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Nonlinear dynamics of the electromagnetic ion cyclotron structures in the inner magnetosphere

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[1] Electromagnetic ion cyclotron waves, called EMICs, are widely observed in the inner magnetosphere and can be excited through various plasma mechanisms such as ion temperature anisotropy. These waves interact with magnetospheric particles, which they can scatter into the loss cone. This paper investigates how nonlinearities in the ion fluid equations governing the electromagnetic ion cyclotron waves cause large-amplitude EMIC waves to evolve into coherent nonlinear structures. Both planar soliton structures and also two-dimensional vortex-like nonlinear structures are found to develop out of these nonlinearities.

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1. Introduction

[2] Electromagnetic ion cyclotron waves, commonly called EMICs, appear to play a multifaceted role in the Earth's inner magnetosphere. These waves can be excited by spontaneous plasma instability when plasma is injected via substorms from the geomagnetic tail into the inner magnetosphere [e.g., *Cornwall*, 1965]. This plasma injection conserves the magnetic momentum of the hot ions in the plasma sheet, producing an energetic ion plasma component with energy up to 30 keV near L shell values of 4 or 5. Owing to its anisotropy in velocity space, this population of energetic ions can drive the EMIC waves through Landau cyclotron resonances. Another common source of electromagnetic ion cyclotron waves is the compression of the magnetopause, which leads to an anisotropic ion distribution, where the anisotropy is the instability source [*Anderson and Hamilton*, 1993].

[3] Electromagnetic ion cyclotron waves have been investigated for a long time with both ground-based instruments and satellite experiments. Initial observations by satellite were made by *Russell et al.* [1970], *Gurnett* [1976], and *Kintner and Gurnett* [1977]. EMIC waves have been seen in association with intense electron fluxes on the auroral field lines with the S3-3 satellite [*Kintner et al.*, 1979; *Temerin and Lysak*, 1984], the Freja satellite [*Hamrin et al.*, 2002], the Fast satellite [*Chaston et al.*, 2002], and the Polar satellite [*Santolík et al.*, 2002]. The typical spatial distribution of intense EMICs and whistlers in the magnetosphere is shown in Figure 1, which is taken from *Summers et al.* [1998]. EMIC waves have also been observed in the equatorial plane at various L values by the GEOS satellites [*Young et al.*, 1981], the Akebono satellite [*Sawada et al.*, 1991; *Kasahara et al.*, 1994; *Liu et al.*, 1994], the Equator-S satellite [*Mouikis et al.*, 2002], and the CRESS satellite [*Fraser and Nguyen*, 2001; *Meredith et al.*, 2003]. In this equatorial region, EMIC waves are generated by the ion cyclotron instability driven by the anisotropic distribution of ring current energetic ions during magnetic storms [see *Mouikis et al.*, 2002; *Summers and Thorne*, 2003, and references therein]. They are characterized by wave structures with the electric and magnetic field power below the first multiple of the proton cyclotron frequency. EMIC waves are known to take part in the precipitation of electrons [e.g., *Lorentzen et al.*, 2000; *Summers and Ma*, 2000; *Summers and Thorne*, 2003; *Meredith et al.*, 2003]. *Parrot et al.* [2006] reported the observation of electromagnetic harmonic emissions with the microsatellite DEMETER. These emissions were detected in the ELF range (500 Hz to 2 kHz) in the upper ionosphere during large magnetic storms, and it was shown that they could be related to the electromagnetic ion cyclotron waves.

[4] In the present work, we assume that electromagnetic ion cyclotron waves are present with sufficiently large amplitude so that their nonlinear structure and the conditions for parametric instabilities become important. After displaying the linear dispersion relation with a kinetic theory response for the ions, we develop a nonlinear two-component fluid description of the electromagnetic ion cyclotron waves and explore their properties with respect to propagation

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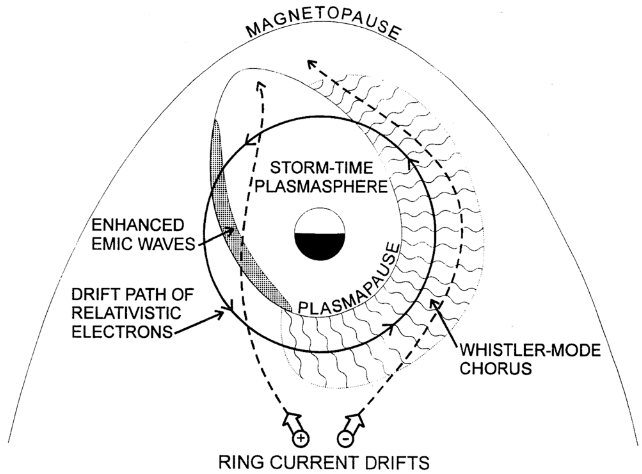

Figure 1. Typical spread of the electromagnetic ion cyclotron waves (EMIC) and whistler waves in the inner magnetosphere [from *Summers et al.*, 1998].

speed versus amplitude. We also consider how the resulting coherent nonlinear structures may accelerate ions to MeV energies.

[5] In section 2 we develop the basic equations for the linear EMICs in the magnetosphere. In section 3 we give the nonlinear two-component fluid equations for the EMICS and consider the parametric decay instability of a large-amplitude plane wave solution. In section 4 we consider that the amplitudes are well above the parametric instability limit and thus look for soliton and vortex solutions of the nonlinear equations. In section 5 we summarize our results.

2. Basic Equations for Electromagnetic Ion Cyclotron Waves

[6] We consider the nonlinear dynamics of electromagnetic fields in the inner magnetosphere. Our investigation is also applicable to the upper regions of the Earth's ionosphere. We consider the propagation of circularly polarized nonlinear electromagnetic waves propagating along the dipole geomagnetic field in the equatorial region of the inner magnetosphere. We assume that the electromagnetic emission frequency ω_0 is close to the proton gyrofrequency ω_{cp}, i.e.,

$$\omega_0 \simeq \omega_{cp} = 2.98 L^{-3} \text{kHz}, \quad (1)$$

where $L = r_{eq}/R_E$ is the shell parameter of the dipole geomagnetic field, r_{eq} is the radial distance to the magnetic field line in the equatorial plane, and $R_E = 6.37 \times 10^6$ m is the Earth's radius. Further, we consider regions around $L = 2$. The circularly polarized electric field is given by

$$E_- = E_x - iE_z = E(x, y, t) \exp[i(k_0 y - \omega_0 t)]. \quad (2)$$

Here k_0 is the y component of the wave number of the electromagnetic emission. Thus the pumping electromagnetic waves are assumed to propagate almost along the homogeneous and constant geomagnetic field $\mathbf{B}_0$ oriented along the y axis. The amplitude $E(x, y, t)$ in equation (2) becomes a slowly varying function of spatial coordinates and time due to the nonlinear interaction of the electromagnetic waves with plasma. A schematic of the local coordinate system used for the EMIC waves is shown in Figure 2. The electromagnetic wave fluctuations are in the x and z directions, transverse to the geomagnetic field (y direction), where x is the azimuthal symmetry direction around the Earth.

[7] Taking the inner magnetosphere plasma to be collisionless and nondegenerate for the frequencies of equation (1), we have the following dispersion equation [*Alexandrov et al.*, 1984],

$$N^2 = \frac{k_0^2 c^2}{\omega_0^2} = 1 - \frac{\omega_{pe}^2}{\omega_0 \omega_{ce}} - \frac{\omega_{pp}^2}{\omega_0(\omega_0 - \omega_{cp})} I_+\left(\frac{\omega_0 - \omega_{cp}}{k_0 v_{tp}}\right), \quad (3)$$

where

$$I_+(\zeta) = \zeta e^{-\zeta^2/2} \int_{-\infty}^{i\zeta} d\tau e^{-\tau^2/2}, \quad (4)$$

with $\zeta = (\omega_0 - \omega_{cp})/v_{tp}$. Here ω_{pe} and ω_{pp} are the electron and proton Langmuir frequencies, respectively; ω_{ce} is the electron gyrofrequency; v_{tp} is the proton thermal velocity; and c is the speed of light in vacuum. For the range of frequencies $\omega_0 \gg |\omega_0 - \omega_{cp}| \gg k_0 v_{tp}$, the function I_+ can be approximated as

$$I_+(\zeta) \cong 1 + \frac{1}{\zeta^2} + \cdots - i\sqrt{\frac{\pi}{2}} \zeta \exp(-\zeta^2/2). \quad (5)$$

It follows from the dispersion equation (3) that the refractive index N is complex, i.e., $N = N_0 + i\eta$, where

$$N_0^2 = \frac{k_0^2 c^2}{\omega_0^2} = 1 - \frac{\omega_{pe}^2}{\omega_0 \omega_{ce}} - \frac{\omega_{pp}^2}{\omega_0(\omega_0 - \omega_{cp})}, \quad (6)$$

$$\eta = \sqrt{\frac{\pi}{8}} \frac{\omega_{pp}^2}{N_0 k_0 \omega_0 v_{tp}} \exp\left(-\frac{c^2(\omega_{cp} - \omega_0)^2}{2\omega_0^2 N_0^2 v_{tp}^2}\right). \quad (7)$$

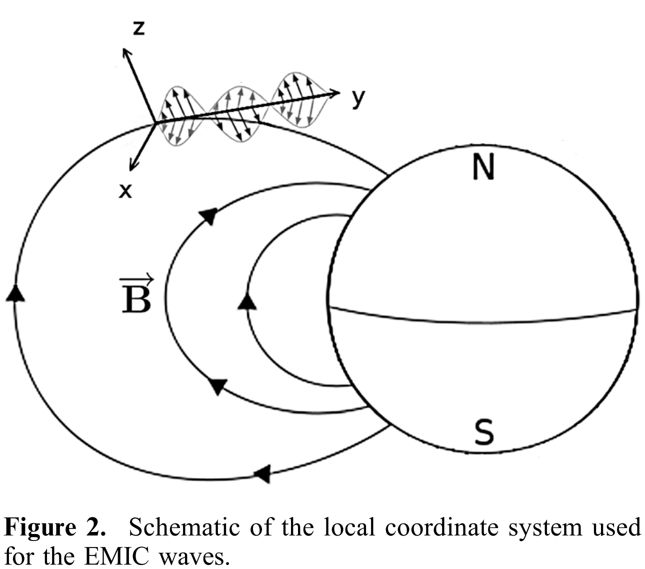

Figure 2. Schematic of the local coordinate system used for the EMIC waves.

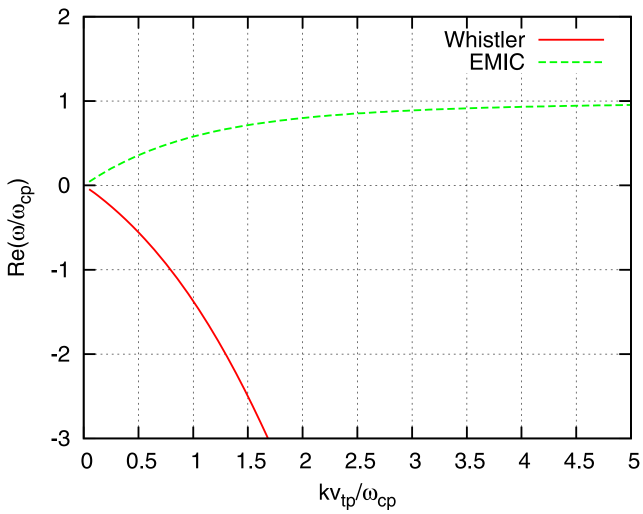

Figure 3. Dispersion curves for EMIC wave (upper dashed curve) and whistler wave (lower solid curve).

[8] Outside the resonance region $|\omega_{cp} - \omega_0| \gg k_0 v_{tp}$, the absorption of proton cyclotron waves is exponentially small. From equation (7), by maximizing η with respect to k_0, we find that the limit to the resonance occurs at $k_0 = |\omega_{cp} - \omega_0|/\sqrt{2}v_{tp}$. By using this expression for the frequency difference in the third term on the right-hand side of equation (6), which dominates near resonance ($\omega_0 \leq \omega_{cp}$), we obtain the maximum value for the refraction index as

$$N_{0\,\max} = \left(\frac{c}{\sqrt{2}v_{tp}} \frac{\omega_{pp}^2}{\omega_{cp}^2} \right)^{1/3}. \tag{8}$$

[9] Neglecting the wave absorption and solving equation (6) as the dispersion relation $N_0^2 = (k_0 c/\omega_0)^2$ for the frequency, we obtain the result shown in Figure 3. We used the following typical parameters: $L = 2$, $B_0 = 0.3\ L^{-3} \approx 0.04$ G, background densities (plasmasphere) $n_e = n_p = 10^4\ \mathrm{cm}^{-2}$, and energetic particle energies (ring current) $T_p = 10$ keV and $T_e = 1$ keV. The upper branch in Figure 3 is the dispersion curve for EMIC waves and the lower branch is for whistler waves. The EMIC waves are left-hand circularly polarized and resonate with energetic ions, whereas whistler waves are right-hand circularly polarized and resonate with high-energy electrons. The formation of coherent structures, including vortices and solitons, requires the balancing of the dispersion in the $\omega(k)$ waves with nonlinear terms in the wave dynamics, as is explained in sections 3 and 4.

[10] For the EMIC waves and solitons, the typical spatial scale is $c/\omega_{cp} \sim 500$ km (at $L = 2$) and the typical timescale is $(\omega_0 \Delta\omega)^{-1/2} \sim 30$ ms, with $\Delta\omega = |\omega_0 - \omega_{cp}|$. Hence the appropriate dimensionless space and time variables are $x' = x(\omega_{cp}/c)$ and $t' = t(\omega_0 \Delta\omega)^{1/2}$. The normalized electric field-related quantity (dimensionless) is $a = (\omega_0/\Delta\omega)(eA/m_p c^2)$, where A is the amplitude of the vector potential. In terms of a, the electric and magnetic field amplitudes are given by $E = (\Delta\omega m_p c/e)a$ and $\delta B = (\Delta\omega/\omega_0)E$.

3. Nonlinear Two-Component Fluid Description of EMICs

[11] In this section we will discuss the modulational instability that occurs when the waves under consideration propagate along the external geomagnetic field. The slow motion will be described in the MHD approximation. We will show that if the plasma has appropriate properties, the growth rates for the modulational instability become much higher at oblique propagation angles to B_{0y}. For ion cyclotron waves, we derive a two-dimensional nonlinear Schrödinger equation and show that in the stationary case, it has two naturally distinct solutions: for sufficiently small amplitudes, there exists a bright soliton solution, and for larger amplitudes, the amplitude of the electromagnetic waves oscillates. The generation of quasi-static magnetic fields and vortices and the acceleration of protons by these vortices are also discussed.

3.1. Vector Potential Description and the Pondermotive Force

[12] Here we will discuss the basic equations that describe the nonlinear modulation of electromagnetic ion cyclotron waves propagating along the magnetic field B_{0y}. We assume that the modulation frequency is much smaller than the ion (proton) gyrofrequency. Then the slow motion of the plasma can be described by the following time-averaged single-fluid MHD equations for the plasma mass density ρ and velocity $\mathbf{v}$, valid under the condition that the electrons and ions are isothermal, with temperatures such that $T_e \gg T_i$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{9}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{v_s^2 \nabla \rho}{\rho} - \frac{1}{4\pi\rho} \mathbf{B} \times \nabla \times \mathbf{B} + \frac{\mathbf{f}}{m_p}. \tag{10}$$

Here, $\mathbf{B}$ is the total magnetic field; $\mathbf{f}$ is the ponderomotive force incorporating the motion of the protons; $\rho = m_p n_p + m_e n_e \cong n_p m_p$ since $m_p \gg m_e$, where n_p is the proton density; and $v_s = \sqrt{T_e/m_p}$ is the proton sound velocity. The first term on the right-hand side of equation (10) represents the plasma pressure gradient contribution.

[13] In our consideration, the ponderomotive force $\mathbf{f}$ has two components [*Nishikawa et al.*, 1980],

$$f_x = -\frac{m_p \omega_0}{\omega_0 - \omega_{cp}} \frac{\partial}{\partial x} \frac{e^2 \mid E \mid^2}{m_p^2 \omega_0^2}, \tag{11}$$

$$f_y = -\frac{m_p \omega_0}{\omega_0 - \omega_{cp}} \frac{\partial}{\partial y} \frac{e^2 \mid E \mid^2}{m_p^2 \omega_0^2} + \frac{m_p k_0 \omega_{cp}}{(\omega_0 - \omega_{cp})^2} \frac{\partial}{\partial t} \frac{e^2 \mid E \mid^2}{m_p^2 \omega_0^2}, \tag{12}$$

where $|E| = (\omega_0/c)|A|$, with the electric field $\mathbf{E}$ being related to the vector potential $\mathbf{A}$ as $\mathbf{E} = -\frac{1}{c}(\partial \mathbf{A}/\partial t)$.

3.2. Nonlinear Schrödinger Equation for the Complex Envelope Function $A(x, y, t)$

[14] Now we will derive the nonlinear Schrödinger equation, using the Maxwell equation for the ion cyclotron waves,

$$\nabla^2 E_- - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_- = \frac{4\pi e n_p}{m_p c^2}\frac{\partial p_-}{\partial t}. \quad (13)$$

For our problem, the contribution of the electron current density is smaller than of that of the protons. Here $p_- = p_x - ip_z$ is the ion momentum due to a rapidly varying electromagnetic field. A simple calculation of p_- follows from the equation of motion, equation (10):

$$p_- = \left[\frac{ieA}{\omega - \omega_{cp}} + \frac{e\omega_{cp}}{(\omega - \omega_{cp})^2}\frac{1}{\omega_0}\frac{\partial A}{\partial t}\right]e^{i(k_0 y - \omega_0 t)}. \quad (14)$$

Recall that according to equation (2), we have assumed that all quantities have both fast and slow temporal-spatial scales.

[15] Substituting equation (14) into equation (13), we obtain for the ion cyclotron waves a nonlinear Schrödinger equation of the following form:

$$2i\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial y}\right)A + \frac{c^2}{\omega_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A - \frac{\omega_0 \triangle\omega}{\Gamma}A - \frac{\omega_{pp}^2\omega_0}{\Gamma(\omega_0 - \omega_{cp})}\frac{\delta n_p}{n_{0p}}A = 0, \quad (15)$$

where the group velocity is

$$v_g = 2\left(\frac{\omega_{cp} - \omega_0}{k_0}\right), \quad (16)$$

and

$$\Gamma = \frac{\omega_{pp}^2\omega_{cp}}{2(\omega_{cp} - \omega_0)^2}, \qquad \Delta\omega = \frac{k_0^2c^2 + \frac{\omega_{pp}^2\omega_0}{\omega_0 - \omega_{cp}} - \omega_0^2}{\omega_0}. \quad (17)$$

3.3. Parametric Decay Instability of Large-Amplitude Plane Waves

[16] In order to consider modulational instabilities (leading to the excitation of magnetosonic waves), we linearize equations (9)–(12) and (15) and search for plane wave solutions of the form $\exp[i(\mathbf{qr} - \Omega t)]$. We obtain the following dispersion equation:

$$\left[(\Omega - q_y v_g)^2 - \left(\frac{c^2q^2}{2\Gamma}\right)^2\right](\Omega^2 - \Omega_+^2)(\Omega^2 - \Omega_-^2) = B\left(\frac{eA}{m_pc^2}\right)^2, \quad (18)$$

where

$$B = \left[\left(\Omega^2 - q_y^2V_A^2\right)\left(q_y^2c^2 + \frac{q_yk_0\Omega\omega_{cp}c^2}{\omega_0(\omega_0 - \omega_{pi})}\right) + \Omega^2q_x^2c^2\right] \times \frac{2\omega_0^2}{(\omega_0 - \omega_{cp})^2}\frac{\omega_{pp}^2q^2c^2}{\Gamma^2}. \quad (19)$$

Here V_A is the Alfvén speed, and $q^2 = q_x^2 + q_y^2$. In equation (18), $\Omega_\pm$ are the fast and slow magnetosonic frequencies:

$$\Omega_\pm^2 = \frac{1}{2}q^2(V_A^2 + v_s^2) \pm \frac{q^2}{2}\sqrt{(V_A^2 + v_s^2)^2 - 4V_A^2v_s^2\cos^2\theta}, \quad (20)$$

where $\theta = \tan^{-1}(q_y/q_x)$.

[17] Equation (18) has several types of complex solutions. For example, when $\Omega \sim q_yv_g \approx \Omega_+$ or $\Omega \sim q_yv_g \approx \Omega_-$, we have the excitation of fast or slow magnetosonic waves by the electromagnetic waves. These instabilities also lead, in general, to the existence of two different solutions.

4. Coherent Nonlinear EMIC Structures

[18] In the previous section, we mentioned that the amplitude modulation of the electromagnetic waves leads to an excitation of magnetosonic waves whose amplitude grows exponentially. Eventually, the wave amplitude stops growing due to the influence of nonlinear terms that were ignored in the linear analysis. Now we take into account nonlinear terms due to the ponderomotive force, which redistributes the protons and changes the density of the plasma. The convective derivative term $(\mathbf{v}_p \cdot \nabla)\mathbf{v}_p$ continues to be ignored, at least as long as the wave does not steepen too much.

4.1. Envelope Solitons

[19] Here we show that there are soliton solutions for the nonlinear Schrödinger equation in equation (15). We look for solutions that propagate as $\xi = y - v_gt$ and assume that any dependence on this variable is faster than that on the spatial coordinate x or the time t. Hence for the ponderomotive force in equations (11) and (12), we assume that $|f_y| \gg |f_x|$. Thus we can simplify equations (11) and (12) and the ponderomotive force f_y may be written as

$$f_y = -m_pc^2\frac{\omega_{cp}}{\omega_{cp} - \omega_0}\frac{\partial}{\partial\xi}\frac{e^2 \mid E \mid^2}{m_p^2c^2\omega_0^2}. \quad (21)$$

In this limit, we obtain the density perturbation as follows:

$$\frac{\delta n}{n_0} = \frac{1}{(v_g^2 - v_s^2)}\frac{\omega_{cp}}{\omega_{cp} - \omega_0}\frac{e^2A^2}{m_p^2\omega_0^2}. \quad (22)$$

Substituting the expression of equation (22) into the nonlinear Schrödinger equation, equation (15), we obtain

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial x^2}\right)A - \frac{\omega_0\triangle\omega}{c^2}A + \frac{\omega_{pp}^2\omega_{cp}^2}{(v_g^2 - v_s^2)}\frac{1}{(\omega_{cp} - \omega_0)^2} \cdot \frac{e^2}{m_p^2c^2\omega_0^2}A^3 = 0. \quad (23)$$

Introducing the new variables $\xi' = \frac{c}{\sqrt{\omega_0 \triangle\omega}}\xi$ and $x' = \frac{c}{\sqrt{\omega_0 \triangle\omega}}x$ and the dimensionless function

$$U = \left(\frac{c^2}{\omega_0 \triangle\omega}\right)^{1/2} \frac{\omega_{pp}}{\sqrt{v_g^2 - v_s^2}} \frac{\omega_{cp}}{(\omega_{cp} - \omega_0)} \frac{e}{m_p c \omega_0} A, \qquad (24)$$

we can rewrite the nonlinear Schrödinger equation as

$$\left(\frac{\partial^2}{\partial \xi'^2} + \frac{\partial^2}{\partial x'^2}\right) U - U + U^3 = 0. \qquad (25)$$

[20] In general, equation (25) describes asymmetrical two-dimensional solitons. For the case of symmetrical solitons written in terms of the cylindrical coordinate $\varrho = \sqrt{\xi'^2 + x'^2}$, equation (25) has the form

$$\frac{1}{\varrho}\frac{\partial}{\partial \varrho}\left(\varrho \frac{\partial}{\partial \varrho} U\right) - U + U^3 = 0. \qquad (26)$$

The solutions of this equation have been described by *Gurevich and Shvartsburg* [1973], with the boundary conditions $U \to 0$ for $\rho \to \infty$ and either U = const or $dU/d\rho = 0$ at $\rho = 0$. The boundary condition $dU/d\rho = 0$ at $\rho = 0$ leads to U having discrete values U_n at $\rho = 0$. The fundamental mode has $U_1 = 2.2$; this is a soliton solution, peaked at $\rho = 0$ and falling off as $\rho^{1/2} e^{-\rho}$ as $\rho \to \infty$. The higher-order modes for $n > 1$ are damped oscillatory wave functions with $(n - 1)$ zero crossings and have initial values $U_2 = 3.3$, $U_3 = 4.1$, $U_4 = 4.6$, etc. Hence equation (26) has a soliton solution only for moderate initial values of U. For comparatively large initial values of U, this equation describes nonlinear stationary waves propagating with a group velocity v_g.

4.2. Vortex Solutions and the Acceleration of Ion Jets

[21] In this section we show that in a homogeneous plasma, electromagnetic ion cyclotron waves can lead to the generation of vortices and a quasi-static magnetic field. The acceleration of ions by these vortices are also considered. Previously, these problems have been considered in laser plasma physics by *Tsintsadze et al.* [2002, 2006].

[22] Now we will investigate the acceleration of protons (as light ions) by the vortex that is generated by the ponderomotive force. To derive the equation for vortices, we use the momentum equation for the proton species,

$$\frac{\partial}{\partial t}\left(m_p \mathbf{v} + \frac{e}{c}\mathbf{A}\right) = -e\nabla\varphi + \frac{e}{c}\mathbf{v} \times \mathbf{B} + \mathbf{v} \times (\nabla \times m_p \mathbf{v}) - \frac{1}{n_p}\nabla\mathcal{P} - \nabla\left(\frac{m_p v^2}{2}\right) + \mathbf{f}\ . \qquad (27)$$

The expression for $\mathbf{f}$ is given in equations (11) and (12). Here, $\mathcal{P}$ is the proton pressure, and $\mathbf{A}$ and φ are the vector and scalar potentials, respectively, related to the existence of the ponderomotive force $\mathbf{f}$:

$$\mathbf{B} = B_{0y}\hat{\mathbf{e}}_y + \nabla\times\ \mathbf{A}. \qquad (28)$$

Taking the curl of both sides of equation (27), for isothermal protons we obtain

$$\frac{\partial \mathbf{\Omega}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{\Omega}) + \nabla \times \mathbf{f}, \qquad (29)$$

where

$$\mathbf{\Omega} = \nabla \times \left(m_p \mathbf{v} + \frac{e}{c}\mathbf{A}\right) \qquad (30)$$

is the vorticity of the canonical ion momentum.

[23] If the characteristic spatial scale length and the characteristic timescale satisfy the inequality $l > vt$, we may then neglect the first term on the right-hand side in equation (29) compared to the term on the left-hand side and obtain the following simple relationship between the vorticity and the source:

$$\Omega = \frac{\omega_{cp}}{(\omega_{cp} - \omega_0)^2}\ \mathbf{k}_0 \times \nabla \frac{e^2 A^2}{m_p c^2}\ . \qquad (31)$$

We now introduce the canonical momentum circulation $\mathbf{G}$, defined as

$$\mathbf{G} = \oint_l \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right) \cdot d\mathbf{l} = \int_S \mathbf{\Omega} d\mathbf{S}. \qquad (32)$$

Here the integral is taken along a closed contour. Owing to the existence of the last term in equation (12), the flux of $\mathbf{\Omega}$ through a surface bounded by any closed ion fluid contour, as well as the canonical momentum circulation, is not conserved, namely,

$$\frac{d\mathbf{G}}{dt} = \frac{d}{dt}\oint \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right) \cdot d\mathbf{l} = \frac{\omega_{cp}}{(\omega_{cp} - \omega_0)^2} \frac{\partial}{\partial t} \oint_l \frac{e^2 A^2}{m_p c^2} \mathbf{k}_0 \cdot d\mathbf{r}. \qquad (33)$$

Hence if there are initially no vortices, they will be generated by the electromagnetic ion cyclotron waves over time.

[24] Note that since the wave number $\mathbf{k}_0$ is directed along the y-axis and the ponderomotive force is a function of (x, y), then the vorticity has only one component, Ω_z. Thus we can rewrite equation (29) as

$$\frac{d}{dt}\ln\frac{\Omega_z}{n} = \frac{k_0 \omega_{cp} m_p}{(\omega_{cp} - \omega_0)^2} \frac{1}{\Omega_z} \frac{\partial}{\partial t} \frac{\partial}{\partial x} \frac{e^2 A^2}{m_p^2 c^2}. \qquad (34)$$

Equation (34) is a generalization of the Hasegawa-Mima equation, and it clearly breaks the frozen-in condition [*Nishikawa et al.*, 1980], given that $d[\ln(\Omega_z/n)]/dt = 0$.

[25] A simple expression for the magnetic field that is generated by the nonstationary ponderomotive force can be obtained from equation (31) if we assume that $|\nabla \times m_p \mathbf{v}| \ll eB_z/c$. We then obtain

$$\frac{eB_z}{m_p c} = \frac{\omega_{cp} c^2}{(\omega_{cp} - \omega_0)^2} k_0 \frac{\partial}{\partial x}\left(\frac{eA}{m_p c^2}\right)^2. \qquad (35)$$

[26] Let us estimate this quantity for the following numerical values: $\omega_{cp} \sim 0.37 \times 10^3$ Hz, $\omega_0 - \omega_{cp} \approx k_0 v_{tp}$,

$\partial/\partial x \sim 1/L_{0x} \sim 1.6 \times 10^{-9}$ cm^{-1}, $k_0 \sim 10^{-7}$ cm^{-1}, $eA/m_pc^2 \sim 10^{-6}$, $v_{tp} \sim 10^5$ cm/sec. Substituting these parameters into equation (35), we obtain the following estimate for the normalized magnetic field perturbation

$$\frac{eB_z}{m_pc} \cong 3.7 \text{ Hz}, \quad (36)$$

or, in unnormalized form, $B_z \cong 0.5 \times 10^{-4}$ G.

[27] We now demonstrate that there exists a vortex ring that provides a mechanism for proton acceleration. Essentially, this is a consideration of the inverse problem, namely, that given a vortex, we can define the canonical fluid momentum **P** at any point of the plasma. To this end, we suppose that the plasma is at rest until the vortex is generated, $\mathbf{\Omega} = \nabla \times \mathbf{P}$. Let **D** be some vector for which $\nabla \cdot \mathbf{D} = 0$, and $\mathbf{P} = \nabla \times \mathbf{D}$. Then for **D** we obtain the following equation:

$$\nabla^2 D_z = -\Omega_z, \quad (37)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

[28] The solution of equation (37) in our two-dimensional case is

$$D_z = \frac{1}{2\pi}\int \Omega_z(x', y') \ln \frac{1}{\sqrt{(x-x')^2 + (y-y')^2}} dx'dy'. \quad (38)$$

To carry out the integration in equation (38), we suppose that $|A(x,y)|^2$ has the form of a step function,

$$|A|^2 = A_0^2 \Theta(x_0 - x)\Theta(y_0 - y) \ , \quad (39)$$

where the Heaviside function $\Theta(w) = 1$ if $w \geq 0$, and $\Theta(w) = 0$ when $w < 0$.

[29] In this case, the expression in equation (31) for the z component of the vorticity becomes

$$\Omega_z = Q\frac{\partial}{\partial x}|A|^2 = -Q\delta(x - x_0)\Theta(y_0 - y'), \quad (40)$$

where

$$Q_0 = \frac{\omega_{cp} m_p c^2}{(\omega_{cp} - \omega_0)^2} k_0 \left(\frac{eA_0}{m_p c^2}\right)^2. \quad (41)$$

Substituting equation (40) into the equation, equation (38), for the function D_z, we obtain the following integral:

$$D_z = \frac{1}{2} Q_0 \int_0^{y_0} dy' \ln\left((x_0 - x)^2 + (y - y')^2\right). \quad (42)$$

[30] Knowing the z component of the vector **D**, we can define the components of the canonical momentum:

$$P_x = \frac{\partial D_z}{\partial y} \quad \text{and} \quad P_y = -\frac{\partial D_z}{\partial x}. \quad (43)$$

Simple calculations with equation (43) lead to the following expressions for P_x and P_y:

$$P_x = \frac{Q_0}{2\pi} \frac{(x_0 - x)^2}{(x_0 - x)^2 + (y_0 - y)^2}, \quad (44)$$

$$P_y = \frac{Q_0}{\pi} \frac{(y_0 - y)(x_0 - x)}{(x_0 - x)^2 + (y_0 - y)^2}, \quad (45)$$

where x_0 and y_0 are the width and length of the vortex belt, respectively.

[31] Let us consider the limiting case of $y = x = 0$ and suppose that $|(x_0 - x)| \ll |(y_0 - y)|$. Then, from equations (44) and (45), we derive the following expressions:

$$P_x \cong \frac{Q_0}{2\pi}\left(\frac{x_0}{y_0}\right)^2 \quad \text{and} \quad P_y \cong \frac{Q_0}{\pi}\frac{x_0}{y_0}, \quad (46)$$

where Q_0 is defined by equation (41).

[32] The vortex structures here derived could be used to compute the extent of the acceleration of ions by following particle orbits in the wavefields. We defer this numerical calculation to a future publication.

5. Summary

[33] In this paper we have studied the nonlinear interaction of the electromagnetic ion cyclotron frequency waves with plasma particles in the inner magnetosphere. The emission is considered to be circularly polarized electromagnetic waves propagating along the almost constant dipole geomagnetic field in the equatorial region of the inner magnetosphere. We studied excitation of the magnetosonic waves through the amplitude modulation of the electromagnetic ion cyclotron waves, and obtained a two-dimensional nonlinear Schrödinger equation for the EMIC waves. In the stationary case, we found two solutions of the nonlinear Schrödinger equation with distinct natures. For sufficiently small amplitudes of the EMIC field, there exists a two-dimensional bright soliton, whereas for larger amplitudes the solution is oscillating. The generation of both vortices and of a quasi-static magnetic field across the geomagnetic field lines was discussed. The possible relationship of EMIC waves and their nonlinear properties to various geophysical source mechanisms is a subject for future studies.

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