Medium Access Control Game with An Enhanced Physical-Link Layer Interface

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***Abstract*—We consider distributed medium access control in a wireless network where each link layer user (transmitter) is equipped with multiple transmission options as opposed to the classical binary options of transmitting/idling. In each time slot, a user randomly chooses a transmission option according to a "transmission probability vector". Packets sent by the users are either received or lost depending on whether reliable decoding is supported by the communication channel. We propose a game theoretic model for distributed medium access control where each user adapts its transmission probability vector to maximize a utility function. Condition under which the medium access control game has a unique Nash equilibrium is obtained. Simulation results show that, when multiple transmission options are provided, users in a distributed network tend to converge to channel sharing schemes that are consistent with the well-known information theoretic understandings.**

I. Introduction

Classical medium access control (MAC) protocols assume that a link-layer user (transmitter) should choose either to idle in a time slot or to transmit a packet with pre-determined communication parameters. Under this assumption, when users in a distributed wireless network experience packet collision, the only approach to control contention is to reduce and randomize their transmission activities [1]. While such a model and its derived contention control approaches are widely adopted in distributed MAC protocols such as the DCF protocol in 802.11, they do not permit exploitation at the link layer of the well known information theoretic result that parallel transmission with carefully controlled rates achieves the optimal sum throughput of a multiple access system.

Recently, an extension of Shannon-style channel coding theory to distributed communication systems was presented in [2][3][4]. The new channel coding framework gave each transmitter multiple transmission options corresponding to different channel codes, which imply different values of communication parameters such as power and rate. A transmitter can choose an arbitrary option, denoted by a code index parameter, to send its message in a time slot without sharing such a decision with other transmitters or with the receiver. It was shown that fundamental performance limitation of the system can be characterized using an achievable region defined in the vector space of code index parameters of all users [2]. If the code index vector happens to locate inside the achievable region, the receiver will decode the messages reliably, while if the code index vector is outside the achievable region, the receiver will report a collision reliably.

The new channel coding framework makes it possible to extend the classical physical-link layer interface to give a link layer user multiple transmission options corresponding to different communication settings such as different rates and powers. It also enables the derivation and analysis of link layer channel model and medium access control performance using physical layer channel properties. Consequently, link layer users can now exploit advanced communication adaption approaches, such as rate adaptation, to improve channel sharing efficiency in distributed networking. Understanding the impact of the enhanced physical-link layer interface on the strategy of link layer communication adaptation and contention control therefore becomes an important research topic.

For a wide range of distributed networks, game theoretic problem formulations and analysis have proven to provide new insights to reverse/forward engineering of existing MAC protocols for improved fairness and higher throughput, and for decoupling contention control from handling failed packets [5][6]. The key idea of a game theoretic MAC algorithm is to control contention by distributively adapting transmission probabilities of the users to optimize their individual utilities, each being carefully chosen as a function of the transmission cost and the experienced contention level of the corresponding user. Utility function design often requires a good understanding on the expected contention level and the desired transmission probability, derived from performance objectives of the users. While contention control in a distributed wireless network has been rigorously investigated using the classical link layer model, extending the understandings to cases when each link layer user has a handful or more transmission options is a new research direction that deserves careful and rigorous exploration.

As a first step effort, in this paper, we consider the problem of distributed medium access control in a wireless network with an enhanced physical-link layer interface. Under the assumption that users are backlogged with messages, we model the medium access control problem as a non-cooperative game where each user adapts its communication to maximize an individual utility function. We show that existing understandings on stability and throughput of random access communication over collision and multi-packet reception channels can be exploited to design utility functions in the new system. Conditions under which the distributed medium access control game has a unique Nash equilibrium are obtained. Computer simulations show that, in a multiple access environment with a large number of users each being equipped with multiple communication rate options, the game theoretic medium access control algorithm does favor low rate and parallel channel access options over high rate and exclusive channel access options. This is consistent with the well-known understandings in information theory.

II. The Link Layer Model and A Game Theoretic Problem Formulation

Consider a wireless network with K users. Each user is equipped with $M+1$ transmission options corresponding to a set of $M+1$ channel coding schemes, denoted by $G_k = \{g_{k0}, g_{k1}, \cdots, g_{kM}\}$. Each element g_{km}, $m = 0, \cdots, M$, represents a particular transmission setting of User k that includes the specifications of transmission power, communication rate, etc. We assume that the first element g_{k0} always represents the "idling" option. Time is slotted with each slot equalling the length of a fixed number of channel symbols, and this is also the length of a codeword. In each time slot, User k chooses one of the transmission options and sends a packet with encoded message to its receiver. The choice of transmission option of a user is shared neither with other users nor with the receiver. We assume that the communication channel is memoryless and static. Depending on whether reliable message decoding is supported by the channel or not, a transmitted packet is either received successfully or lost. In the latter case, a collision report is fed back to the transmitter. Note that, the link layer model degrades to a classical one if each G_k only contains two elements.

We assume that users are backlogged with messages. In each time slot, User k randomly chooses a transmission option according to an M-length vector $\boldsymbol{p}_k = [p_{k1}, \cdots, p_{kM}]^T$ termed the "transmission probability vector". Here $p_{km} \geq 0$, $m = 1, \cdots, M$, denotes the probability that User k chooses option g_{km}, and $1 - \sum_{m=1}^{M} p_{km} \geq 0$ is the probability that User k idles. We use $\boldsymbol{P} = [\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_K]$ to denote the transmission probability vectors of all users, and use $\boldsymbol{P}_{-k} = [\boldsymbol{p}_1, \cdots, \boldsymbol{p}_{k-1}, \boldsymbol{p}_{k+1}, \cdots, \boldsymbol{p}_K]$ to denote the transmission probability vectors of all users but User k. Conditioned on User k transmitting with option g_{km}, let $0 \leq q_{km} \leq 1$ be the probability that the message (or packet) is successfully received. We define $\boldsymbol{q}_k = [q_{k1}, \cdots, q_{kM}]^T$ as the "conditional success probability vector" of User k. Clearly, given the communication channel, $\boldsymbol{q}_k$ is a function of $\boldsymbol{P}_{-k}$.

We model medium access control as a non-cooperative game where users distributively adapt their transmission probability vectors to maximize individual utility functions. The utility function of User k is denoted by $U_k(\boldsymbol{p}_k, \boldsymbol{q}_k)$, which is a function of the transmission probability vector $\boldsymbol{p}_k$ and the conditional success probability vector $\boldsymbol{q}_k$. Given $\boldsymbol{P}_{-k}$ and consequently $\boldsymbol{q}_k$, the utility maximization problem of User k is represented by

$$\max_{\boldsymbol{p}_k} U_k(\boldsymbol{p}_k, \boldsymbol{q}_k), \quad \text{s.t. } \boldsymbol{p}_k \geq \boldsymbol{0}, \boldsymbol{p}_k^T \boldsymbol{1} \leq 1, \tag{1}$$

where $\boldsymbol{0}$ and $\boldsymbol{1}$ are vectors of all zeros and all ones, respectively.

We say $\boldsymbol{P}$ is a Nash equilibrium of the medium access control game if for all $k = 1, \cdots, K$, $\boldsymbol{p}_k$ maximizes $U_k(\boldsymbol{p}_k, \boldsymbol{q}_k)$ given $\boldsymbol{P}_{-k}$. The following theorem gives a sufficient condition for the existence of Nash equilibrium.

Theorem 1: The medium access control game admits at least one Nash equilibrium if, for all $k = 1, \cdots, K$, utility function $U_k(\boldsymbol{p}_k, \boldsymbol{q}_k)$ is concave in $\boldsymbol{p}_k$.

Theorem 1 is implied by [7, Theorems 1].

Given $\boldsymbol{P}$, define $\boldsymbol{G}_{kl}(\boldsymbol{P})$ as the second order partial derivative of $U_k(\boldsymbol{p}_k, \boldsymbol{q}_k)$ with respect to $\boldsymbol{p}_k$ and $\boldsymbol{p}_l$,

$$\boldsymbol{G}_{kl}(\boldsymbol{P}) = \frac{\partial^2 U_k(\boldsymbol{p}_k, \boldsymbol{q}_k)}{\partial \boldsymbol{p}_k \partial \boldsymbol{p}_l}. \tag{2}$$

The following theorem gives a sufficient condition under which Nash equilibrium of the medium access control game is unique.

Theorem 2: Assume that the medium access control game has at least one Nash equilibrium. Let $\boldsymbol{P}^{(1)}$ and $\boldsymbol{P}^{(2)}$ be two Nash equilibria. For any $0 \leq \theta \leq 1$, let $\boldsymbol{P} = \theta \boldsymbol{P}^{(1)} + (1 - \theta)\boldsymbol{P}^{(2)}$. If $\boldsymbol{P}^{(1)} \neq \boldsymbol{P}^{(2)}$ implies

$$\sum_{k=1}^{K} \sum_{l=1}^{K} (\boldsymbol{p}_k^{(1)} - \boldsymbol{p}_k^{(2)})^T \boldsymbol{G}_{kl}(\boldsymbol{P}) (\boldsymbol{p}_l^{(1)} - \boldsymbol{p}_l^{(2)}) < 0, \tag{3}$$

then Nash equilibrium of the medium access control game must be unique.

Theorem 2 is implied by [7, Theorems 2, 6].

III. Utility Design with A Classical Physical-Link Layer Interface

To help explaining the utility function design with a relatively simple notation, in this section, we will first consider wireless networks with the classical physical-link layer interface where each user only has binary transmission options. We choose to skip subscripts of the variables if this causes no confusion.

Consider a multiple access system with a symmetric channel and homogeneous users. Each user only has two transmission options, $G = \{g_0, g_1\}$, where g_0 is the idling option. According to the achievable region result [4][2], if multiple users transmit in parallel, the packets should be received successfully so long as sum rate of the users is supported by the channel. Assume that packet transmissions should be successful in a time slot if and only if no more than N users transmit in parallel, and the value of N is known to all users. Assume that $K \gg N \geq 1$ and users want to maximize their symmetric throughput. With binary transmission options, the system described is a random multiple access system over a multi-packet reception channel [8][9]. Optimal sum throughput of the system is approached when each user transmits at probability x^*/K where x^* is the solution of the following maximization problem [9].

$$x^* = \operatorname*{argmax}_x e^{-x} \sum_{i=1}^{N} \frac{x^i}{(i-1)!}. \tag{4}$$

When all users set their transmission probabilities at x^*/K, conditional success probability experienced by each user can be approximated by

$$q^* = e^{-x^*}\sum_{i=0}^{N-1}\frac{x^{*i}}{i!}. \quad (5)$$

Assume that a user estimates the total number of users to be $\tilde{K}$. According to the above understanding, in the medium access control game, we design the utility function of each user as follows.

$$U(p,q) = \frac{p}{x^*}t(q) - h\frac{p}{x^*}\log\frac{p}{ex^*/\tilde{K}}. \quad (6)$$

The utility function contains two parts. The first part $\frac{p}{x^*}t(q)$ is a linear function in p that intends to control the conditional success probability q above its desired value shown in (5). We require that $t(q)$, which is a function of q, should satisfy $\frac{dt(q)}{dq} \geq 0$. We say that function $t(q)$ is unbiased if $t(q^*) = 0$. Note that, with an unbiased $t(q)$ function, the $\frac{p}{x^*}t(q)$ term alone tells a user to increase p when $q < q^*$ and to decrease p when $q > q^*$[1]. The second part $h\frac{p}{x^*}\log\frac{p}{ex^*/\tilde{K}}$ in the utility function, with h being a scaling parameter, is a convex function in p that intends to keep the transmission probability p around its targeted value $x^*/\tilde{K}$. Note that $h\frac{p}{x^*}\log\frac{p}{ex^*/\tilde{K}}$ is minimized at $p = x^*/\tilde{K}$.

According to Theorem 3, if parameter h is chosen appropriately, the non-cooperative medium access control game should have a unique Nash equilibrium. Furthermore, if $\tilde{K} = K \gg 1$, and the $t(q)$ function is unbiased, then the Nash equilibrium is represented by $p = x^*/K$ for all users since

$$\left.\frac{\partial U(p,q)}{\partial p}\right|_{p=x^*/K, q=q^*} = \frac{t(q^*)}{x^*} - \frac{h}{x^*}\left[\log\frac{1}{e}+1\right] = 0. \quad (7)$$

To understand the idea behind the utility function design of (6), we can think about the distributed channel sharing game as a social event. Let us regard transmission probability p and conditional success probability q as the "behavior" and the measured "feedback" of a user. To participate in the social event, each user chooses a behavior target $x^*/\tilde{K}$ and a feedback target q^*, which are calculated via utility maximization in an envisioned network. In the above discussion for example, the targets are computed via sum throughput optimization in a random multiple access network with homogeneous users. Once the targets are obtained, each user chooses a utility function that specifies how the user should try to keep his behavior around the behavior target, and how the user should respond to the social force if the measured feedback differs from the feedback target.

IV. Utility Design with An Enhanced Physical-Link Layer Interface

Let us now consider the same system investigated in Section III, but with each user having $M \geq 2$ transmission options. Assume that, each user, say User k, keeps an estimated total number of users, denoted by $\tilde{K}_k$. For each user and each non-idling transmission option, say option g_{km}, User k chooses two parameters: a targeted conditional success probability q^*_{km} and a targeted transmission probability $x^*_{km}/\tilde{K}_k$. Given these parameters, utility function of User k is designed as the summations of two parts each is the summation of M items corresponding to the M non-idling transmission options.

$$U_k(\boldsymbol{p}_k, \boldsymbol{q}_k) = \sum_{m=1}^{M}\frac{p_{km}}{x^*_{km}}d_{km}t_{km}(q_{km}) - h_k\sum_{m=1}^{M}\frac{p_{km}}{x^*_{km}}\log\frac{p_{km}}{s_{km}ex^*_{km}/\tilde{K}_k}. \quad (8)$$

The first part $\sum_{m=1}^{M}\frac{p_{km}}{x^*_{km}}d_{km}t_{km}(q_{km})$ is a linear function in $\boldsymbol{p}_k$ that intends to control the conditional success probability vector $\boldsymbol{q}_k$ above its desired values. We require that $\frac{dt_{km}(q)}{dq} \geq 0$. Differs from the case of binary transmission options, we introduce a "steering vector" $\boldsymbol{d}_k = [d_{k1}, d_{k2}, \cdots, d_{kM}]^T$, with $\boldsymbol{d}_k \geq \boldsymbol{0}, \boldsymbol{d}_k^T\boldsymbol{1} \leq 1$, to allow each user to assign different weights to terms corresponding to different transmission options. The second part $h_k\sum_{m=1}^{M}\frac{p_{km}}{x^*_{km}}\log\frac{p_{km}}{s_{km}ex^*_{km}/\tilde{K}_k}$ is a convex function in $\boldsymbol{p}_k$ that intends to keep the transmission probability vector $\boldsymbol{p}_k$ around a targeted value $\boldsymbol{p}^*_k$. We introduce another "steering vector" $\boldsymbol{s}_k = [s_{k1}, s_{k2}, \cdots, s_{kM}]^T$, with $\boldsymbol{s}_k \geq \boldsymbol{0}, \boldsymbol{s}_k^T\boldsymbol{1} \leq 1$, and construct the targeted transmission probability vector $\boldsymbol{p}^*_k$ as $\boldsymbol{p}^*_k = [s_{k1}x^*_{k1}/\tilde{K}_k, s_{k2}x^*_{k2}/\tilde{K}_k, \cdots, s_{kM}x^*_{kM}/\tilde{K}_k]^T$.

Without specifying how the q^*_{km} and $x^*_{km}/\tilde{K}_k$ parameters are determined, our key result is presented in the following theorem, which shows that if the scaling parameters h_k are chosen appropriately, then the non-cooperative medium access control game has a unique Nash equilibrium.

Theorem 3: Given the steering vectors $\boldsymbol{d}_k$, $\boldsymbol{s}_k$, $k = 1, \cdots, K$, the medium access control game has a unique Nash equilibrium if the following inequality is satisfied for all $k = 1, \cdots, K$ and $m = 1, \cdots, M$

$$\frac{\tilde{K}_k}{K}\frac{h_k}{x^{*2}_{km}}e^{-\frac{t^{(\max)}_{km}}{h_k}} \geq \max\left\{\left(\frac{t'^{(\max)}_{km}}{x^*_{km}}\right)^2, 1\right\}, \quad (9)$$

where $t^{(\max)}_{km} = \max_q t_{km}(q)$ and $t'^{(\max)}_{km} = \max_q \frac{dt_{km}(q)}{dq}$.

Proof: According to Theorem 1, the medium access control game has at least one Nash equilibrium. We will use Theorem 2 to prove that the Nash equilibrium must be unique.

Assume that $\boldsymbol{P}^{(1)}$ and $\boldsymbol{P}^{(2)}$ are two different equilibria of the medium access control game. Let $0 \leq \theta \leq 1$. Define $\boldsymbol{P} = \theta\boldsymbol{P}^{(1)} + (1-\theta)\boldsymbol{P}^{(2)}$. Because $\boldsymbol{P}^{(1)}$ is a Nash equilibrium, the following inequality holds for all $k = 1, \cdots, K$ and $m = 1, \cdots, M$,

$$d_{km}t_{km}(q^{(1)}_{km}) - h_k\left(\log\frac{p^{(1)}_{km}}{s_{km}x^*_{km}/\tilde{K}_k}\right) \geq 0, \quad (10)$$

where $q^{(1)}_{km}$ is the conditional success probability corresponding to equilibrium $\boldsymbol{P}^{(1)}$. Because $d_{km} \leq 1$, we get from (10) that

[1]For various reasons, users may prefer a biased $t(q)$ function over an unbiased one. Discussions on this issue is skipped in this paper due to page limitation.

$p_{km}^{(1)} \leq \frac{s_{km}x_{km}^*}{\tilde{K}_k} e^{\frac{t_{km}^{(\max)}}{h_k}}$. Since $\boldsymbol{P}^{(2)}$ is also a Nash equilibrium and therefore has to satisfy a similar inequality, from $\boldsymbol{P} = \theta \boldsymbol{P}^{(1)} + (1-\theta)\boldsymbol{P}^{(2)}$, we get

$$p_{km} \leq \frac{s_{km}x_{km}^*}{\tilde{K}_k} e^{\frac{t_{km}^{(\max)}}{h_k}}. \tag{11}$$

Let $\boldsymbol{G}_{kl}(\boldsymbol{P})$ be defined in (2). We first obtain the following inequality according to (11).

$$\begin{aligned}
&\sum_{k=1}^{K} [\boldsymbol{p}_k^{(1)} - \boldsymbol{p}_k^{(2)}]^T \boldsymbol{G}_{kk}(\boldsymbol{P}) [\boldsymbol{p}_k^{(1)} - \boldsymbol{p}_k^{(2)}] \\
&= -\sum_{k=1}^{K}\sum_{m=1}^{M} (p_{km}^{(1)} - p_{km}^{(2)})^2 \frac{h_k}{x_{km}^* p_{km}} \\
&\leq -\sum_{k=1}^{K}\sum_{m=1}^{M} (p_{km}^{(1)} - p_{km}^{(2)})^2 \frac{\tilde{K}_k h_k}{s_{km} x_{km}^{*2}} e^{-\frac{t_{km}^{(\max)}}{h_k}} \\
&\leq -\sum_{k=1}^{K} \left(\sum_{m=1}^{M} |p_{km}^{(1)} - p_{km}^{(2)}| \sqrt{\frac{\tilde{K}_k h_k}{x_{km}^{*2}} e^{-\frac{t_{km}^{(\max)}}{h_k}}} \right)^2 \\
&\leq -\left(\sum_{k=1}^{K}\sum_{m=1}^{M} |p_{km}^{(1)} - p_{km}^{(2)}| \sqrt{\frac{\tilde{K}_k}{K} \frac{h_k}{x_{km}^{*2}} e^{-\frac{t_{km}^{(\max)}}{h_k}}} \right)^2
\end{aligned} \tag{12}$$

Next, we show that, for any $k, l = 1, \cdots, K$, $l \neq k$, and for any $m, n = 1, \cdots, M$, we have $-1 \leq \frac{\partial q_{km}}{\partial p_{ln}} \leq 0$. Define $\xi_{(k)}(\boldsymbol{P}|g_{km}, g_{ln})$ as the probability that the packet from User k is received successfully conditioned on that User k chooses transmission option g_{km} and User l chooses transmission option g_{ln}. Define $\xi_{(k)}(\boldsymbol{P}|g_{km}, g_{l0})$ as the probability that the packet from User k is received successfully conditioned on that User k chooses transmission option g_{km} and User l idles. Because idling causes no more interference than transmitting a packet, we have $\xi_{(k)}(\boldsymbol{P}|g_{km}, g_{l0}) \geq \xi_{(k)}(\boldsymbol{P}|g_{km}, g_{ln})$. Note that

$$\begin{aligned}
\frac{\partial q_{km}}{\partial p_{ln}} &= \frac{\partial \left[p_{ln}\xi_{(k)}(\boldsymbol{P}|g_{km}, g_{ln}) \right]}{\partial p_{ln}} \\
&\quad + \frac{\partial \left[\left(1 - \sum_{i=1}^{M} p_{li}\right) \xi_{(k)}(\boldsymbol{P}|g_{km}, g_{l0}) \right]}{\partial p_{ln}} \\
&= \xi_{(k)}(\boldsymbol{P}|g_{km}, g_{ln}) - \xi_{(k)}(\boldsymbol{P}|g_{km}, g_{l0}) \\
&\in [-1, 0].
\end{aligned} \tag{13}$$

From (13), we get

$$\begin{aligned}
&\sum_{k=1}^{K}\sum_{l=1, l\neq k}^{K} [\boldsymbol{p}_k^{(1)} - \boldsymbol{p}_k^{(2)}]^T \boldsymbol{G}_{kl}(\boldsymbol{P}) [\boldsymbol{p}_l^{(1)} - \boldsymbol{p}_l^{(2)}] \\
&\leq \sum_{k=1}^{K} \left[\sum_{m=1}^{M} \frac{d_{km}}{x_{km}^*} {t'}_{km}^{(\max)} |p_{km}^{(1)} - p_{km}^{(2)}| \right] \\
&\quad \times \left[\sum_{l=1, l\neq k}^{K}\sum_{m=1}^{M} |p_{lm}^{(1)} - p_{lm}^{(2)}| \right] \\
&< \left[\sum_{k=1}^{K}\sum_{m=1}^{M} \max\left\{ \frac{{t'}_{km}^{(\max)}}{x_{km}^*}, 1 \right\} |p_{km}^{(1)} - p_{km}^{(2)}| \right]^2.
\end{aligned} \tag{14}$$

Combining (12), (14) and assumption (9), we obtain

$$\sum_{k=1}^{K}\sum_{l=1}^{K} (\boldsymbol{p}_k^{(1)} - \boldsymbol{p}_k^{(2)})^T \boldsymbol{G}_{kl}(\boldsymbol{P}) (\boldsymbol{p}_l^{(1)} - \boldsymbol{p}_l^{(2)}) < 0. \tag{15}$$

According to Theorem 2, the Nash equilibrium must be unique. ■

To understand the utility function design and the significance of Theorem 3, we can again think about the distributed channel access game as a social event, and use $\boldsymbol{p}_k$, $\boldsymbol{q}_k$ to represent the "behavior" and the measured "feedback" of User k. We assume that User k should choose a targeted transmission probability $x_{km}^*/\tilde{K}_k$ and a targeted conditional success probability q_{km}^* for each of the non-idling transmission options. Since a user now has M non-idling options, the behavior target $\boldsymbol{p}_k^*$ is constructed using a steering vector $\boldsymbol{s}_k$ as $\boldsymbol{p}_k^* = [s_{k1}x_{k1}^*/\tilde{K}_k, s_{k2}x_{k2}^*/\tilde{K}_k, \cdots, s_{kM}x_{kM}^*/\tilde{K}_k]^T$. The term $h_k \sum_{m=1}^{M} \frac{p_{km}}{x_{km}^*} \log \frac{p_{km}}{s_{km} e x_{km}^*/\tilde{K}_k}$ in the utility function, which we call the "self-behavior preference" term, intends to keep the behavior of User k around the targeted behavior $\boldsymbol{p}_k^*$. On the other hand, we construct the feedback target $\boldsymbol{q}_k^*$ as $\boldsymbol{q}_k^* = [q_{k1}^*, q_{k2}^*, \cdots, q_{kM}^*]^T$. The term $\sum_{m=1}^{M} \frac{p_{km}}{x_{km}^*} d_{km} t_{km}(q_{km})$ in the utility function, which we call the "social-behavior preference" term, specifies how User k should adapt his behavior according to the social forces represented by the measured feedback. Here the steering vector $\boldsymbol{d}_k$ is introduced to allow User k to emphasize or ignore social forces corresponding to different transmission options. Online adaptations of the steering vectors $\boldsymbol{d}_k$ and $\boldsymbol{s}_k$ will be illustrated using an example in Section V. Note that, if the the scaling parameters h_k are large enough, then the self-behavior preference term will dominate the utility function of each user. Consequently, users will keep their behaviors around their pre-determined targets, and this can easily lead to a unique Nash equilibrium for the distributed channel access game. Theorem 3 showed that, to achieve such an effect, so long as the estimated total number of users $\tilde{K}_k$ is not too far from the true value K, the values of h_k do not need to scale in the total number of users K or the total number of non-idling transmission options M.

V. Simulation Results

In this section, we use computer simulations to show that, once multiple transmission options are provided for each user, distributed networks often prefers low rate and parallel channel access options over high rate and exclusive channel access options. Such a property, being consistent with the well-known information theoretic understanding, can demonstrate the potential impact of the enhanced physical-link layer interface on the design of medium access control algorithms.

Example: Consider a multiple access system where $K = 100$ users stays on a circle centered around the receiver. The multiple access channel is memoryless with additive Gaussian noise of zero mean and variance N_0. The channel gains from all the users to the receiver are assumed to be unit-valued. Each user has three non-idling transmission options, denoted by g_1,

g_2, and g_3. The three non-idling options all correspond to random Gaussian block channel codes at the physical layer with transmission power P, but with rates $r_1 = \frac{1}{14}\log\left(1+\frac{7P}{N_0}\right)$, $r_2 = \frac{1}{6}\log\left(1+\frac{3P}{N_0}\right)$, $r_3 = \frac{1}{2}\log\left(1+\frac{P}{N_0}\right)$, respectively. We set $P/N_0 = 10$. Assume that the total number of users is known, i.e., $\tilde{K} = 100$. For every transmission option g_i, $i = 1, 2, 3$, targeted conditional success probability q_i^* and transmission probability $x_i^*/100$ are chosen to maximize the sum throughput of a classical system with each user having binary transmission options of $\{g_0, g_i\}$. In other words, (q_1^*, x_1^*), (q_2^*, x_2^*), (q_3^*, x_3^*) are determined using (5) and (4) by setting N at 7, 3, 1, respectively. We choose $t_i(q_i) = x_i^* d_i r_i q_i$, which is a biased function since $t_i(q_i^*) \neq 0$. We also set the scaling parameter h at the minimum value satisfying (9).

We initialize the transmission probability vectors of all users at $\boldsymbol{p} = [1/4, 1/4, 1/4]^T$, and their steering vectors at $\boldsymbol{d} = \boldsymbol{s} = [1/3, 1/3, 1/3]^T$. During the distributed channel sharing game, each user first uses 200 time slots to measure the conditional success probability vector $\boldsymbol{q}$. If during this time interval a user does not have a sufficient number of transmission attempts using a particular option g_i, then q_i is set to a small but none-zero value. After measuring the conditional success probability vector $\boldsymbol{q}$, each user then updates its transmission probability vector $\boldsymbol{p}$ in the gradient direction that maximizes the utility function. Steering vector $\boldsymbol{s}$ is updated in the gradient direction that minimizes the term $\sum_{m=1}^{M} \frac{p_m}{x_m^*} \log \frac{p_m}{s_m e x_m^*/\tilde{K}}$. Steering vector $\boldsymbol{d}$ is updated to increase the weights of feedback terms with larger values of $t_i(q_i)$. The procedure iterates till transmission probability vectors of all users converge. Figure 1 illustrated the sum throughput of the system in bits/symbol in each iteration (one iteration takes 200 time slots). The three dashed-red lines respectively correspond to the targeted sum throughput of the systems where each user only has binary transmission options of $\{g_0, g_1\}$, $\{g_0, g_2\}$, $\{g_0, g_3\}$. In this example, transmission probability vectors of all users converge quickly to $\boldsymbol{p} = [0.0507, 0, 0]^T$. In other words, users will only use the low rate option to share the multiple access channel. Note that the resulting sum throughput is slightly higher than the top dashed-red line because the targeted probability x_1^*/K is only approximately optimal for a finite K.

In this example, we update steering vector $\boldsymbol{s}$ to minimize the "self-behavior preference" term. Let us define the following region of the transmission probability vector,

$$R_{\boldsymbol{p}} = \Big\{\boldsymbol{p} | \exists \tilde{\boldsymbol{s}}, \tilde{\boldsymbol{s}} \geq \boldsymbol{0}, \tilde{\boldsymbol{s}}^T \boldsymbol{1} = 1, \text{such that} \quad \boldsymbol{p} = [\tilde{s}_1 x_1^*/\tilde{K}, \tilde{s}_2 x_2^*/\tilde{K}, \cdots, \tilde{s}_M x_M^*/\tilde{K}]^T \Big\}. \quad (16)$$

Note that, if we take the adaptation of $\boldsymbol{s}$ into consideration, the self-behavior preference term $\min_{\boldsymbol{s}, \boldsymbol{s} \geq \boldsymbol{0}, \boldsymbol{s}^T \boldsymbol{1} \leq 1} \sum_{m=1}^{M} \frac{p_m}{x_m^*} \log \frac{p_m}{s_m e x_m^*/\tilde{K}}$ achieves the same minimum value of $-\tilde{K}$ at any $\boldsymbol{p} \in R_{\boldsymbol{p}}$. Therefore, with the help of the steering vector adaptation, the self-behavior preference term only intends to keep the behavior $\boldsymbol{p}$ of a user around region $R_{\boldsymbol{p}}$. It however does not provide any preference on which transmission option should be more favorable to the user. On the other hand, the "social-behavior preference" term intends to help a user to find the best transmission option based on feedback received from the system. If we fix steering vector $\boldsymbol{d}$ at $\boldsymbol{d} = [1/3, 1/3, 1/3]^T$, then each user will assign positive probabilities to all entries of the transmission probability vector. Adaptation of steering vector $\boldsymbol{d}$ helps a user to favor the transmission option with the best feedback, which maximizes $t_i(q_i)$.

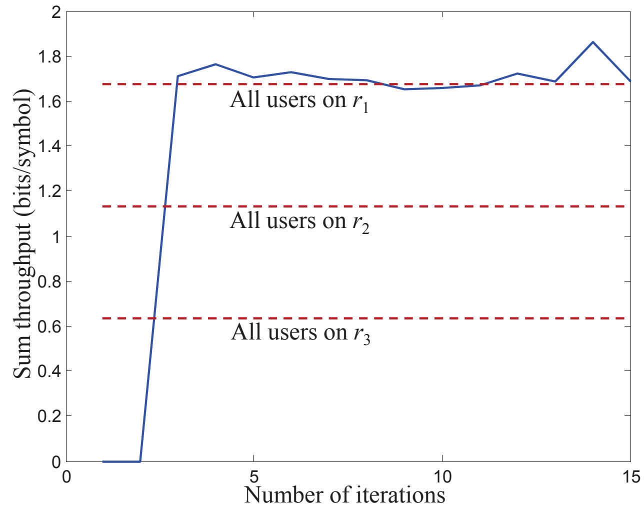

Fig. 1. Convergence of the sum throughput in bits/symbol. $P/N_0 = 10$. One iteration takes 200 time slots.

VI. Conclusion

We investigated the problem of distributed medium access control with an enhanced physical-link layer interface where each link layer user is provided with a handful of transmission options. We formulated the problem as a non-cooperative game and obtained a condition under which the game has a unique Nash equilibrium.

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