



# Proposal of a probabilistic model for multi-hazard risk assessment of structures in seismic zones subjected to blast for the limit state of collapse

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## ABSTRACT

It is desirable to verify the structural performance based on a multi-hazard approach, taking into account the critical actions the structure in question could be subjected to during its lifetime. This study presents a proposal for a probabilistic model for multi-hazard risk associated with the limit state of collapse for a reinforced concrete (RC) structure subjected to blast threats in the presence of seismic risk. The annual risk of structural collapse is calculated taking into account both the collapse caused by an earthquake event and the blast-induced progressive collapse. The blast fragility is calculated using a simulation procedure for generating possible blast configurations, and verifying the structural stability under gravity loading of the damaged structure, using a kinematic plastic limit analysis. As a case study, the blast and seismic fragilities of a generic four-storey RC building located in seismic zone are calculated and implemented in the framework of a multi-hazard procedure, leading to the evaluation of the annual risk of collapse.

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## 1. Introduction

A strategic structure could be subject to more than one critical action during its service life. A performance-based design aims to ensure the satisfactory performance of the structure during its lifetime. Therefore, it needs to consider all the possible critical actions, which the structure could be subjected to in the future. Given the uncertainty involved in characterizing these elements, it seems inevitable to address the performance-based design based on a probabilistic framework. The target structural reliability in such probabilistic framework is represented by the *probability of failure* or more specifically by the mean annual frequency that the structural response exceeds a certain limit threshold identified based on the design performance objectives.

This study aims to evaluate the probability of failure following a multi-hazard approach. In particular, it considers the case of a strategic structure located in a highly seismic zone, which is to be subjected to blast actions during its lifetime. The attention is focused on the structural collapse as the limit threshold for which the mean annual frequency of exceedance is calculated. By structural collapse, it is intended the loss of ability to withstand gravity loads. The multi-hazard approach proposes consideration of the blast actions in the form of blast fragilities in addition to the seismic fragilities. The blast fragility, defined as the probability of collapse given that a blast event has taken place in the structure, is evaluated

using an advanced simulation method. It is assumed that a possible blast scenario is identified by the quantity of the explosive and the location of the center of the blast within or close to the structure. For each possible blast scenario realization, generated by the simulation, the stability of the structure is verified by performing a plastic limit analysis on the damaged structure [1]. The seismic fragility, defined as the probability of structural collapse given a specified level of ground motion intensity, is calculated using a non-linear static analysis (pushover) approach and integrated with the corresponding seismic hazard. Finally, the mean annual frequency of collapse can be evaluated and benchmarked against the acceptable risk threshold. As a case study, the blast and seismic fragilities of a generic four-storey RC building located in the seismic zone are calculated and implemented in the framework of a multi-hazard procedure, leading to the evaluation of the annual risk of collapse.

## 2. Multi-hazard assessment/design

The probability-based multi-hazard design of a structure is performed taking into account all possible events that could potentially cause significant damage. In particular, for the limit state of collapse, the probability of collapse can be written as [2]:

$$P(C) = \sum_A P(C|A)P(A) \quad (1)$$

where  $A$  stands for a critical event, such as, earthquake, Blast, etc. Formally,  $A$  can be written as the logical union of the potential critical events, that is:

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$$A \equiv EQ + \text{Wind} + \text{Gas Explosion} + \text{Blast} + \text{MISC} \quad (2)$$

Eq. (1) is written using the total probability theorem assuming that the critical events  $A$  are mutually exclusive (i.e., they cannot happen simultaneously) and collectively exhaustive (i.e., all of the potential  $A$  are considered). Obviously, the events that contribute to  $A$  vary based on the type, location, and function of the structure to be designed or assessed. That is, depending on the particulars of each problem, some of the terms in  $A$  might be dominant with respect to the others. The *de minimis* risk  $v_{dm}$ , which defines that risk below which society normally does not impose any regulatory guidance, is in the order of  $10^{-7}$ /year [3]. Therefore, if the annual risk of occurrence of any critical event  $A$  is considerably less than the *de minimis* level, it could be omitted from the critical events considered in Eq. (2). Therefore, the multi-hazard acceptance criteria can be written as following:

$$P(C) = \sum_A P(C|A)P(A) \leq v_{dm} \quad (3)$$

The above-mentioned criteria could be used both for probability-based design and assessment of structures for limit state of collapse.

Considering a particular case in which the critical events are earthquake and blast, the design/assessment criterion can be written as:

$$v_C = P(C|EQ)v_{EQ} + P(C|Blast)v_{Blast} \leq v_{dm} \quad (4)$$

where  $v_C$  stands for the annual rate of collapse and  $v_{EQ}$  and  $v_{Blast}$  stand for the annual rates of occurrence of earthquake and blast events of significance, respectively.  $P(C|EQ)$  and  $P(C|Blast)$  represent seismic and blast fragilities. In this case, considering earthquake and blast hazards as mutually exclusive implies assuming that just one of them can induce structural collapse. Furthermore, it is assumed that after each critical event, there is enough time to repair the strategic structure back to its intact state. Note that  $v_C$  is a rate of exceedance and not a probability, however, for very rare events, the probability is approximately equal to the annual rate. The annual rate of an earthquake event of interest can be calculated using probabilistic seismic hazard analysis (PSHA) for the site of the project. On the other hand, estimation of the annual rate of the occurrence of a blast event caused by terrorist attack cannot be easily quantified and defined analytically. In other words, the estimation of  $v_{Blast}$  is not entirely an engineering problem since it depends on socio-political considerations and how strategically vulnerable the structure is against such events. However, in order to facilitate calculations, it is assumed here that  $v_{Blast}$  is a known quantity.

Alternatively, in cases where  $v_{Blast}$  cannot be identified, one could perform a scenario-based calculation of the probability of collapse and compare it against an acceptable threshold that is larger than *de minimis* level (e.g.,  $10^{-2}$  is the conditional collapse probability necessary to achieve the *de minimis* level of less than  $10^{-6}$ /yr, see [2]). It should be noted that employing the multi-hazard formulation makes it possible to consider the rehabilitation strategies with respect to both blast and earthquake. That is, the risk reduction techniques for blast and earthquake can be similar (i.e., composite wrapping of columns, steel bracing installations). In fact, the authors have verified such correlation in a different paper [4], in which it has been demonstrated that a seismic retrofit intervention (e.g., steel bracing installations) can lead to a reduction in the risk of blast-induced progressive collapse.

### 3. Blast loading

An explosion induces mainly a quick and significant increase of pressure in the medium where it occurs, i.e., air or water. Such overpressure propagates as a wave, the so called “blast wave”, and is characterized by its speed, intensity and duration. These parameters are fundamental in order to evaluate the actions that

an explosion can induce in the structural elements in its vicinity. The numerical values of these parameters depend on several aspects, such as, type and the amount of the exploding mass, distance of the target of interest from the explosion, geometry of the target, type of reflecting surfaces (e.g., the ground in case of external explosions or walls or slabs in case of closed-in explosions). In the past decades, several investigations have been performed on such aspects and they have provided reliable numerical procedures for quantification of the overpressure time-histories. In the case of blast explosion, the induced overpressure follows a trend over time similar to that shown in Fig. 1, where a positive decaying phase is followed by a weaker negative phase. The duration of the blast wave depends on the amount of the exploding charge and its distance from the target; however, the phenomenon is very quick and can last up to  $10^{-2}$  s.

The primary effect of a blast explosion on civil structures is caused by such rapid and intense action that is able to induce severe local structural damages. In fact, the applied loads are so fast that they are unable to activate the global vibration modes of the structure, since the inertia corresponding to such modes has no sufficient time to react. Therefore, in case of an RC framed structure, the blast-induced overpressures hit directly the single frame elements, which behave as independent structures, and can be modeled as fixed ends elements [5].

An indirect effect of blast explosion on civil structure is progressive collapse. The progressive collapse can be defined as a mechanism involving a large part of a structure, triggered by local less-extensive damage in the structure. In fact, a blast explosion occurring within or near an RC framed building can cause the loss of one or more single frame elements. Having lost some elements, the whole structure can become unstable, failing under the present vertical loads. That is, the structure can eventually develop a global mechanism, which is widely referred to as the progressive collapse mechanism [6–8]. Design and/or assessment of structures accounting for such failure mechanism can follow a direct approach or an indirect approach [9]. In the indirect approach, resistance to progressive collapse is pursued guaranteeing minimum levels of strength, continuity and ductility, whereas in the direct approach progressive collapse scenarios are directly analyzed. Actually, the progressive collapse mechanism is most often identified as the predominant mode of failure after a blast event [10] and it is already the subject of wide research related to the protection of critical infrastructures [7,8,10–12].

## 4. Blast fragility

### 4.1. Using simulation-based reliability methods for risk assessment

The blast fragility denoted by  $P(C|Blast)$ , in the context of this work, can be defined as the conditional probability for the event

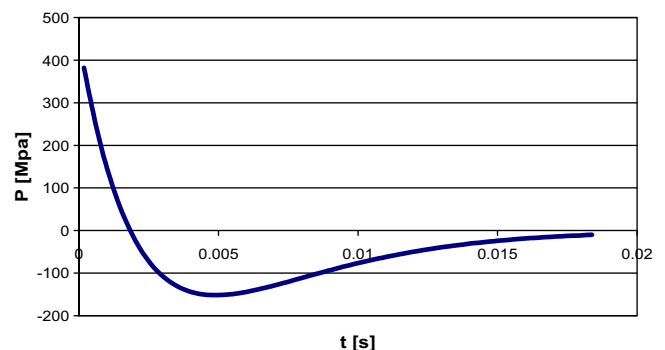


Fig. 1. Blast overpressure in air.

of progressive collapse given that a blast event takes place near or inside the strategic structure in question.

Consider that real vector  $\underline{\theta}$  represents the uncertain quantities of interest, related to structural modeling and loading conditions. Let  $p(\underline{\theta})$  represent the probability density function (PDF) for the vector  $\underline{\theta}$ . The  $P(C|Blast)$  can be written as follows:

$$P(C|Blast) = \int I_{C|Blast}(\underline{\theta})p(\underline{\theta})d\underline{\theta} \quad (5)$$

where  $I_{C|Blast}(\underline{\theta})$  is an index function which is equal to unity in the case where  $\underline{\theta}$  leads to blast-induced progressive collapse and zero otherwise. Here, the probability of progressive collapse  $P(C|Blast)$  is calculated using standard Monte Carlo (MC) simulation for generating  $N_{sim}$  samples  $\underline{\theta}_i$  from PDF  $p(\underline{\theta})$ . The event of progressive collapse is identified by the ratio index  $\lambda_c(\theta_i)$  which is the factor by which the gravity loads should be multiplied in order to create a global collapse mechanism. In case it assumes a value less than unity, the event of progressive collapse is actually activated, since the acting loads are sufficient to induce instability in the structure. Moreover, the uncertain quantities of interest here are the amount of explosive and its position with respect to the structure. Obviously, any other uncertain quantity such as those related to structural modeling can be added to vector of uncertain parameters  $\underline{\theta}$ . For each simulation realization  $\underline{\theta}_i$ , the following two steps are performed:

- (1) A local dynamic analysis is performed on the column elements affected by the blast in order to verify whether they can resist the explosion and keep their vertical load carrying capacity.
- (2) After identifying the damaged columns to be removed, a kinematic plastic analysis is performed on the damaged structure in order to evaluate the progressive collapse index  $\lambda_c(\theta_i)$  and to control whether the structure is able to carry the gravity loads in its post-explosion state.

#### 4.2. Closed-form solution for the local dynamic analysis

As mentioned in a previous section, the blast action can be modeled by a quick decay pressure time-history curve. This curve can be approximated by a triangular shape identified by two parameters, namely, the initial peak pressure  $p_0$  and the duration  $t_{plus}$  of the positive phase. These parameters, which depend on the amount of explosive and the distance from the charge, can be evaluated according to empirical formulas available in literature (e.g., [5,13]). Since the blast-induced action is very rapid and consequently the structural inertia does not have sufficient time to respond, the individual elements react to it as if they were fixed-end elements. Moreover, for the same reason, the structural damping can be ignored.

For each simulation realization, the step 1 described above is conducted, performing the dynamic analysis of an un-damped distributed-mass fixed-end beam subject to triangular impact loading, for all the columns on the same floor as the explosion. Moreover, for the sake of simplicity in calculations, it is assumed that the blast action is constant across the length of the columns. Consider that the beam in question has constant  $EI$ , constant distributed mass  $\bar{m}$  and length  $L$ . It can be shown [14] that the period of first-mode vibration of a fixed-end beam with the above-mentioned properties is equal to:

$$T = 2\pi \left( \frac{L}{4.73} \right)^2 \sqrt{\frac{\bar{m}}{EI}} \quad (6)$$

It happens that the equation of motion of the beam in response to impact loading is identical to that of a single degree of freedom

(SDOF) system with period of vibration equal to  $T$ . An analytic closed-form solution can be found for an un-damped SDOF system subject to triangular impulse loading [14]. It turns out that the impulse duration is much smaller than the natural period of vibration of the SDOF system; therefore, the maximum response will most likely be in the free-vibration response phase. The free-vibration response  $Y(t)$  of an SDOF oscillator with angular frequency  $\omega = 2\pi/T$  can be derived from following:

$$Y(t) = Y(t_{plus}) \cos \omega t + \frac{\dot{Y}(t_{plus})}{\omega} \sin \omega t \quad (7)$$

Where  $Y(t_{plus})$  and  $\dot{Y}(t_{plus})$  are the displacement and velocity initial conditions for the free-vibration response evaluated at the end of the triangular impulse loading. Thus, the maximum response will be equal to:

$$\rho = \sqrt{Y(t_{plus})^2 + \left( \frac{\dot{Y}(t_{plus})}{\omega} \right)^2} \quad (8)$$

It can be shown that the maximum bending moment and shear will take place at the fixed ends and will be calculated as following:

$$M_{max} = 1.26 \left( \frac{4.73}{L} \right)^2 EI \rho \quad (9)$$

$$V_{max} = 1.24 \left( \frac{4.73}{L} \right)^3 EI \rho$$

In order to verify whether the individual column can resist the explosion, the maximum blast-induced bending moment and shear  $M_{max}$  and  $V_{max}$  are compared against the ultimate bending and shear capacity of the element at its ends. The fact that the linear elastic analysis method incorporated for the local dynamic analysis of each column arrives at a closed-form solution, makes it particularly easy to quickly check the affected columns and identify those which needed to be removed for each blast scenario generated inside the Monte Carlo simulation scheme. The accuracy of the checking phase could be improved by using the non-linear time-step methods in order to solve the equation of motion under the blast impact loading.

#### 4.3. Kinematic plastic analysis on damaged structure

After identifying and removing the damaged elements, it should be verified whether the damaged structure can withstand the applied vertical loads. This is essentially a global stability analysis of the damaged structure. A possible approach to performing such analysis would be to conduct a plastic limit analysis. A plastic limit analysis [1,15] involves finding the load factor  $\lambda_c$  on the applied loads for which the following effects occur:

- (1) Equilibrium conditions are satisfied.
- (2) A sufficient number of plastic hinges are formed in the structure in order to activate a collapse mechanism in the whole structure or in a part of it.

It is assumed that the non-linear behavior in the structure is concentrated at the element ends and the member ends are capable of developing their fully plastic moment (i.e., the brittle failure modes such as axial and shear failure or the ultimate rotational failure do not take place before the member has developed its plastic bending capacity).

It has been shown [16] that the procedure for the plastic limit analysis can be defined as a linear optimization programming with the objective of minimizing the load factor  $\lambda_c$ .

This linear programming problem could be resolved by employing a simplex algorithm.

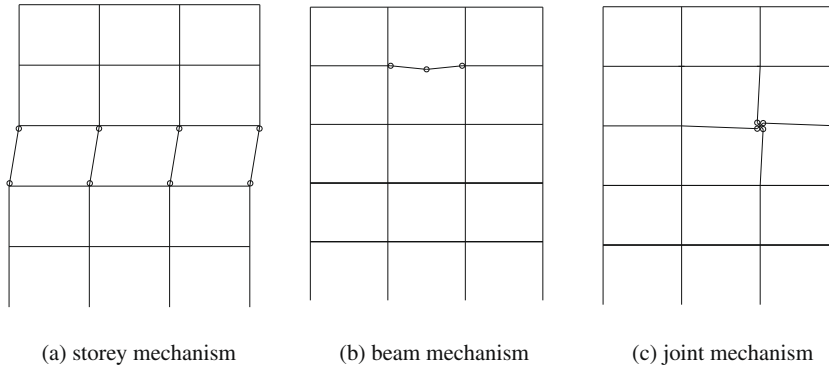


Fig. 2. Principal mechanisms.

For example, in the particular case of a RC framed structure, the independent mechanisms are classified as follows [16] (Fig. 2): (a) the soft-story mechanisms in which the plastic hinges at both ends of all the columns within a given storey are activated, (b) the beam mechanisms in which (at least) three hinges are formed in given beam, and (c) the joint mechanisms in which the end hinges of all the frame elements converging into a given joint are activated.

In static loading problems, a  $\lambda_c$  less than or equal to unity indicates that the structure is already unstable under the applied loads. On the other hand, in instantaneous dynamic loading problems, the threshold for  $\lambda_c$  is equal to 2. In case of progressive collapse, it has been shown that a value 2 is probably conservative and the actual value of  $\lambda_c$  causing instability in the structure is between 1 and 2 [17]. It should be mentioned that the plastic limit analysis algorithm presented herein ignores some second-order non-linear actions that could prevent a mechanism from forming (e.g., the catenary actions and the arch effects).

#### 4.4. Calculating the blast fragility implementing the MC simulation

As mentioned in a previous section, the blast fragility is defined as the probability of progressive collapse event given that a blast event takes place inside or in the vicinity of the structure in question. The progressive collapse event can be characterized by a Bernoulli-type variable that is equal to unity in the event of progressive collapse and equal to zero otherwise. Using the kinematic plastic limit analysis described in the previous section, the Bernoulli collapse variable denoted by  $I_{C|Blast}(\theta)$  can be determined as a function the collapse load factor  $\lambda_c$ :

$$I_{C|Blast}(\theta) = \begin{cases} 0 & \text{if } \lambda_c > \lambda_{c,th} \\ 1 & \text{if } \lambda_c \leq \lambda_{c,th} \end{cases} \quad (10)$$

Where  $\lambda_{c,th}$  is the threshold value for the load factor indicating the onset of progressive collapse varying between 1 and 2. The MC procedure can be used to generate  $N_{sim}$  realizations of the uncertain vector  $\theta_i$  according to its probability density function (PDF)  $p_{\theta}$ . Finally, the conditional probability of progressive collapse in Eq. (5) can be solved numerically as the expected value of the Bernoulli collapse index variable  $I_{C|Blast}(\theta)$ :

$$P(C|Blast) \equiv \frac{\sum_{i=1}^{N_{sim}} I_{C|Blast}(\theta_i)}{N_{sim}} \quad (11)$$

It can be shown that the coefficient of variation of the conditional progressive collapse probability can be calculated as follows:

$$C.O.V._{P(C|Blast)} = \sqrt{\frac{1 - P(C|Blast)}{N_{sim} \cdot P(C|Blast)}} \quad (12)$$

## 5. Numerical example

A possible application of the methodology described in the previous section can refer to the calculation of the mean annual risk for progressive collapse of a generic RC framed building. A numerical example is here presented; the characteristics of the case-study structure are outlined in the following.

### 5.1. Structural model description

The case-study building is a generic five-story RC framed structure designed according to the European seismic provisions. The structural model is illustrated in Fig. 3, presenting a plan of the generic storey; column sections are all  $0.60 \times 0.30 \text{ m}^2$  at the first and the second floor and  $0.50 \times 0.30 \text{ m}^2$  at other floors, whereas two types of beam are present, Type A and Type B, whose width and height are  $0.30 \text{ m}$  per  $0.50 \text{ m}$  and  $0.80 \text{ m}$  per  $0.24 \text{ m}$ , respectively; the floors are supposed to be one-way joist slabs,  $0.24 \text{ m}$  thick.

Fig. 4 shows a 3D view of the model. Each storey is  $3.00 \text{ m}$  high, except the second one, which is  $4.00 \text{ m}$  high. The non-linear behavior in the sections is assumed to be only flexural and is modeled based on the concentrated plasticity concept. It is assumed that the plastic moment in the hinge sections is equal to the ultimate moment capacity in the sections which is calculated using the Mander [18] model for concrete and elastic–plastic model for steel rebar. Materials parameters and RC sections properties are outlined in Tables 1 and 2, respectively. The period for small-amplitude fundamental mode of vibration is equal to  $0.58 \text{ s}$  and the first-mode damping ratio is equal to 5%.

### 5.2. Characterization of the uncertainties

As mentioned in the methodology, the uncertain quantities of interest in this study are the amount of explosive  $W$  and its position with respect to a fixed point within the structure denoted by  $R$ . Formally, the vector of uncertain parameters contains two uncertain quantities:  $\underline{\theta} = \{W, R\}$ . The following assumptions<sup>1</sup> are made in order to determine the possible values of  $\theta$ :

- The access to the structure is allowed to people at each floor whereas at the first floor, representing an underground garage, the access is permitted to cars; at the second floor, corresponding to the ground level, a fence system is present at  $10 \text{ m}$  from the structural perimeters, providing a stand-off distance for cars and trucks.

<sup>1</sup> The blast scenarios in the article are merely for illustrative purposes. Care has been taken to use plausible scenarios and to assign them relative frequencies that would roughly reflect the corresponding relative plausibility.

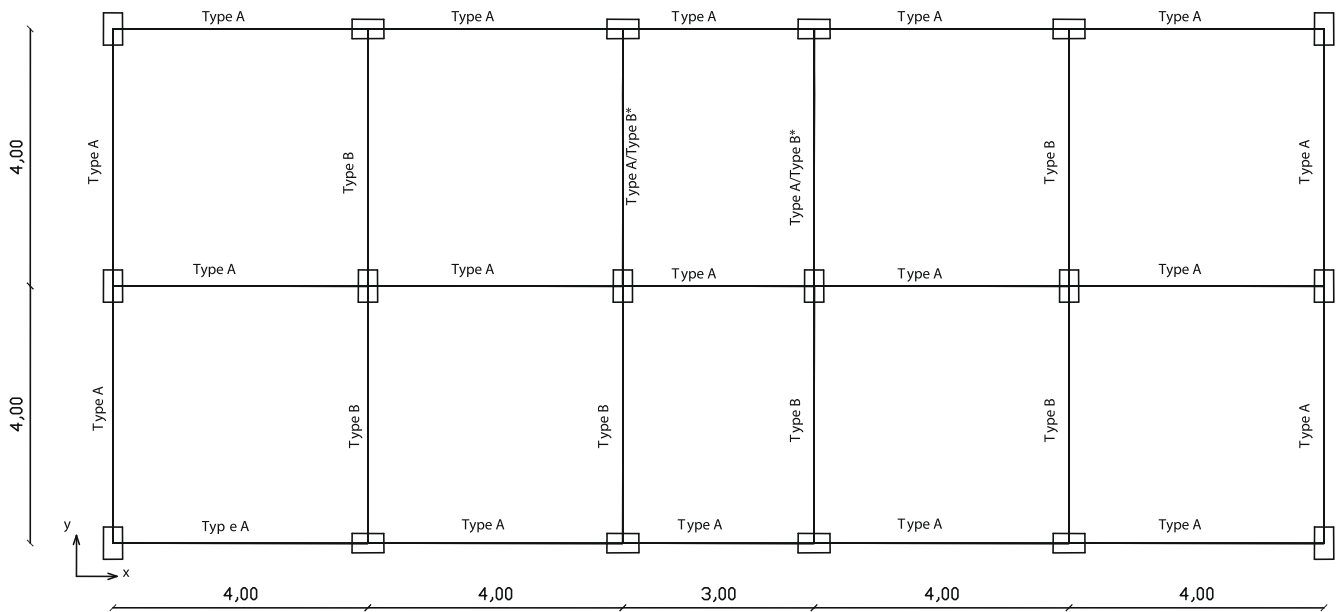


Fig. 3. Storey view (dimensions in m).

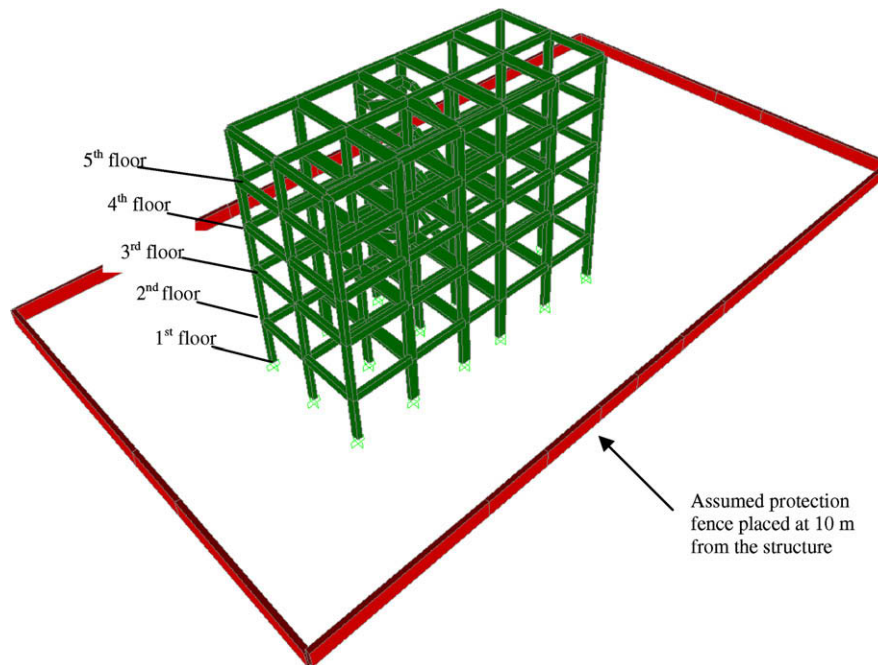


Fig. 4. 3D model view.

Table 1  
Materials parameters.

| Concrete strength (MPa) | Concrete strain corresponding to maximum stress | Concrete ultimate strain (MPa) | Steel yielding design stress (MPa) | Steel Young modulus (GPa) |
|-------------------------|---|--------------------------------|------------------------------------|---------------------------|
| 11                      | 0.02  | 0.04                           | 382.6                              | 210                       |

- Consequently, a backpack bomb can explode from the second to the fifth floor of the structure, a car bomb can explode at the first floor and, in addition a car bomb or a truck bomb can explode at a variable point, at 10 m from the structural perimeter, in correspondence to the second floor.
- For each simulation realization, the center of explosion is determined assuming that the explosion occurs within the structure or outside, with a probability of 30% and 70%, respectively. Once the explosion scenario occurs inside the structure, with the same probability it can take place at one of the five floors of the building. Then the amount of explosive is defined assuming that it can vary between 15 and 35 kg of equivalent TNT (simulating a backpack bomb), if the explosion takes place within the structure from the second to the fifth floor, and between 200 and 500 kg of equivalent TNT (simulating a car bomb), if the explosion occurs at the first floor, corresponding to the underground level. Furthermore, in case the explosion occurs outside, at the

**Table 2**  
RC sections properties.

| Element type | Section  | Tensile rebar (mm <sup>2</sup> )  | Compression rebar (mm <sup>2</sup> ) | Stirrups (mm)             | Rebar cover (mm) |
|--------------|--|---|--------------------------------------|---------------------------|------------------|
| Columns      | x direction – 1st and 2nd floor, entire length + 3rd floor, bottom section – columns 1, 6, 7, 8, 9, 10, 11, 12, 13, 18 | 8.04  | 8.04                                 | φ8/10 at the end sections | 30               |
|              | y direction – 1st and 2nd floor, entire length + 3rd floor, bottom section – columns 1, 6, 7, 8, 9, 10, 11, 12, 13, 18 | 6.03  | 6.03                                 | φ8/10 at the end sections | 30               |
|              | x direction – 1st and 2nd floor, entire length + 3rd floor, bottom section – columns 2, 3, 4, 5, 14, 15, 16, 17        | 6.03  | 6.03                                 | φ8/10 at the end sections | 30               |
|              | y direction – 1st and 2nd floor, entire length + 3rd floor, bottom section – columns 2, 3, 4, 5, 14, 15, 16, 17        | 8.04  | 8.04                                 | φ8/10 at the end sections | 30               |
|              | x and y directions – 3rd floor, top section + 4th and 5th floor, entire length – all columns                           | 6.03  | 6.03                                 | φ8/10 at the end sections | 30               |
|              | Beams  | 50 × 30 beams – positive bending moment, midspan section + negative bending moment, ends sections | 8.04                                 | 4.02                      | Not specified    |
|              | 24 × 100 beams – positive bending moment, midspan section + negative bending moment, ends sections                     | 9.04  | 4.52                                 | Not specified             | 30               |

ground level, the amount of TNT has the 10% of probability to vary between 15,000 and 25,000 kg of equivalent TNT (simulating a truck bomb) and the remaining probability to vary between 200 and 500 kg of equivalent TNT. All uncertain quantities are assumed to be uniformly distributed (i.e., the possible values for the uncertain quantity are all equally likely).

The process in determining the realization of  $\theta$  vector is clarified in Fig. 5.

It should be noted that the vector  $\theta$  ideally needs to also include the uncertainties in the structural modeling parameters and the structural component capacities. However, the overall effect of these sources of uncertainty seems not to drastically affect the overall structural risk compared to the uncertainties in blast loading parameters (see [19] for further discussion of the effect on blast risk and [20] for the effect on seismic risk). Hence, the uncertainties in structural modeling and component capacity have not been considered in the present work.

### 5.3. Characterization of the parameters defining the local dynamic analysis

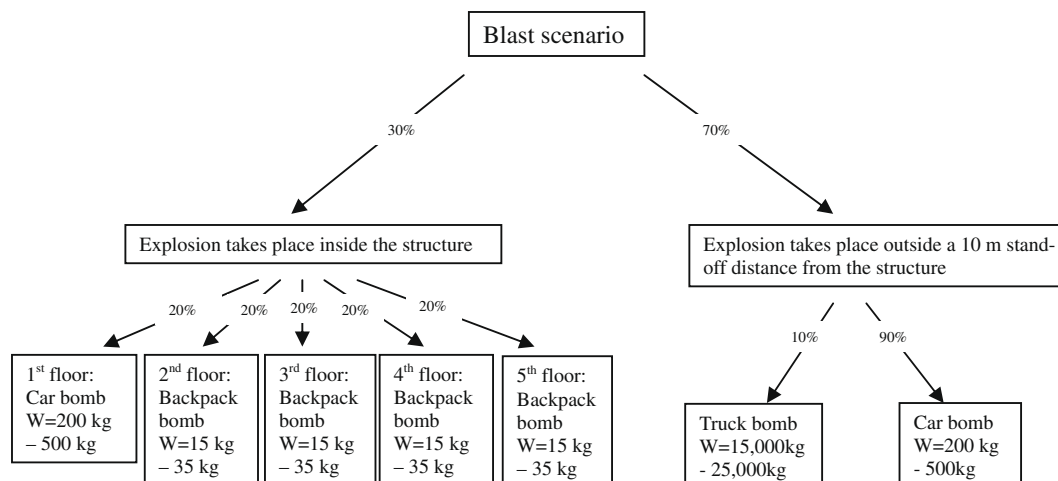
In case of inside explosion, it is assumed that only the columns on the same floor as that of the explosion are affected by it. This assumption is supported by the fact that the columns on the other floors and the floor beams are sheltered from the blast wave by the

floor slab system [5]. Therefore, in case of outside explosion, only the external columns directly viewable from the charge location are affected by the explosion, since the internal ones are sheltered by the perimeter walls [5].

Then, for each of the columns hit by the explosion at the distance  $r$  from the center of the charge, given the amount of explosive  $W$ , the reduced distance  $Z = r/\sqrt[3]{W}$  is calculated. Then, a triangular impulse loading is considered to be acting on the columns (Fig. 6), whose parameters  $p_0$  (maximum initial pressure) and  $t_{\text{plus}}$  (duration of the impulse) can be obtained based on the semi-empirical formulas available in the literature [13]. It is further assumed that the intensity of the impact loading is uniform across the column height. Furthermore, since such load generally acts in a direction that is not parallel to local axes of the column, it is divided into two components and the maxima for bending moment and shear force are evaluated. These values are then used to verify whether the column fails; in particular, in case of bending moment a convex domain criterion is employed, using the following formulation [21]:

$$\left(\frac{M_x}{M_{xu}}\right)^\alpha + \left(\frac{M_y}{M_{yu}}\right)^\alpha < 1 \quad (13)$$

where  $M_x$ ,  $M_{xu}$ ,  $M_y$  and  $M_{yu}$  represents the acting and the ultimate bending moment on both local axes directions, respectively, and  $\alpha$  is a parameter defining the convex domain shape, equal to 1.5 in the present case [21]. On the contrary, in order to verify if a shear



**Fig. 5.** Blast realization logic tree.

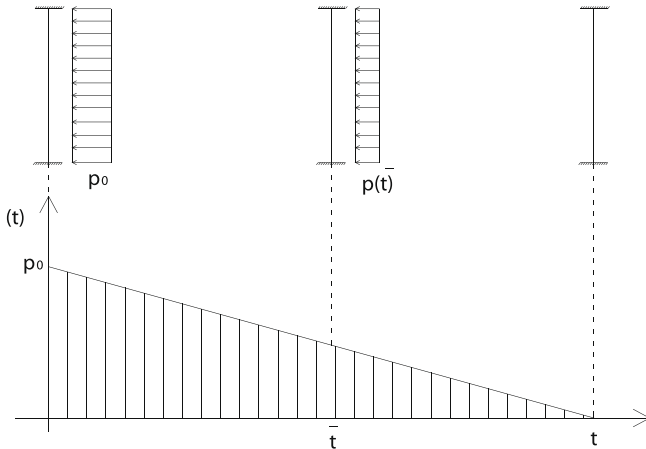


Fig. 6. Blast impulse loading.

failure occurs, the acting forces in the two directions are compared separately with their respective ultimate shears. It should be mentioned that it is conservatively assumed that the concrete and steel mechanical properties do not increase significantly due to the strain-rate effect.

#### 5.4. Blast fragility

A standard MC simulation technique was used to generate 500 blast scenario realizations, assuming that the structure was subjected to its gravity loads and to the 30% of the characteristic live loads equal to  $2.0 \text{ kN/m}^2$  (the live load considered in the analysis is equal to  $0.6 \text{ kN/m}^2$ ). For each of these realizations, the collapse load factor  $\lambda_c$  was calculated. The cumulative distribution function for the load factor denoted by  $P(\lambda \leq \lambda_c | \text{Blast})$  is plotted for possible values of  $\lambda_c$  in Fig. 7. The threshold value, identifying progressive collapse region, is  $\lambda_{c,th} = [1.2]$ , as marked in Fig. 7. However, considering a conservative value equal to 2, it can be observed that probability  $P(C | \text{Blast})$  that a blast event leads to progressive collapse of the case-study structure is around 0.18. On the contrary, the value  $\lambda_c = 4.22$  corresponds to the case that none of the columns is eliminated due to the blast; in other words, it is the load factor corresponding to the original structure. This explains why the

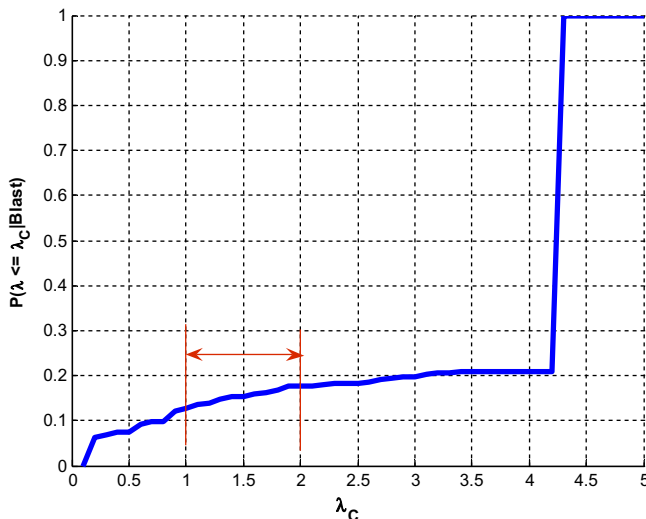


Fig. 7. Blast fragility.

probability that a blast event leads to a collapse load factor less than  $\lambda_c \leq 4.22$  is equal to unity.

In order to gain further insight about the simulation results, the blast scenarios leading to progressive collapse, identified by  $\lambda_c \leq 2$ , are plotted in Figs. 8 and 9. Fig. 8 illustrates the location of the blast charges that lead to progressive collapse on the structure's original geometry together with the histogram for the storey in which the explosion takes place. This kind of plot is very helpful for identifying the critical zones within which, an explosion could most likely lead to progressive collapse. It can be observed that the collapse scenarios take place predominantly on the ground level (second storey) at selected stand-off distance of 10 m from the structure. Fig. 9 shows the histograms for the quantity of the explosive, distinguishing the three blast categories (backpack bomb, car bomb and truck bomb) which seems to indicate that the event of progressive collapse is not very sensitive to the quantity of explosive, within each blast category (the histograms seem to imply uniform distribution with respect to the quantity of explosive). This is to be expected since the induced blast pressure depends on the explosive quantity by the cubic root, as indicated in [5,13].

#### 5.5. Seismic fragility

The seismic fragility for the case-study structure is calculated in two steps. In the first step, a non-linear static analysis is performed on the structure model using SAP2000 (version 10) software utilizing beam-column elements with concentrated plastic hinges at the points of maximum moment. The pushover curve or roof displacement versus the base shear for the analyzed structure is evaluated and the point at which the first element in the structure reaches its ultimate rotation capacity is determined, according to European seismic guideline [22]. The equivalent elastic-perfectly plastic SDOF system corresponding to the above-mentioned pushover curve is then approximated using a procedure recommended in [22] and is illustrated in Fig. 10. The point on the figure that is marked by  $d_{max}$  corresponds to the first instance when ultimate rotation capacity takes place in the structure. In the second step a suite of 50 ground motion accelerations are applied to the equivalent elastic-plastic SDOF system based on the incremental dynamic analysis or the multiple-stripe analysis procedures [23,24]. Based on these non-linear analysis procedures, the suite of ground motion records are scaled to increasing levels of spectral acceleration and applied to the structure. At each spectral acceleration level, the probability of structural failure was estimated with the ratio of number of records that cause maximum displacement in the equivalent SDOF system that are greater than  $d_{max}$ , to the total number of records (i.e., 50). The seismic fragility curve is plotted in Fig. 11 after a Lognormal probability distribution is fitted to the results, showing the probability of failure at each spectral acceleration level versus spectral acceleration. It should be mentioned that the standard deviation for the fragility curve only represents the aleatory record-to-record variability; it does not take into account either the record-to-record variability in structural displacement capacity or the epistemic structural modeling uncertainties.

## 6. Discussion on the case study

In order to calculate the seismic risk, the fragility should be integrated with the hazard for spectral acceleration at a period close to the fundamental period of the structure. Here, the annual rate of exceeding spectral acceleration at  $T_1 = 0.50$  has been extracted from Italian National Institute of Geophysics and Volcanology (INGV) database [25] assuming that the structure in question is located at Naples, Italy. Consequently, the seismic contribution to

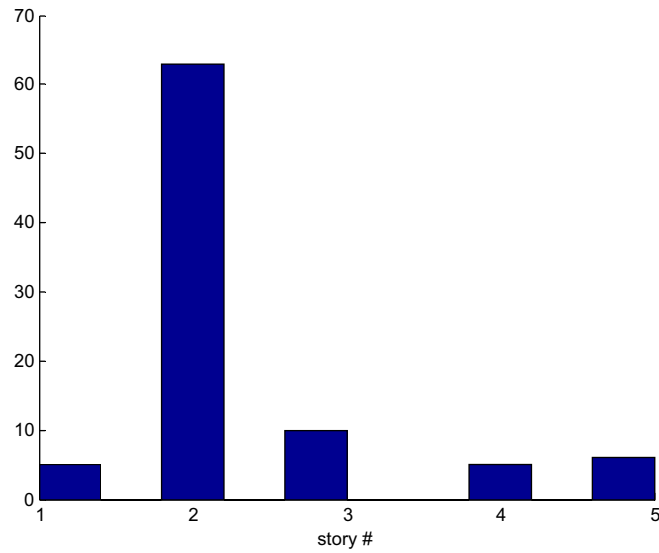
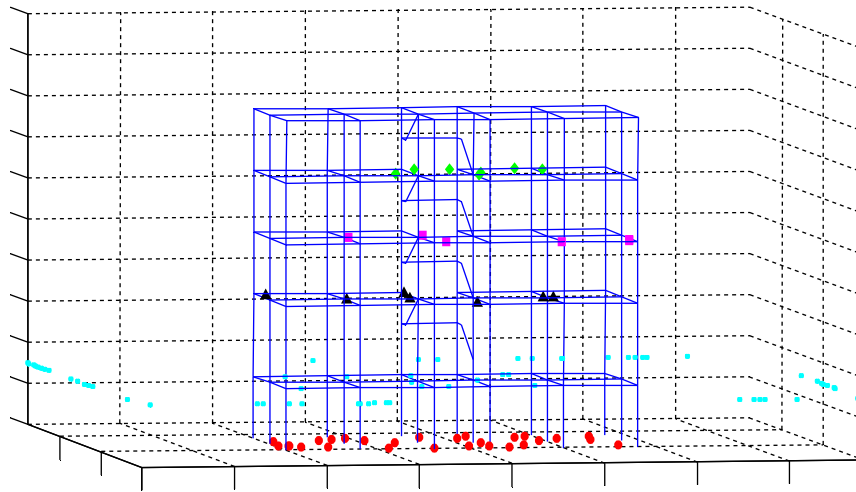


Fig. 8. The blast scenarios that lead to progressive collapse in the structure.

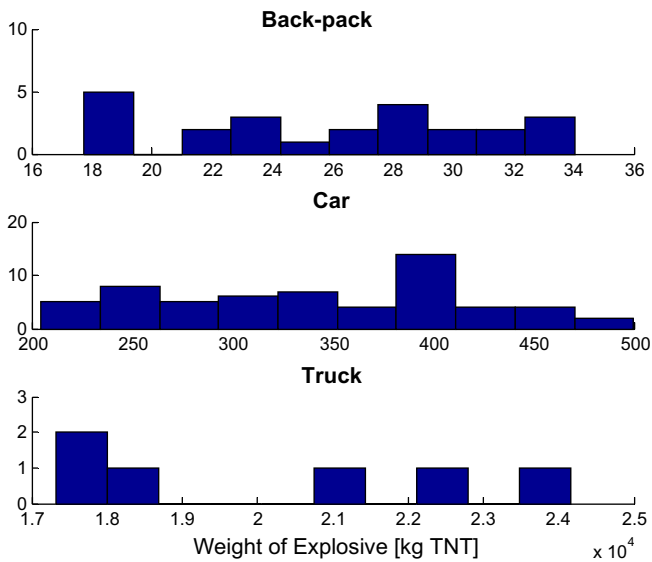


Fig. 9. Weight of explosive in the blast scenarios that lead to progressive collapse in the structure.

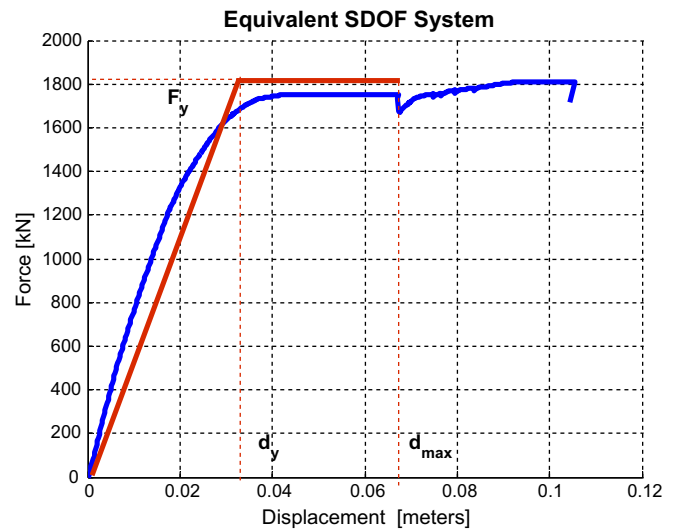


Fig. 10. The pushover curve for the equivalent SDOF system and the simple elastic-plastic curve fitted to it.



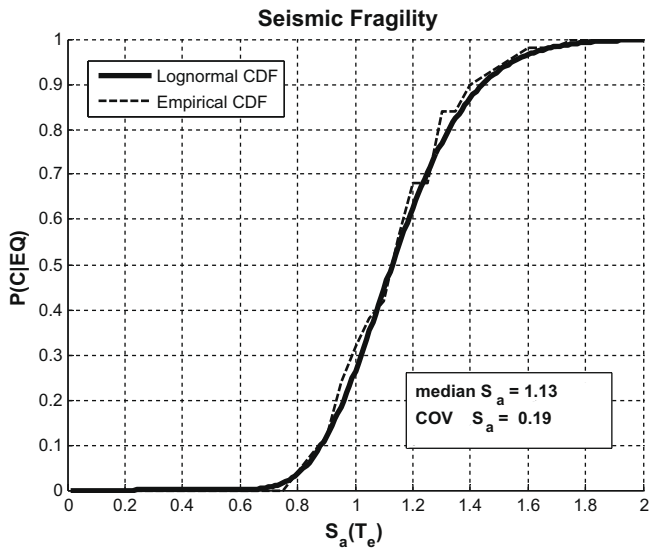


Fig. 11. The seismic fragility curve or the probability of collapse for a given spectral acceleration.

the total risk (the first term in Eq. (4)) is calculated and is equal to  $5.8 \times 10^{-5}$ .

Therefore, the annual risk of collapse, to compare with the *de minimis* threshold, can be calculated from Eq. (4) as follows:

$$v_C = 5.8 \times 10^{-5} + 0.18 \cdot v_{\text{Blast}} \quad (14)$$

where the value of  $P(C|\text{Blast})$ , evaluated with the presented procedure and equal to 0.18, is substituted. As it can be observed from Eq. (14), the blast fragility needs to be multiplied by the annual rate  $v_{\text{Blast}}$  that a significant blast event takes place. However, as mentioned before, this rate is difficult to evaluate as an engineering quantity and it depends more on the socio-political circumstances and the strategic importance of the structure. Nevertheless, the blast contribution to  $v_C$ , equal to  $0.18v_{\text{Blast}}$ , can be compared with the seismic contribution (equal to  $5.8 \times 10^{-5}$ ) in order to assess its relative importance and determine whether blast poses a significant risk of collapse in the investigated structure. For instance, in case of a non-strategic structure  $v_{\text{Blast}}$  can be in the order of  $10^{-7}$  [2], making blast contribution to the annual risk of collapse negligible, compared with that of earthquake. Alternatively, in case of a strategic structure  $v_{\text{Blast}}$  can be as large as  $10^{-4}$ ; in such case, blast hazard will be the dominant term in Eq. (4) for calculating the annual risk of collapse.

It should be noted that the terms blast fragility and earthquake fragility plotted in Figs. 7 and 11 are calculated differently, although both are referred to as fragility. Blast fragility is defined as the probability of progressive collapse given that a significant blast event has taken place and it needs to be multiplied by the annual rate of significant blast event taking place in order to yield the mean annual risk of collapse. On the other hand, seismic fragility is defined as the probability of structural collapse given a specific value of spectral acceleration and it needs to be integrated with the annual rate of exceeding spectral acceleration in order to yield the mean annual risk of collapse.

## 7. Conclusions

A methodology for calculating the annual risk of collapse for a strategic structure located in a seismic zone is presented in the framework of multi-hazard assessment. In this methodology, given that a blast event of interest takes place, the probability of progres-

sive collapse is calculated using a MC simulation procedure. The simulation procedure employs a closed-form solution for local dynamic analysis of structural elements subjected to impulsive blast-induced loads. It also implements an efficient limit state analysis to verify whether progressive collapse mechanisms are activated, under the service vertical loads, on the damaged structure. As a numerical example, a case study is presented, in which the annual rate of collapse of a generic RC frame building is discussed.

Following observations and outcomes can be made:

- The results for the presented case-study seem to justify the choice of a MC simulation procedure for calculating the probability of progressive collapse. That is, given that a blast event takes place, the probability of progressive collapse is found to be around 18%, which is within the range of probabilities calculated efficiently with MC simulation. For example, the 500 realizations generated by conducting the MC simulation herein lead to a reasonably low coefficient of variation in the failure probability estimate (equal to 0.09).
- The methodology presented herein exploits the particular characteristics of the blast action and its effect on the structure in order to achieve maximum efficiency in the calculations. More specifically, the use of plastic limit analysis (formulated as a linear programming problem) instead of a common 3D finite element analysis renders the calculations significantly more rapid and thereby feasible for implementation within a simulation procedure. Moreover, the derivation of an analytic closed-form solution for the problem of dynamic impulse facilitates the damage analysis of individual structural elements for each simulation realization.
- The efficiency and rigor of the presented methodology make it particularly useful as a design and/or retrofit tool for strategic structures. More specifically, the outcome of the MC simulations can be used to mark the location of critical blast scenarios on the structural geometry and identify the risk-prone areas. An example of a simple and effective prevention strategy would be to limit or to deny the access to critical zones within the structure, once they are identified using the presented procedure.
- Once the annual rate of blast  $v_{\text{Blast}}$  is known, the blast fragility  $P(C|\text{Blast})$  evaluated herein can be used to determine the annual risk of collapse  $v_C$  (Eq. (4)).

Moreover, it should be noted that the methodology presented herein for assessment of a case-study RC structure can be extended in order to evaluate the vulnerability of a class of structures, located in a seismic zone, against blast-induced progressive collapse (i.e., masonry buildings, steel-frame buildings, RC bridges).

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