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Static Analysis of Functionally Gradient Material Plate with various Functions

Bhandari Manish[1], Purohit Kamlesh[2] and Sharma Manoj[1]
[2]Jai Naraine Vyas University, Jodhpur, Rajasthan, INDIA
[1]Jodhpur Institute of Technology, Jodhpur, Rajasthan, INDIA

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Abstract

Functionally gradient materials are one of the most widely used materials in various applications because of their adaptability to different situations by changing the material constituents as per the requirement. Nowadays it is very easy to tailor the properties to serve specific purposes in functionally gradient material. Most structural components used in the field of engineeringcan be classified as beams, plates, or shells for analysis purposes. In this paper static analysis of functionally gradient material plate is carried out by sigmoid law and verified with the published results. The plate is modeled in step wise variation of the properties in thickness direction. The convergence study of the results is optimized by changing the mesh size and layer size. Power law and exponential laware applied for the same material and set of conditions. Results have been presented comparing with each other and the published results.

Keyword: Functional composites, elastic properties, finite element analysis (FEA)

Introduction

A huge amount of published literature has been observed for evaluation of thermomechanical behavior of functionally gradient material plate using finite element techniques. It includes both linearity and non linearity in various areas. A few of published literature highlights the importance of topic. E. J. Barbero and J. N. Reddy introduced a laminate theory for a desired degree of approximation of the displacements through the laminate thickness, allowing for piecewise approximation of the in-plane deformation through individual laminae. The solutions are compared with the 3-D elasticity solutions for the simply supported case[1]. G. Bao and L. Wang found that under mechanical loading the effect ofdifferent gradations on the crack driving force is relatively small[2]. S. Suresh and A. Mortensen focused on the processing of functionally graded metal-ceramic composites and their thermo mechanical behavior. They discussed various approximations for determination of properties and their limitations. They focused on various issues related to functionally gradient material manufacturing[3]. G. N. Praveen and J.N. Reddy reported the static and dynamic response of the functionally graded material plates by varying the volume fraction of the ceramic and metallic constituents using a simple power law distribution. Deflection and stresses under thermomechanical loadinghave been reported[4]. J.N. Reddy reported theoretical formulations and finite element analysis of the thermomechanical transient response of functionally graded cylinders and plates with nonlinearity. Numerical results of the deflections, temperature distributions and stress distributions in the cylinder and plates have beenpresente. The problems were studied by varying the volume fraction of a ceramic and a metal using power law distribution[5]. J. N. Reddy gave Navier'ssolutions of rectangular plates and Finite element models based on the third-order shear deformation plate theory for functionally graded plates. The formulation accounts for the thermomechanical coupling, time dependency and von Karman-type geometric non-linearity to show the effects of volume fractions and modulus ratio of the constituents on deflections and transverse shear stresses[6]. J.N. Reddy reported three-dimensional thermomechanical deformations of simplysupported functionally graded rectangular plates. The temperature, displacements and stresses of the plate were computed for different volume fractions of the ceramic and metallic constituents[7]. Jin and Paulino, Power-law function and exponential function are commonly used to describe the variations of material properties of FGMs[8]. However, in both power-law and exponential functions, the stress concentrations appear in one of the interfaces in which the material is continuous but rapidly changing. Therefore, Chung and Chi proposed a sigmoid FGM which is composed of two power-law functions to define a new volume fraction. They indicated that the use of a sigmoid FGM can significantly reduce the stress intensity factors of a cracked body[9]. Bhavani V. Sankar solved the thermoelastic equilibrium equations for a functionally graded beam in closed-form to obtain the axial stress distribution by keeping the Poisson ratio constant. The stresses were calculated for the cases for which the elastic constants vary in the same manner as the temperature and vice versa. The residual thermal stresses are greatly reduced, when the variation of thermoelastic constants are opposite to that of the temperature distribution[10]. Senthil S. Vel and R.C. Batra developed an analytical solution for three-dimensional thermomechanical deformations of a simply supported functionally graded rectangular plate subjected to time-dependent thermal loads[11]. M. Tahani1, M. A. Torabizadeh and A. Fereidoon, reported analytical method to analyze displacements and stresses in a functionally graded composite beam subjected to transverse load and the results obtained from this method were compared with the finite element solution done by ANSYS[12]. Ki-Hoon Shin suggested that the Finite Element Analysis (FEA) is an important step for the design of structures or components formed by heterogeneous objects such as multi-materials, Functionally Graded Materials (FGMs), etc.[13]. Shyang-Ho Chi and Yen-Ling Chung, used sigmoid function to model[14] and analyze[15] FGM plate of various Young's modulus. Hui Wang, Qing-Hua Qin (2008) determined a meshless algorithm to simulate the static thermal stress distribution in two-dimensional functionally graded materials[16]. GH.Rahimi and AR. Davoodinik studied the thermal behavior of functionally graded beam (FGB)[17]. FatemehFarhatnia, Gholam-Ali Sharifi and SaeidRasouli, determined the thermo-mechanical stress distribution for a three layered composite beam having a middle layer of functionally graded material (FGM), by analytical and numerical methods. They found that there is no considerable difference, between stress profiles obtained analytically, from FEM model and ANSYS[18]. S. Momennia and A. H. Akbarzadeh (2011) determinedmechanical behavior of rectangular and circular plates made of functionally graded materials (FGMs). The analysis was based on the finite element approach using Abaqus, and those analytical solutions reported in the literature[19]. M.K. Singha, T.Prakash and M.Ganapathi reported the nonlinear behaviors of functionally graded material (FGM) plates under transverse distributed load. The effective material properties are then evaluated based on the rule of mixture[20]. Mohammad Talha and B N Singh reported formulations based on higher order shear deformation theory with a considerable amendment in the transverse displacement using finite element method (FEM). Convergence and comparison studies have been performed to demonstrate the efficiency of the present model[21]. Srinivas. G and Shiva Prasad. U simulated traditional composites under thermal loads[22]. In present paper static analysis of functionally gradient material plate is carried out by sigmoid law and verified with the published results. The plate is modeled in step wise variation of the properties in thickness direction. The convergence study of the results is optimized by changing the mesh size and layer size. Power law and exponential law are applied for the same material and set of conditions. Results have presented comparing with each other and the published results.

Material properties

An FGM can be defined by the variation in the volume fractions. Volume fraction and effective material properties of FGM's may vary in the thickness direction or in the plane of a plate. Most researchers use the Power-law function, Sigmoid function or Exponential function to describe the volume fractions.

Power Law: Volume fraction and material properties of FGM's may vary in the thickness direction or in the plane of a plate. The FGM modeled usually is done with one side of the material as ceramic and the other side as metal. A mixture of the two materials composes the through-the-thickness characteristics. This material variation is dictated by a parameter called volume fraction exponent, *n*. At n = 0 the plate is a fully ceramic plate while at $n = \infty$ the plate is fully metal. Material properties are dependent on the value of 'n' and the position in the plate.

Here we assume that the material property gradation is through the thickness and we represent the profile for volume fraction variation by the expression of power law, i.e.

$$P(z) = (P_t - P_b)V + P_b \quad (1)$$

$V_f=(z/h+1/2)^n$

At bottom layer, (z/h)=-1/2hence $V_f=0$, and $P(z) = P_b$

At top layer, (z/h)=+1/2hence$V_f=1$, hence $P(z)= P_t$

Where P denotes a generic material property like modulus, P_t and P_b denote the property of the top and bottom faces of the plate respectively, h is the thickness of the plate and n is a parameter that dictates the material variation profile through the thickness. Variation of volume fraction V_f through plate thickness for various values of the volume fraction exponent 'n' is shown in figure-1.

Sigmoid Law: In the case of adding an FGM of a single power-law function to the multi-layered composite, stress concentrations appear on one of the interfaces where the material is continuous but changes rapidly. Therefore, Chung and Chi defined the volume fraction using two power-law functions to ensure smooth distribution of stresses among all the interfaces [8]. The two power law functions are defined by:

$$g_1(z)=1-\frac{1}{2}\left(\frac{\frac{h}{2}-z}{\frac{h}{2}}\right)^p \quad \text{for } 0\le z\le h/2$$

$$g_2(z)=\frac{1}{2}\left(\frac{\frac{h}{2}+z}{\frac{h}{2}}\right)^p \quad \text{for } -h/2\le z\le 0$$

By using the rule of mixture, the Youngs modulus of the S-FGM can be calculated by:

$$E(z)=g_1(z)E_1+[1-g_1(z)]E_2 \text{ for } 0\le z\le h/2$$
$$E(z)=g_2(z)E_1+[1-g_2(z)]E_2 \text{ for } -h/2\le z\le 0 \quad (2)$$

Exponential Law: Many researchers used the exponential function to describe the material properties of FGMs as follows [14]

$$E(z)=E_2 e^{\frac{1}{h}\ln\left(\frac{E_1}{E_2}\right)(z+h/2)} \quad (3)$$

The material distribution in the thickness direction of the E-FGM plates is plotted in figure -3.

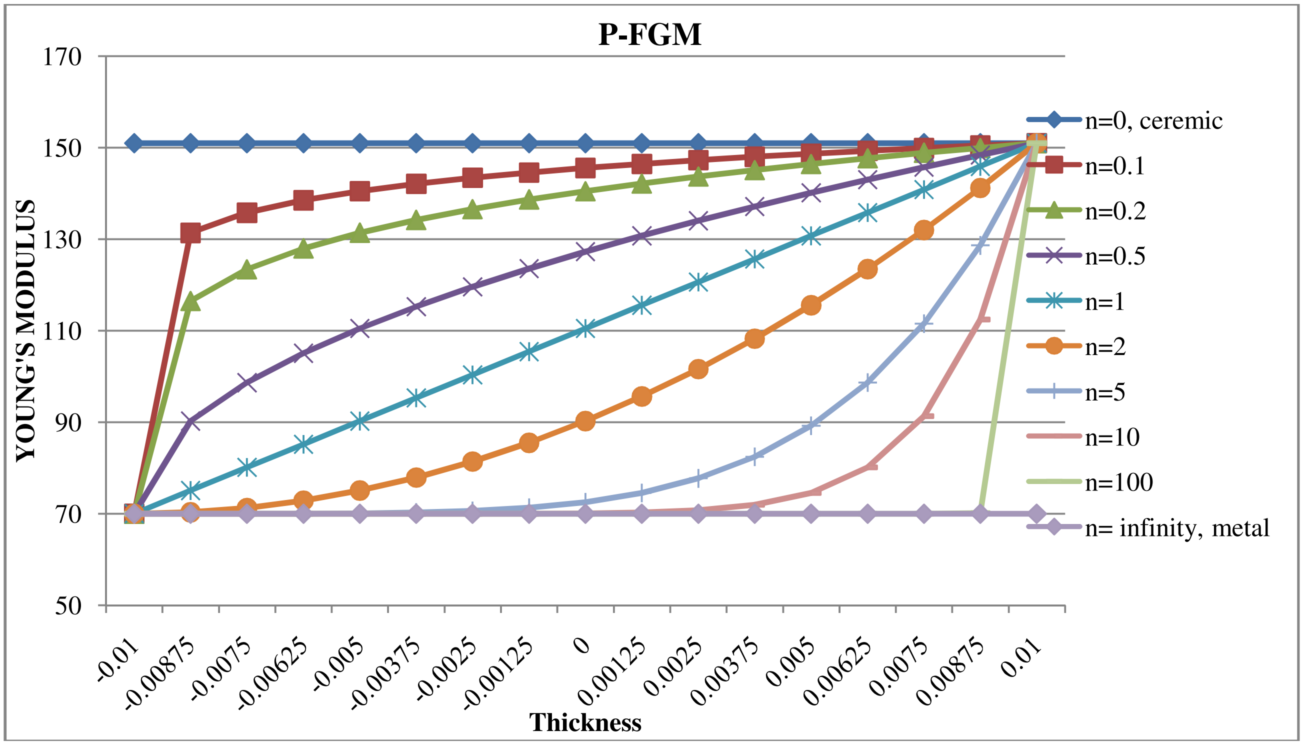

Figure-1
Variation of volume fraction V_f through plate thickness for various values of the 'n' for power law function

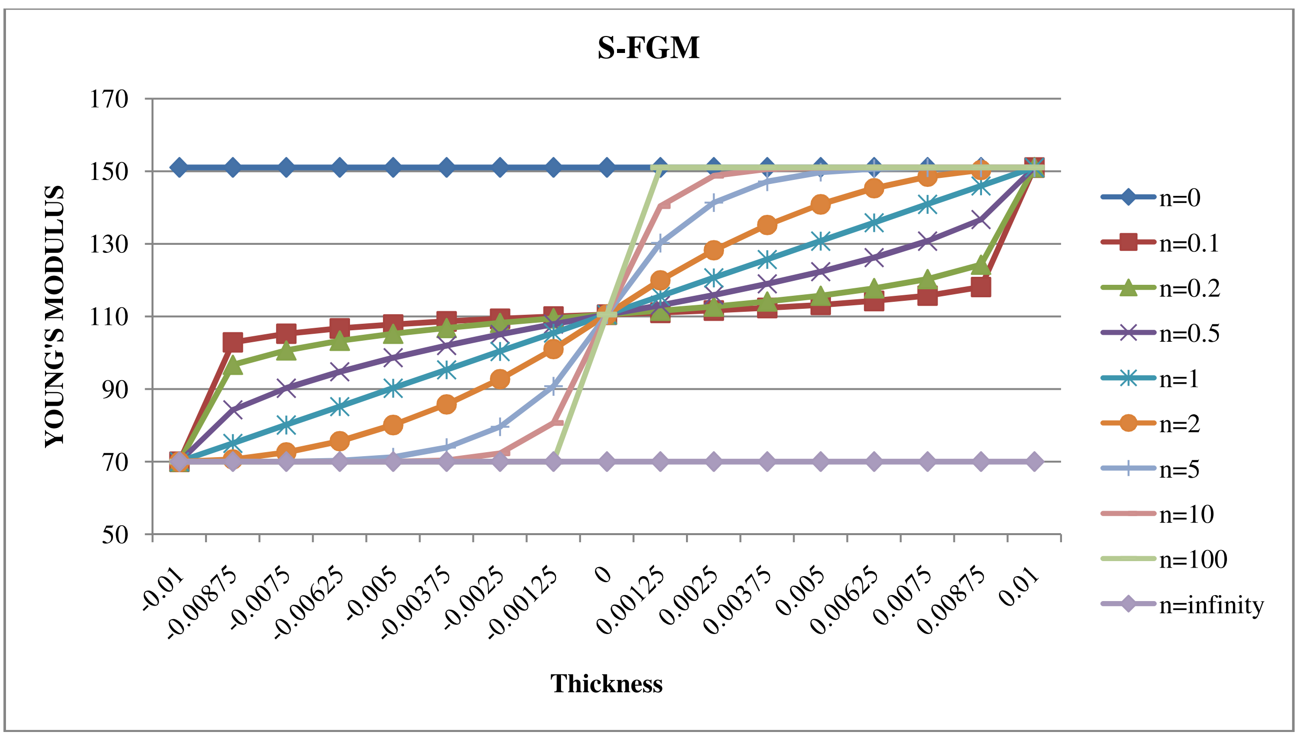

Figure-2
Variation of volume fraction V_f through plate thickness for various values of the 'nfor sigmoid function

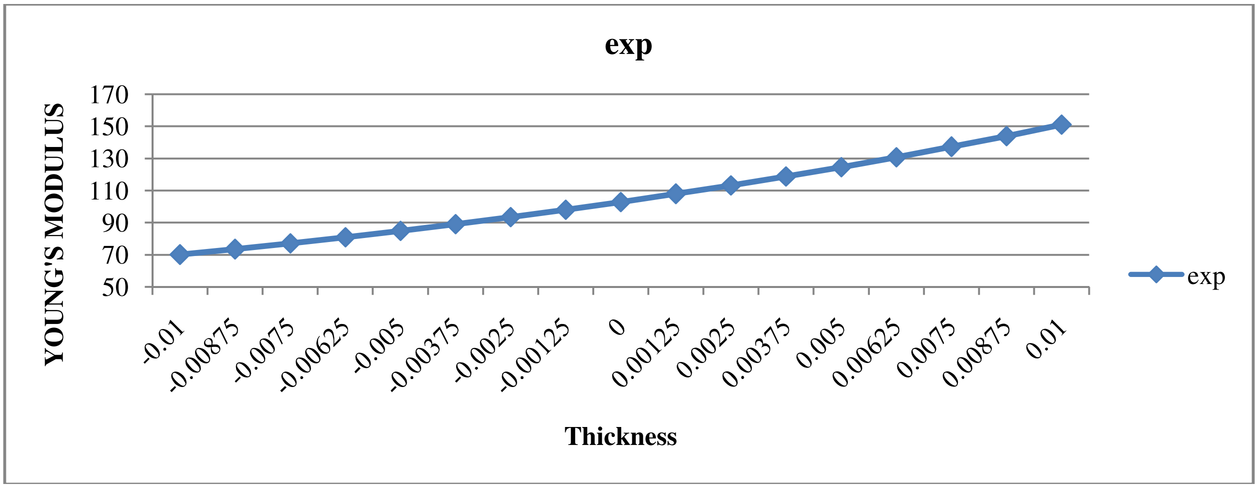

Figure-3
Variation of volume fraction V_f through plate thickness for exponential function

Plate modeling, meshing and boundary conditions

A square plate simply supported at all of its edges is modeled and meshed using the ANSYS APDL mesh tool. Since the plate modeled throughout this work is simply supported at its edges, hence the boundary conditions are as follows:

$u_0 = w_0 = \psi_y = \alpha_x = \alpha_z = \beta_y = \theta_x = 0$, at x = 0 and a
$v_0 = w_0 = \psi_x = \alpha_y = \alpha_z = \beta_x = \theta_y = 0$, at y = 0 and b

Here, 'a'and 'b'refer the length and width of the plate, respectively. Using the AnsysAPDL tool, the model along the thickness is divided into the number of layers desired. FGM plate is modeled with 8 layers and a mesh count ofsize 50×50 along the x-y plane. Once the model is meshed; the model is modified in order to create layers with different material properties. The material properties are then assigned to the respective layers defined along the thickness. It is to be noted that each layer is isotropic in nature.

Numerical examples and discussions: In this section we present several numerical simulations, in order to assess the behavior of functionally graded plates subjected to mechanical loads. The obtained results are compared with those given by Shyang-Ho Chi, Yen-Ling Chung[15]. The plate of thickness (h) 0.02 m with the varying aspect ratio from 0.25 to 8 considered. The plate is simply supported on its four sides and subjected to uniform loading (q_0) of 1 MPa. Poisson's ratio is kept constant i.e. 0.3. The Young's modulus of the plate at bottom (E_1) is 210 GPaand at the top of the plate (E_2) is 21 GPai.e. $E_1/E_2 = 10$ is assumed. The analysis is performed for fix values of the volume fraction exponent i.e. n=2. The results are presented in terms of non-dimensional stress and maximum deflection.

Non dimensional maximum deflection is($\overline{umax}$) = (u_{max}/h)
And non dimensional stress is ($\overline{sxx}$) = (s_{xx}/q_0).

Convergence Study: To make certain the accuracy and proficiency of the present finite element formulation, convergence (mesh convergence and layer convergence) studies have been carried out for static loading of the FGM plates. The convergence study with respect to varying number of layers is shown in Table 1. The FGM plate is modeled considering 4, 8 and 16 layers with a mesh size of 50×50. In table-1 the results for transverse deflection, stress and strain are presented. The difference between the two results for transverse deflection and tensile stress is below 1.5%. It can be observed that as the number of layers increases, the obtained results converge towards the published results. Since higher number of layers gives better results, 16 layers will be used in further work.

Table-1
Layer Convergence for S-FGM with 4, 8 and 16 no. of layers

Parameters	4	8	16	Published data [15]	Difference (%)
u_x, u_y (E-05)	6.88	7.99	8.30	8.20	-1.22
u_z (E-03)	6.77	7.26	7.49	7.38	-1.49
s_x, s_y (E+08)	1.20	1.30	1.33	1.35	1.48
e_x, e_y (E-04)	4.25	4.39	4.49	4.44	-1.12

Table-2
Mesh Convergences for S-FGM with various mesh counts

Parameters	6X6	10X10	20X20	50X50	100X100	Published data [15]	Difference(%)
u_x, u_y (E-05)	7.97	7.95	7.96	7.99	8.28	8.20	-0.97
u_z (E-03)	7.20	7.21	7.22	7.26	7.28	7.38	-1.36
s_x, s_y (E+08)	1.21	1.26	1.29	1.30	1.31	1.35	0.29
e_x, e_y (E-04)	4.10	4.27	4.35	4.39	4.41	4.44	0.68

In table-2 the results for transverse deflection, stress and strain are presented for various mesh size. The difference between the two results for transverse deflection is below 1.5% and the difference between the two results for tensile stress is below 0.5% which shows the agreement between the two results is excellent. The results show that the performance of the present formulation is very good in terms of solution accuracy. It can be observed that as the mesh size increases, the obtained results converge towards the published results. Hence finer mesh will help in attaining accurate results. Since higher mesh gives better results, 100X100 mesh is considered for further work.

After convergence studies a regular 100 X 100 mesh in a full size plate and no of layers 16 are chosen.

Comparative study: The comparison of present results with published results is shown in following figures: figure-4, 5 and 6 show the variations of nondimensional deflection and nondimensional tensile stress (s_x and s_y).

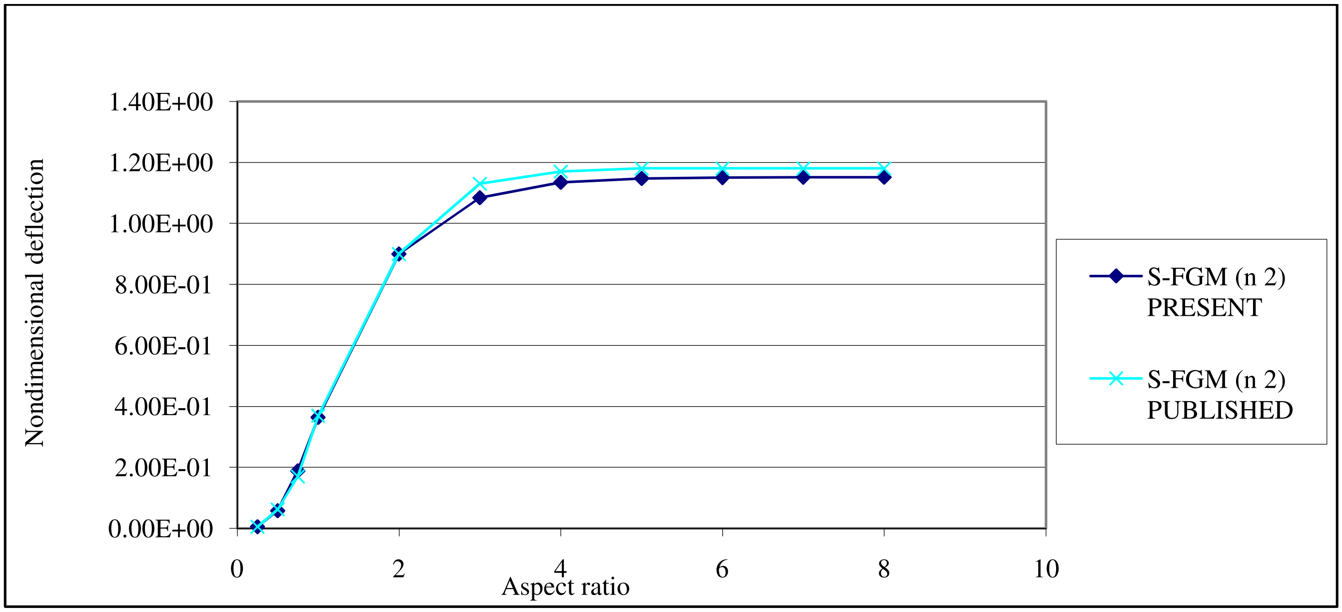

Figure-4
Verification of Non dimensional Deflection using sigmoid law (forn=2)

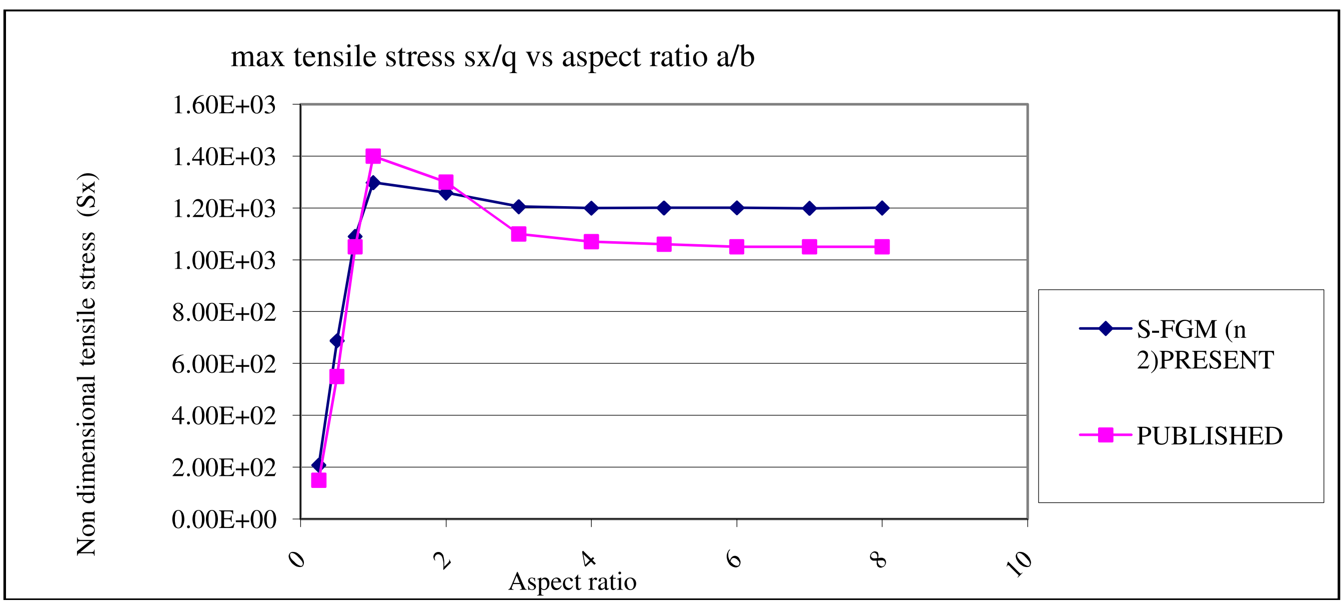

Figure-5
Verification of Non dimensional tensile stress (S_x/q) using sigmoid law (for n=2)

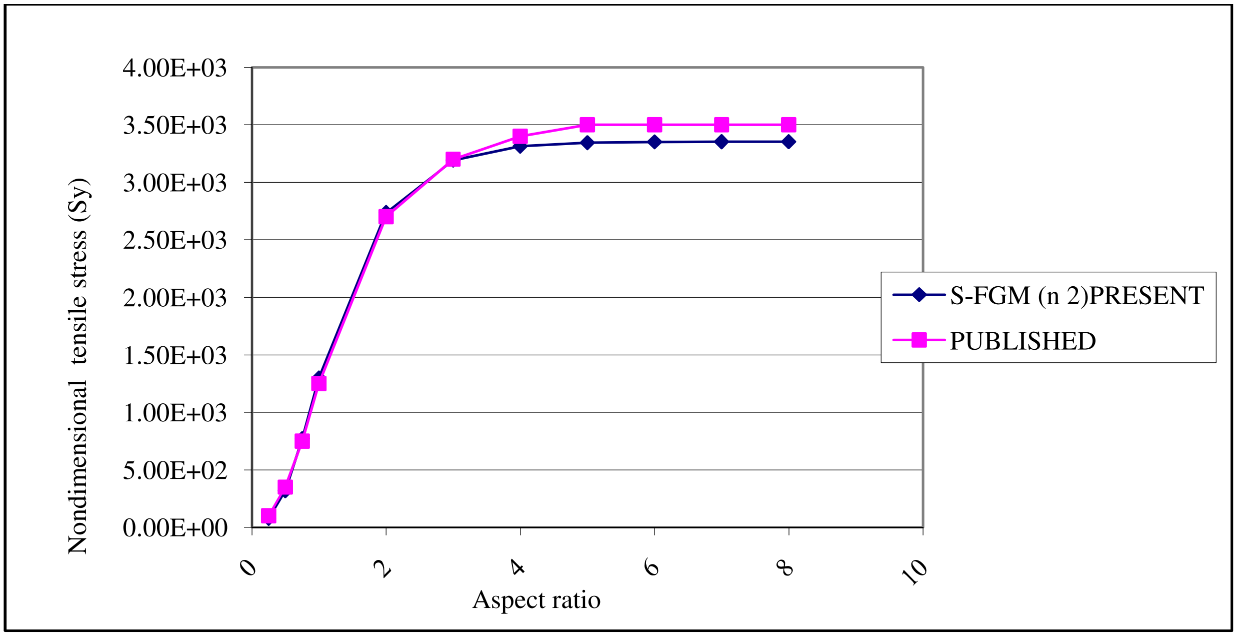

Figure-6
Verification of Non dimensional tensile stress (S_y/q) using sigmoid law (for n=2)

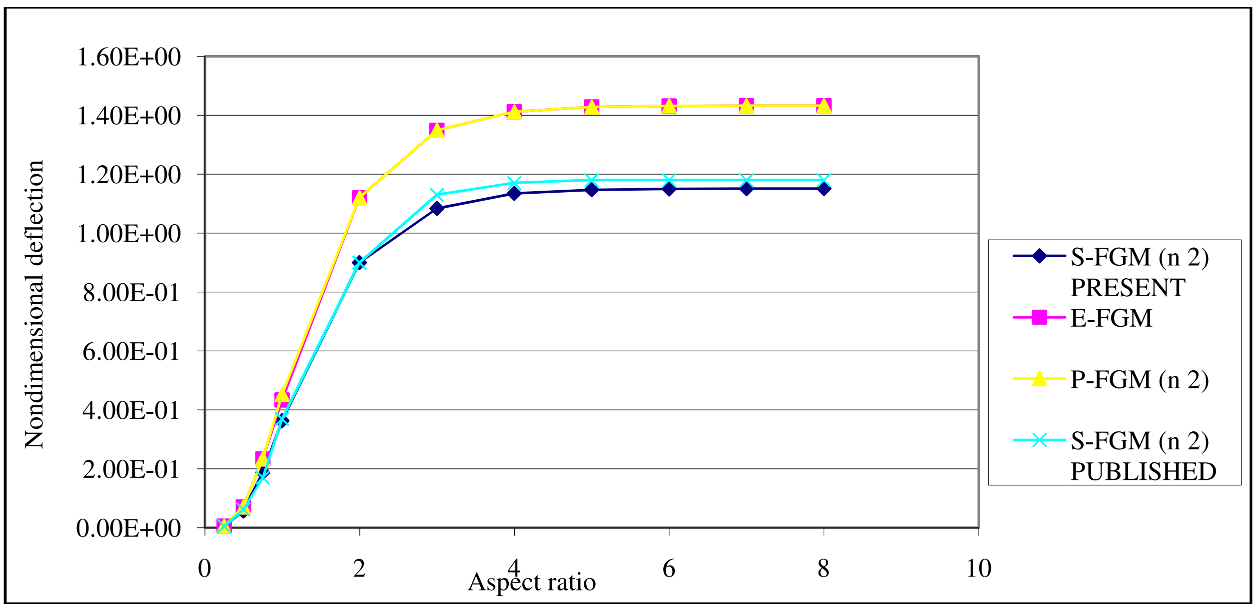

Figure-7
Non dimensional Deflection using sigmoid law, power law, E law and Published (for n=2)

An excellent agreement between the present and published results can be observed. The results show that the performance of the present formulation is very good in terms of solution accuracy.

Results achieved by power and exponential law

The effect of aspect ratio on mechanical behavior of FGM plate has been investigated. Both power law (n=2) and exponential law have been applied for the analysis.The relation between maximum deflection and aspect ratio for various FGM laws has been illustrated in figure-7. The maximum deflection increases upon increasing the aspect ratio, upto the aspect ratio 3. However when aspect ratio is increased beyond 3 the maximum deflection of FGM plate becomes a constant.

To understand the effect of aspect ratio on stresses $(S_{xx})_{max}$ and $(S_{yy})_{max}$ at the centre of the plate with varying aspect ratio for various FGM laws, results are plotted in figure-8 and figure-9. The maximum stress (S_{xx}) increases up to maximum value upto

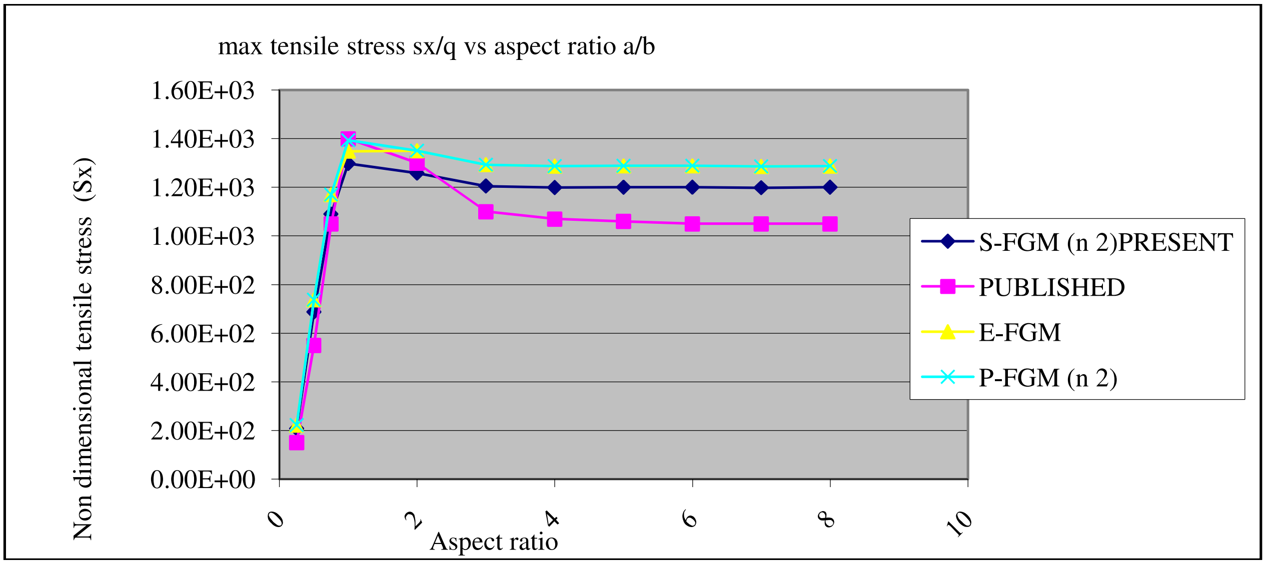

Figure-8
Non dimensional tensile stress (S_x/q) using sigmoid law, power law, E law and Published (for n=2)

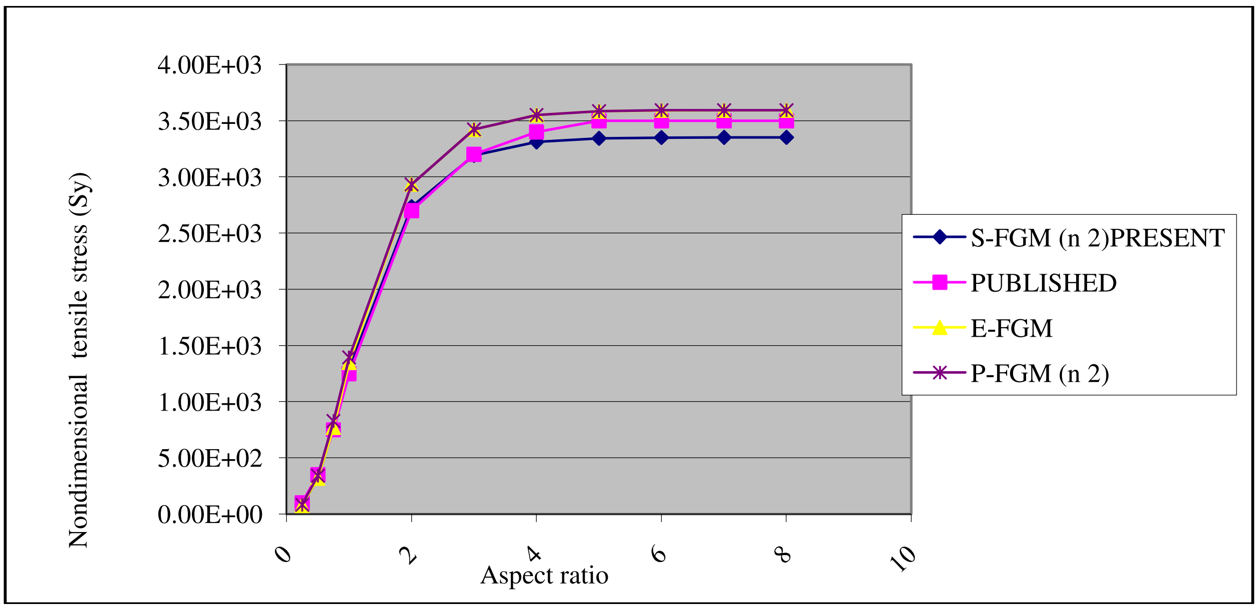

Figure-9
Non dimensional tensile stress (S_y/q) using sigmoid law, power law, E law and Published (for n=2)

aspect ratio 1, again reduces and becomes constant for aspect ratio a/b>5. Similarly the maximum stress (σ_y) increases upto aspect ratio 5 and becomes constant for aspect ratio greater than 5.

Conclusion

In this paper static analysis of functionally gradient material plate is carried out by sigmoid law and verified with the published results. Also power law and exponential law are applied for the same material and set of conditions to achieve the results.It is observed that the value of results is varying approximately 2% compared to the published results.It is also observed that ANSYS 13.0 program used in analysis gives closeresults as in MARC program which has been used in published results. It is found that for material index (n) 2, the power law and exponential law gives same results. The maximum Non dimensional stress (S_{xx}/q) of the FGM plates is located at the aspect ratio (a/b) 1, of the plate and then it is having constant trend. The maximum Non dimensional stress (S_{yy}/q) of the FGM plates is located at the aspect ratio>5 of the plate and then it is having constant trend. All the functions i.e. Power Law, Exponential and Sigmoid give same trend for analysis of FGM plate. The plate modeled here was a step-wise graded structure continuously varying of properties in graded structure is need to be calculated. Also the different boundary conditions and different loading conditions can be added to increase the accuracy of results. Further a thermal and thermo mechanical loading can be performed.

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