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State Estimation Based on Kinematic Models Considering Characteristics of Sensors

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Abstract— **The major benefit of the state estimation based on kinematic model such as the kinematic Kalman filter (KKF) is that it is immune to parameter variations and unknown disturbances and thus can provide an accurate and robust state estimation regardless of the operating condition. Since it suggests to use a combination of low cost sensors rather than a single costly sensor, the specific characteristics of each sensor may have a major effect on the performance of the state estimator. As an illustrative example, this paper considers the simplest form of the KKF, i.e., the velocity estimation combining the encoder with the accelerometer and addresses two major issues that arise in its implementation: the limited bandwidth of the accelerometer and the deterministic feature (non-whiteness) of the quantization noise of the encoder at slow speeds. It has been shown that each of these characteristics can degrade the performance of the state estimation at different regimes of the operation range. A simple method to use the variable Kalman filter gain has been suggested to alleviate these problems using the simplified parameterization of the Kalman filter gain matrix. Experimental results are presented to illustrate the main issues and also to validate the effectiveness of the proposed scheme.**

I. INTRODUCTION

One of fundamental issues in designing mechanical servo systems is how we can estimate the missing states accurately and robustly. Existing state estimation methods often rely on the physical parameters of the plant such as the inertia and the damping to design a state observer. This means that the observer is essentially prone to unexpected errors such as parameter variations and unknown external disturbances. As a result, the robustness and the performance of the closed loop system is generally not guaranteed in the observer-based state feedback control [1]. Further improvement may be achieved through an extensive process to refine the physical model and to identify external disturbances so that we can maximize the robustness margins, thereby extending the limit of performances. Originating from the earlier work in [2], there also have been numerous attempts, to come up with a proper strategy to trade off between the noise rejection and the robustness margin in the observer-based state feedback scheme. However, the fundamental limitation still exists because a physical model cannot be exact in reality.

On the other hand, if other signals can be measured such as the acceleration of the motor shaft or the inertial measurements of the end effector of a robot, the estimator design can be approached from a fundamentally different point of view. Clearly, the more sensors we use, the less model parameters we need. Less dependence on model parameters, in turn, means more robustness of the control and estimation algorithms against parameter uncertainties and external disturbances. So, the performance is not necessarily sacrificed for robustness in such sensor-based approach. From state estimation point of view, one of simple examples of such approaches is to use the (exact) kinematic model instead of (uncertain) physical models so that we can formulate the estimation problem under the ideal condition that optimal estimation algorithms such as the Kalman filter are guaranteed to be optimal. This type of Kalman filter is called the kinematic Kalman filter (KKF) [3][4]. In this approach, the sensor measurements are used not only as output from the system but also as the input to the system. All the information engaged in the estimation process is obtained from sensors, so it is a sensing-rich approach [5]. Besides the main advantage of robust and accurate state estimation, the sensing rich approach such as the KKF suggests that, instead of using a single expensive sensor, we can get more benefits from using multiple low cost sensors without losing the major performance [3]. In fact, the state estimation based on kinematic model has been the main framework in the aided navigation [6] and human motion tracking [7][8] where the system model is described by the kinematics of a particle or a rigid body.

Kinematic model itself is indeed an exact model. However, in reality, the sensor signals engaged in the KKF cannot be ideal due to the limitations and/or characteristics of each sensor used. There are two types of sensors comprising the KKF, which are the absolute position sensor (such as encoder and vision) and inertial sensors (such as accelerometers and gyroscopes). The former is more related to the quantization effect while the latter is more to the bandwidth and noise covariance. Especially, there is an inherent constraint on the signal bandwidth of low cost inertial sensors due to the on-board low pass filter for signal conditioning, which, in fact, realizes the trade off between the signal bandwidth and the noise density of the measured signal. Even though we set the signal bandwidth of the inertial sensor very high (thus sacrificing the noise density), it brings out unnecessary phase lag over the wide frequency range below the cut-off frequency. On the other hand, the absolute position sensor typically has a quantization error and we approximate it with Gaussian white noise in designing the KKF. However, we may encounter situations where this assumption is severely violated, especially when the frequency content of the motion is very low (or the motor is moving very slowly) resulting in the quantization noise of the encoder close to a deterministic

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signal rather than a white noise. This paper addresses these two practical issues in implementing the KKF, i.e., 1) the limitation on the signal bandwidth of the inertial sensors, which results in the excessive phase lag at fast (or high frequency) motion and 2) the manifestation of non-whiteness of the quantization error at slow (or low frequency) motion. It should be noted that the explicit consideration of the sensor characteristics is also one of major issues in systems comprised of large number of low cost sensors such as the sensor network [9][10].

This paper is organized as follows. The velocity estimation using the KKF will be briefly reviewed in Section II. Then Section III explains two practical issues in implementing the KKF. First, the effect of the signal bandwidth is introduced in Subsection III-A and then the problem with the constant (steady state) Kalman filter gain is elaborated in Subsection III-B with some illustrative results from the experiment. To address these issues, a simple strategy to use a variable filter gain is considered in Section IV and finally, the conclusions are given in Section V.

II. Velocity Estimation Based on Kinematic Model: KKF

If the motor encoder is the only measurement available, the simplest method to get the velocity is to numerically differentiate the encoder counts, e.g. in the simplest case,

$$v(k) = \frac{\theta(k) - \theta(k-1)}{T_s}. \tag{1}$$

where $\theta(k)$ is the position at the time step k and T_s is the sampling interval. At high speeds, this method may provide a relatively accurate estimate of the velocity, but at low speeds the estimate becomes highly unreliable. At extremely low speeds, a better approach is to time the interval between two consecutive encoder pulses at the expense of a large phase lag. Such a numerical differentiation is very simple, yet practically important, there have been continuous interests in further improving the accuracy of the estimation of the velocity. The majority of methods in this approach depends on the intelligent use of various filtering and timing mechanisms. The most recent results and reviews can be found in [11] and [12]. The velocity may also be estimated using the model-based state estimation theory [13][14][15][16]. As mentioned above, in the model-based approaches, model parameters and external disturbances must be accurately known for the estimate of velocity to be accurate, which is not trivial. In any case, a high resolution encoder can provide a more accurate velocity estimate, but it greatly increases the implementation cost if we want very high accuracy at low speeds. This is the reason why some of high precision motion control systems use encoders with a resolution much higher than necessary to satisfy the accuracy requirement for positioning.

In contrast, if we have an access to the acceleration signal (potentially noisy), then we can directly use the exact kinematic model instead of the uncertain physical model. The kinematic model relates the angular acceleration $\alpha(t)$ to the position $\theta(t)$ by

$$\ddot{\theta}(t) = \alpha(t). \tag{2}$$

Considering the real angular acceleration $a(t)$ as the sum of the measurement $\bar{a}(t)$ and its noise component $n_a(t)$, i.e.

$$\alpha(t) = \bar{a}(t) + n_a(t) \tag{3}$$

a state space representation of the kinematic model has the acceleration as an input and the encoder measurement as the system output. Since the encoder measurements are obtained only intermittently, it is best to describe the kinematic model in the discrete-time domain and (2) can be rewritten using the zero order hold as

$$x(k+1) = Ax(k) + B(\bar{a}(k) + n_a(k)) \tag{4a}$$

$$y(k) = Cx(k) + q_\theta(k) \tag{4b}$$

$$A = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \; B = \begin{bmatrix} \frac{T_s^2}{2} \\ T_s \end{bmatrix} \; C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{4c}$$

where $q_\theta(k)$ is the quantization noise of the position encoder. The accelerometer noise $n_a(k)$ is correctly modelled as a zero mean Gaussian white noise since it comes from electrical noise. The quantization error $q_\theta(k)$ does not exactly follow the Gaussian white noise property but it is known to behave as an uncorrelated uniform distribution if the signal is sufficiently complicated and the quantization level is sufficiently small [17]. So, $q_\theta(k)$ can be approximated as a Gaussian white noise with its noise variance

$$V = \frac{\Delta_\theta^2}{12}. \tag{5}$$

where Δ_θ is the quantization level of the encoder. Then, the steady-state Kalman filter is an optimal estimator for (4) and its observer gain F can be obtained by the discrete-time algebraic Riccati equation (DARE) as follows.

$$F = \frac{MC^T}{CMC^T + V} \tag{6a}$$

$$M = AMA^T + BWB^T - \frac{AMC^TCMA^T}{CMC^T + V} \tag{6b}$$

where W is the variance of $n_a(k)$ and M is the one-step prediction error covariance matrix. Then it leads to the following state estimator called the kinematic Kalman filter (KKF):

$$\hat{x}_k(k+1) = A_c\hat{x}_k(k) + B_c\bar{a}(k) + Fy(k+1) \tag{7}$$

where $A_c = (I - FC)A$ and $B_c = (I - FC)B$.

III. Sensor Characterization for KKF

A. The Effect of Signal Bandwidth of Accelerometer

In (3), we are assuming that the accelerometer is ideal, i.e. the acceleration can be measured over the entire frequency range. However, in practice, the measurement from the accelerometer will always be limited by a certain bandwidth. This is particularly true for the case with commercial low cost MEMS accelerometers [18]. In fact, the bandwidth should be traded off with the noise density because the output signal is usually conditioned by the first order low pass filter, the cut-off frequency of which is determined by an external capacitor [18]. In this case, the noise density (rms) is related to the cut-off frequency τ_ω [rad/sec] of the bandwidth by

$$\text{rmsNoise} \propto \sqrt{\tau_\omega}. \tag{8}$$

This adds an addition dynamics to the kinematic model in (4) so we need to augment it with the inverse of the low pass filter as shown in Fig. 1 where L_ω is the first order low pass filter given by

$$L_\omega(s) = \frac{1}{s + \frac{1}{\tau_\omega}}. \tag{9}$$

Accordingly, the system matrix B_k in (4c) changes to

$$B_\omega = \begin{bmatrix} \frac{T_s}{\tau_\omega} + \frac{T_s^2}{2} \\ T_s \end{bmatrix}. \tag{10}$$

Also, the noise covariance of the accelerometer will change according to (8) as

$$W = k_\omega \tau_\omega \tag{11}$$

where $k_\omega[(rad/s^2)^2/Hz]$ is the constant coefficient.

Figure 2a shows how the Kalman filter poles change as we set different values for the bandwidth of the accelerometer. It is assumed that $k_\omega = 0.05$ and the resolution of the encoder is 1024 ppr (pulse per revolution). As the covariance of the accelerometer W increases, the observer poles move toward the inside of the unit circle. On the other hand, for the same value of W, the bandwidth limitation results in moving the closed-loop poles of the Kalman filter even further inward the unit circle. This means that we may be using the estimator with the slower dynamics than the desired one if we do not take the bandwidth characteristics into account.

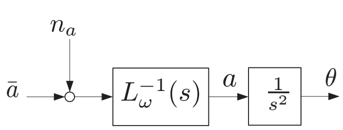
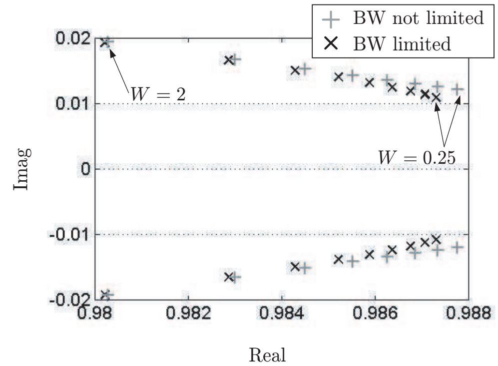


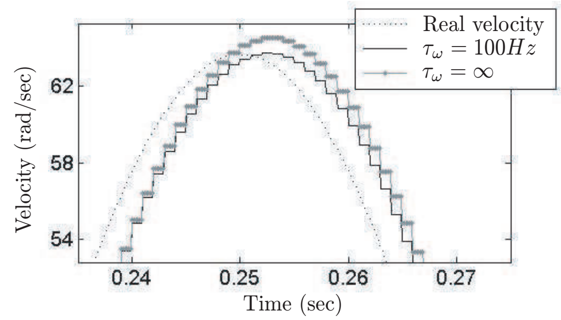
Fig. 1: Augmented kinematic model

This is also confirmed by the estimated velocity profiles in Fig. 2b which simulates different velocity profiles when the system is excited by 10 Hz sinusoidal signal. In this simulation, $\tau_\omega = 100Hz$ is considered as the bandwidth of the accelerometer with $k_\omega = 0.05$ (i.e., $W = 5\ (rad/s^2)^2$). The dashed line is the actual velocity and other lines are the estimated velocities from the KKF. Both estimated velocities show the phase lag due to the limited bandwidth of the accelerometer. However, the velocity estimated by the KKF without considering τ_ω (the grey line with dot marks) introduces additional distortion due to the slower estimation dynamics than the case with the explicit consideration of τ_ω into the design of the KKF (the black solid line).

In fact, as we can see in Fig. 2b, the difference is not very significant between the case that we consider the low pass filter and the case that we do not. More serious problem comes from the fact that there is an inherent phase lag resulting from the low pass filtered acceleromter signal whether we explicitly consider the bandwidth limitation or not. The effect is quite pervasive as shown in Fig 2b. More specifically, even though τ_ω is set to $100Hz$, the phase delay affects the performance of the velocity estimation on signals in much lower frequency ranges as well (10 Hz signal in this case). This problem can actually be mitigated by intentionally increasing the speed of the convergence rate of the observer dynamics. Faster observer dynamics means that we are relying more on the encoder signal than the accelerometer. However, as we rely more on the encoder signal for the velocity estimation, the noise characteristics of the quantization error from the encoder will become more dominant. This issue will be addressed in more detail in the following section when we consider another problem of the KKF, i.e. the manifestation of the non-whiteness of the quantization error for the slow frequency motion when we design the KKF with intentionally fast observer dynamics.



(a) Kalman filter closed-loop poles when $k_\omega = 0.05$



(b) Velocity profiles (10 Hz reference signal with $\tau_\omega = 100Hz$)

Fig. 2: Effect of the bandwidth limitation on KKF

B. The Effect of Deterministic Quantization Error

In formulating the KKF, we assumed that the quantization noise of the encoder can be approximated by the Gaussian white noise as in (5). This is reasonably true when the motor is moving with a motion fast and complicated enough to make the whiteness assumption hold [17], i.e. the asymptotic convergence of the quantization error to the white noise signal. However, this does not hold when the motor is operating in slow frequency ranges due to the increased time correlation between consecutive encoder samples. In a typical case, this may not be significant. However, in such a case that we intentionally increase the convergence rate of the observer (for example, to compensate for the phase lag coming from the bandwidth limitation), the problem of the non-whiteness of the quantization error can become significant.

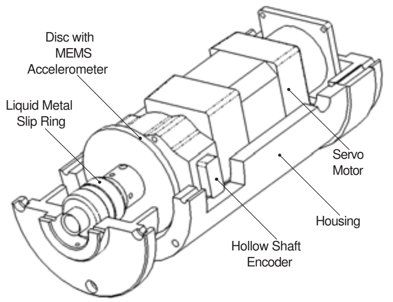


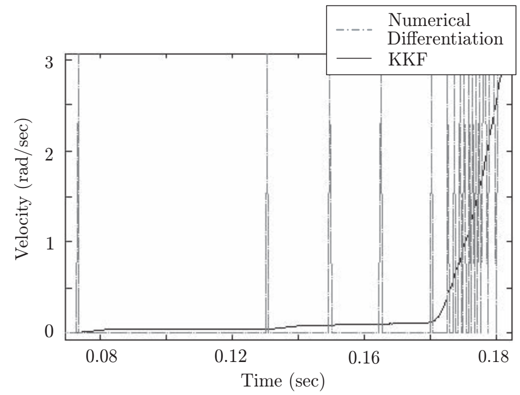
Fig. 3: Experimental setup for KKF test

In order to illustrate this issue, a series of experiments have been performed using the joint actuator unit shown in Fig. 3. It is equipped with the angular accelerometer as well as the motor encoder. It uses two ADXL210E linear MEMS accelerometers (Analog Devices) mounted on the opposite sides of the disc element that rotates with the motor shaft. So it converts the linear acceleration measurement into the angular acceleration. In order to minimize the signal transmission noise, the liquid metal slip ring (Mercotec) has been adopted to transmit the measured acceleration signals. Table I shows the parameters of the encoder and the accelerometer for the design of the KKF.

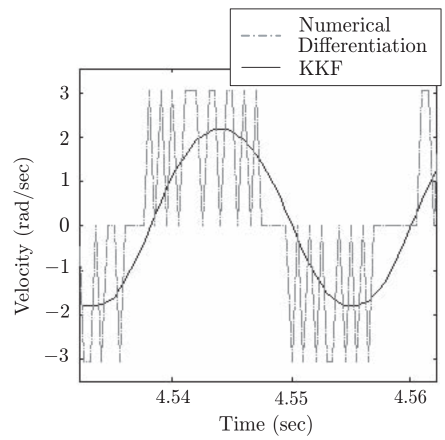
TABLE I: Experimental Conditions

Encoder Spec.	Value	Unit
Encoder Counts N	2^{12}	$[ppr]$
Sampling Time T_s	0.002	$[sec]$
Accelerometer Spec.	Value	Unit
Resolution $\|w_a\|_{max}$	8	$[rad/sec^2]$
Bandwidth	500	[Hz]
k_δ	0.005	$[(rad/sec^2)^2/Hz]$
Maximum Range	5144	$[rad/sec^2]$

Fig. 4a shows the velocity profiles when we designed the steady state KKF with the nominal values given in Table I. The motor was run with the chirping signal (starting from 0 Hz to 50 Hz in 5 second) to see the performance of the velocity estimation both at the slow motion and at the fast motion. The solid line is the velocity from the KKF and the grey dashed line is the velocity estimation by the numerical differentiation. Note the the discontinuous velocity profile from the numerical differentiation is due to the velocity resolution given by $\Delta_\theta/Ts = 3.068rad/sec$. Figure 4a shows the initial motion of the motor. Due to the Coulomb friction in the shaft, the motor moves very slowly in the beginning and as it overcomes the stiction level, the velocity starts ramping up. As we can see, the KKF provides a smooth velocity profile compared to the numerical



(a) Velocity estimation at low frequency motion

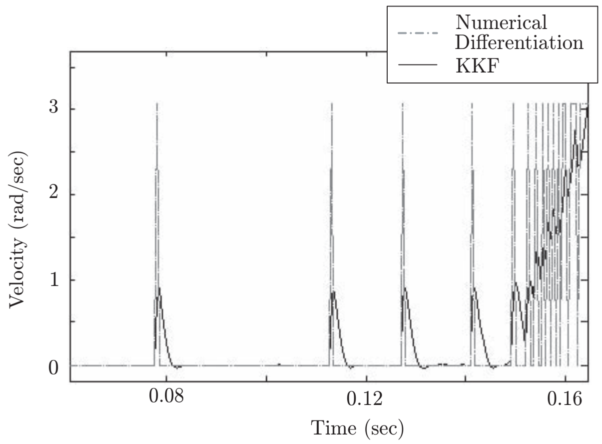


(b) Velocity estimation at high frequency motion

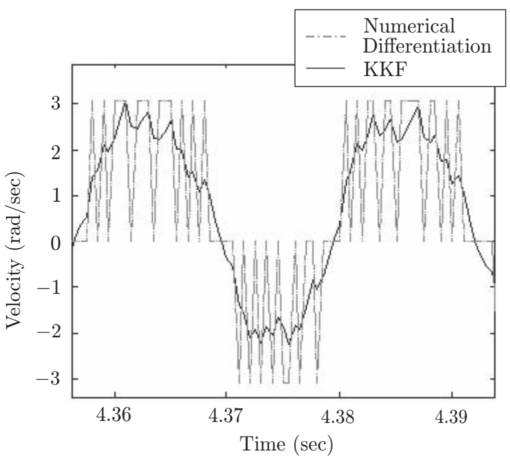
Fig. 4: Estimated velocities with the observer of slow convergence rate

differentiation. However, as Fig. 4b illusrates, during the high frequency motion, the estimated velocity from the KKF introduces an additional phase lag compared to the numerical differentiation. Note that the velocity estimation by the numerical differentiation has the default phase lag corresponding approximately to the size of the sampling time T_s. Therefore, the phase lag in the estimated velocity for the KKF in Fig. 4b is quite significant. As was explained in the previous section, this is primarily due to the the inherent bandwidth limitation of the accelerometer which is set to 500 Hz in this case.

To get around the additional phase lag shown in Fig. 4b, one might try to intentionally increase the convergence rate of the observer dynamics by using excessively large values for W. This means that the velocity estimation will more depend on the encoder measurement than the accelerometer. The results in such a case are shown in Fig. 5. We can see in Fig. 5b that the phase lag has been considerably reduced compared to Fig. 4b. Although the velocity profile now has glitches due to the excessive effect of the encoder, it still shows smoother profile compared to the velocity estimation by the numerical differentiation. In this case, however, we sacrificed the velocity estimation at the slow frequency motion as we can see in Fig. 5a. Due to the quantization error of the encoder, now we have a deterministic signal for the velocity estimation at this slow frequency motion and the velocity profile is directly responding to the discontinuous encoder signal.



(a) Velocity estimation at low frequency motion



(b) Velocity estimation at high frequency motion

Fig. 5: Estimated velocities with the observer of fast convergence rate

IV. THE KKF WITH VARIABLE FILTER GAIN

From Fig. 4 and 5, we can conclude that the steady state gain for the KKF, (i.e. the constant filter gain) may not be suitable for all the frequency ranges of the motion, which suggests to use a variable filter gain depending on the frequency component of the motion. In general, if we want to make the filter gain variable, the DARE in (6) must be solved in real time. However, due to the simplicity of the kinematic model (i.e. the double integrator model), we can have the closed (parameterized) form of the Kalman filter gain. The system matrices in (4) has only one parameter T_s and we can expect that the solution to the DARE in (6) will be parameterized by the sampling time T_s, the accelerometer noise covariance W and the encoder noise covariance V. Following equation provides the parameterized solution for M [19].

$$M(1,1) = V\frac{\sqrt{1+2r}(\sqrt{1+2r}+1)^2}{r^2} \tag{12a}$$

$$M(1,2) = \frac{WT_s^3}{8}(\sqrt{1+2r}+1)^2 \tag{12b}$$

$$M(2,2) = \frac{WT_s^2}{2}\left(\sqrt{1+2r}+1\right) \tag{12c}$$

where $r = \frac{4}{T_s^2}\sqrt{\frac{V}{W}}$. Accordingly, from the Eq. (6a), the KKF gain $F = \begin{bmatrix} f_1 & f_2 \end{bmatrix}^T$ can be written as

$$f_1 = \frac{\sqrt{1+2r}(\sqrt{1+2r}+1)^2}{\sqrt{1+2r}(\sqrt{1+2r}+1)^2 + r^2} \tag{13a}$$

$$f_2 = \frac{2}{T_s}\frac{(\sqrt{1+2r}+1)^2}{\sqrt{1+2r}(\sqrt{1+2r}+1)^2 + r^2} \tag{13b}$$

Equation (13) is reasonable to implement for the real time computation but we can actually further simplify the parameterization by using the equivalent formulation of the so called alpha-beta filter with respect to Kalman filter [20]. From the relations between the Kalman filter version of the alpha-beta filter and its continuous-time impulse-invariant inverse, the KKF gain can be reparameterized as follows.

$$f_1 = \eta T_s\left(\sqrt{2} - \frac{\eta T_s}{2}\right), \quad f_2 = \eta^2 T_s \tag{14}$$

where η is the main parameter to be adjusted and it satisfies

$$\eta T_s \leq \pi, \quad 0 \leq \sqrt{2} - \frac{\eta T_s}{2} \leq 1. \tag{15}$$

The former is needed to satisfy the Nyquist criterion the letter is to guarantee the stability of the Kalman filter.

As mentioned before, we can adjust the KKF gain with respect to frequency component of the motion. Ideally, it can be obtained by the Fourier series of the velocity samples within a fixed time frame. However, to reduce the computational load, we choose to approximate the frequency component of the motion by the ratio between the magnitude of the acceleration and that of the velocity. Denoting N_η as the number of samples within which we want to extract the frequency component of the motion, the index for the frequency content can be obtained as

$$u(k) := \frac{\max\limits_{i=0,..,N_\eta-1} |\bar{a}(N_\eta(m-1)+i)|}{\max\limits_{i=0,..,N_\eta-1} |\hat{\dot{\theta}}(N_\eta(m-1)+i)| + c_v}, \; m = \left\lfloor \frac{k}{N_\eta} \right\rfloor \tag{16}$$

where $\hat{\dot{\theta}}(k)$ is the estimated velocity and c_v is a constant to avoid the zero at the denominator. Since $u(k)$ is updated every N_η sample times, N_η should be chosen considering the expected largest frequency component within the motion range.

Then the variable KKF gain can be determined by the following linear mapping.

$$\eta(k) = \eta_{max}, \quad \text{if}\, u(k) \geq u_{max} \tag{17a}$$

$$\eta(k) = \eta_{min}, \quad \text{if}\, u(k) \leq u_{min} \tag{17b}$$

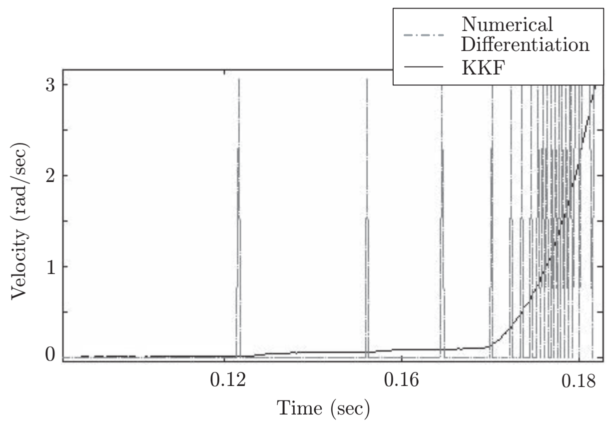
$$\eta(k) = \frac{\eta_{max} - \eta_{min}}{u_{max} - u_{min}}\left(u_{max} - u(k)\right) + \eta_{min} \tag{17c}$$

where the subscripts either max or min indicates the maximum and the minimum values chosen for the corresponding variable. Since the $u(k)$ is defined for $k \geq N_\eta$ in (16), we can use $\eta(k) = \eta_{min}$ for $0 \leq k \leq N_\eta - 1$.

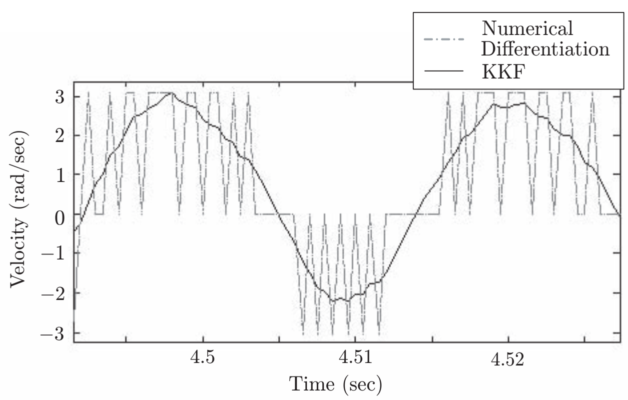
Fig. 6 shows the result when we implement the variable gain for the KKF as described above. Table II lists the parameters chosen for the adaptation rule. We can see that it provides smooth velocity profile both at the slow motion and the fast motion ranges without the additional phase lag.

TABLE II: Constants for variable Kalman filter gain

Variable	Value	Unit	Variables	Value	Unit
N_η	50	samples	c_v	1	rad/sec
η_{max}	400	Hz	u_{max}	500	Hz
η_{min}	30	Hz	u_{min}	0	Hz



(a) Velocity estimation at slow frequency motion



(b) Velocity estimation at fast frequency motion

Fig. 6: Estimated velocities with variable Kalman filter gain

V. CONCLUDING REMARKS

Sensor-based state estimation is an attractive option for its superior robustness and accuracy compared to the conventional model-based methods. However, the use of low cost sensors can introduce various sources of performance degradation coming from the inherent characteristics or the limitations of each sensor. The KKF for velocity estimation is investigated as an illustrative example. The bandwidth limit of the MEMS accelerometer and the deterministic feature of the quantization noise at slow velocity are addressed as major characteristics of sensors used in the KKF. A simple filter gain adaptation rule is suggested to mitigate the above mentioned problems in sensors and its performance has been verified through the experimental results.

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