of ODE version of 2D Navier-Stokes equation in Riccati form, and computational solutions have been obtained elsewhere [1][2][3].

The magnetic field is a source of isotropic and anisotropic pressure, with both contributing to the Lorentz force. At the ideal-MHD limit, the latter recasts the Navier-Stokes equation into [5]:

$$\rho A_a = -\partial_a p - \partial^b \pi_{ab} - \frac{1}{2}\partial_a B^2 + B^b \partial_b B_a, \tag{3}$$

Which replaces the standard expression of covariant form of Navier-Stokes equation as follows:

$$\rho A_a = -\partial_a p - \partial^b \pi_{ab}. \tag{4}$$

Note that the third term on the right-hand side is due to the magnetic pressure and the fourth comes from the field's tension. When these two stresses balance each other out, the Lorentz force vanishes and the B-field can reach equilibrium. [5]

Extended Raychudhury equation

Zalaletdinov has shown that Raychaudhury evolution equation can yield Friedman equation at certain limits. His expression of Raychaudhury evolution equation is as follows: [8, p. 26]

$$\dot{\theta} + \frac{1}{3}\theta^2 + 4\pi G\rho - \Lambda = 0. \tag{5}$$

When the vorticity vector, shear tensor and tidal force tensor vanish, then (11) is equivalent to Friedman equation [8]:

$$\theta = \frac{3}{R}\frac{dR}{dt}. \tag{6}$$

While Zalaletdinov already gives Newton-Poisson version of Raychaudhury equation, according to Vasiliou and Tsagas the Newtonian version of Raychaudhury equation has different form, that is [5]:

$$\dot{\theta} = -\frac{1}{3}\theta^2 - \frac{1}{2}\kappa\rho + \Lambda + \partial^a A_a - 2(\sigma^2 - \omega^2),. \tag{7}$$

With

$$\sigma^2 = \sigma_{ab}\sigma^{ab}/2 = shear, \text{ and} \tag{8}$$

$$\omega^2 = \omega_{ab}\omega^{ab}/2 = vorticity. \tag{9}$$

How to write down an extended Navier-Stokes equations on Cantor Sets

As I have discussed in a previous paper, it is possible to write down the Navier-Stokes equations on Cantor Sets, by keeping in mind their possible applications in cosmology. By defining some operators as follows:

1. In Cantor coordinates [9]:

$$\nabla^\alpha \cdot u = div^\alpha u = \frac{\partial^\alpha u_1}{\partial x_1^\alpha} + \frac{\partial^\alpha u_2}{\partial x_2^\alpha} + \frac{\partial^\alpha u_3}{\partial x_3^\alpha}, \tag{10}$$

$$\nabla^\alpha \times u = curl^\alpha u = \left(\frac{\partial^\alpha u_3}{\partial x_2^\alpha} - \frac{\partial^\alpha u_2}{\partial x_3^\alpha}\right)e_1^\alpha + \left(\frac{\partial^\alpha u_1}{\partial x_3^\alpha} - \frac{\partial^\alpha u_3}{\partial x_1^\alpha}\right)e_2^\alpha + \left(\frac{\partial^\alpha u_2}{\partial x_1^\alpha} - \frac{\partial^\alpha u_1}{\partial x_2^\alpha}\right)e_3^\alpha \,. \tag{11}$$

2. In Cantor-type cylindrical coordinates [10, p.4]:

$$\nabla^\alpha \cdot r = \frac{\partial^\alpha r_R}{\partial R^\alpha} + \frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} + \frac{r_R}{R^\alpha} + \frac{\partial^\alpha r_z}{\partial z^\alpha}, \tag{12}$$

$$\nabla^\alpha \times r = \left(\frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha r_\theta}{\partial z^\alpha}\right)e_R^\alpha + \left(\frac{\partial^\alpha r_R}{\partial z^\alpha} - \frac{\partial^\alpha r_z}{\partial R^\alpha}\right)e_\theta^\alpha + \left(\frac{\partial^\alpha r_\theta}{\partial R^\alpha} + \frac{r_R}{R^\alpha} - \frac{1}{R^\alpha}\frac{\partial^\alpha r_R}{\partial \theta^\alpha}\right)e_z^\alpha \,. \tag{13}$$

Then Yang, Baleanu and Machado are able to obtain a general form of the Navier-Stokes equations on Cantor Sets as follows [9, p.6]:

$$\rho \frac{D^{\alpha} \upsilon}{D t^{\alpha}} = -\nabla^{\alpha} \cdot (pI) + \nabla^{\alpha} \left[2\mu \left(\nabla^{\alpha} \cdot \upsilon + \upsilon \cdot \nabla^{\alpha} \right) - \frac{2}{3} \mu \left(\nabla^{\alpha} \cdot \upsilon \right) I \right] + \rho b \tag{14}$$

Now we can extend equation (14) further to an extended covariant Navier-Stokes equation on Cantor Sets, as follows:

$$\rho A_a = -\partial_a^{\alpha} p - \partial_{\alpha}^{b} \pi_{ab} - \frac{1}{2} \partial_a^{\alpha} B^2 + B^b \partial_b^{\alpha} B_a , \tag{15}$$

Where α term takes the same definition as with (10)-(13).

It can be expected that the cosmological magnetic fields have played some roles to cause cosmic vorticity. But cosmic vorticity is another mysterious subject in cosmology, and it would need further research to clarify the connection between cosmological magnetic fields and cosmic vorticity [7]. The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.

Concluding remarks

Traditionally it has been argued that due to the electric neutrality of the universe on large scales, the only relevant interaction in cosmology should be gravitation. However, the behavior of electromagnetic fields on astrophysical and cosmological scales is still far from clear, the most evident example being the unknown origin of magnetic fields observed in galaxies and galaxy clusters.

This paper discusses a possible route to covariant Navier-Stokes equations to extended covariant Navier-Stokes Cosmology on Cantor Sets. I also discuss Newtonian version of Raychaudhury equation. It can be expected that the cosmological magnetic fields have played some roles to cause cosmic vorticity. But cosmic vorticity is another mysterious subject in cosmology, and it would need further research to clarify the connection between cosmological magnetic fields and cosmic vorticity. The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.

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