A new measure of skills mismatch:

theory and evidence from the Survey of Adult Skills (PIAAC) *

Anne Fichen Michele Pellizzari

OECD University of Geneva

This version: February, 2014

(First Version: April, 2013)

Abstract

This paper proposes a new measure of skills mismatch that combines information about skill proficiency, self-reported mismatch and skill use. The theoretical foundations underling this measure allow identifying minimum and maximum skill requirements for each occupation and to classify workers into three groups, the well-matched, the under-skilled and the over-skilled. The availability of skill use data further permit the computation of the degree of under and over-usage of skills in the economy. The empirical analysis is carried out using the first wave of the OECD Survey of Adult Skills (PIAAC), allowing comparisons across skill domains, labor market statuses and countries.

*We would like to thank Andrea Bassanini, Stéphane Carcillo, Rodrigo Fernandez, Paulina Granados-Zambrano, Alex Hijzen, Mark Keese, Glenda Quintini, Stefano Scarpetta and Stefan Sperlich for their comments and suggestions. Michele Pellizzari is also affiliated to IZA, fRDB and the Dondena Centre for Research on Social Dynamics. *Corresponding author*: Michele Pellizzari, Department of Economics, University of Geneva, Uni-Mail, 40 Bd. du Pont d'Arve, CH-1211 Geneva 4, Tel: +41 (0)22 37 98 915. Email: michele.pellizzari@unige.ch

JEL codes: J0, J20, J24.

Keywords: mismatch, skills.

1 Introduction

A rich literature defines mismatch as the discrepancy between the characteristics of employed workers and the requirements of the jobs that they occupy (Quintini, 2011a). For example, several papers compare the formal education qualifications held by employed workers with the requirements of their jobs, commonly finding large numbers of workers being more qualified than required by their employment (Chevalier, 2003; Dolton and Vignoles, 2000; Groot and Maassen van den Brink, 2000; Quintini, 2011b; Rubb, 2003; Sicherman, 1991; Sloane, Battu, and Seaman, 1999). This finding can be rationalized by arguing, for example, that over-qualified workers may not have benefited from formal education as much as they could and that their actual competencies are less advanced than those one would normally expect them to possess based on their formal educational qualifications. At the same time workers who are found to be under-qualified for their jobs may have acquired the necessary skills to satisfactorily perform their jobs outside formal schooling, through experience, on-the-job learning and adult education (Chevalier and Lindley, 2009; Green and McIntosh, 2007). Hence, it is interesting to contrast qualification mismatch with skill mismatch, namely the discrepancy between the skills possessed by a workers and those required to perform his/her job (Allen and van der Velden, 2001; Desjardins and Rubenson, 2011). Over-skilled workers are those who are more skilled than required by their jobs, the opposite for under-skilled workers.

Unfortunately, measuring skill mismatch is particularly challenging, mostly due to the lack of direct information about workers' skills as well as job requirements. Some authors construct indicators of skills mismatch using information from surveys asking employed workers whether they have the skills to do a more demanding job than what they currently do or whether they need training to carry out their job tasks satisfactorily (Allen and van der Velden, 2001; Green and McIntosh, 2007). However, such data are likely subject to various forms of mis-measurement, the most obvious being people's overconfidence. An alternative approach consists in comparing directly indicators of skill proficiency and skill use at work, thus considering over-skilled those workers who do not make full use of their competencies on the job (CEDEFOP, 2010; Desjardins and Rubenson, 2011). Such an alternative approach is also subject to a number of serious problems. First of all, it implicitly assumes that skill use, which is either self-reported by the worker or derived from occupational titles, can be interpreted as a measure of job requirements, whereas it rather is the outcome of an endogenous effort choice. Second, proficiency and use are very different theoretical concepts and they can hardly be represented along the same metrics. In fact, they are derived from structurally different pieces of information: indicators of skill use normally exploit survey questions about the frequency (and/or the importance) with which specific tasks are carried out in a certain job, whereas skills proficiency is usually measured through cognitive tests.

This paper proposes a new methodology to measure skill mismatch and it applies it to data from the newly released *Survey of Adult Skills (PIAAC)*, which includes a rich battery of questions on skill use at work and direct indicators of workers' skill proficiency derived from a purposely designed assessment exercise. Additionally, the survey covers a large number of countries, whose data are highly comparable thanks to the harmonized sampling procedures and the common questionnaire (OECD, 2013a). The indicator of skill mismatch described in this paper is officially adopted by the OECD in the context of the *Programme for the International Assessment of Adult Competencies (PIAAC)*, of which the *Survey of Adult Skills* is a key element, and hereafter it will be labelled *OECD measure of skill mismatch*. For simplicity, the acronym PIAAC will be used in this paper to refer to both the overall programme and the survey.

In summary, the proposed methodology makes limited but efficient use of self-reported information and processes it according to a simple theoretical framework to overcome the fundamental problem of defining the skill requirements of jobs from a survey of workers. Specifically, for each available skill domain and each job, minimum and maximum requirements are defined as the minimum and the maximum proficiency of self-reported well-matched workers, i.e. workers who report that they do not feel they *"have the skills to cope with more demanding duties than those they are required to perform in their current job"* and they do not feel they *"need further training in order to cope well with their present duties"*.[1] [2] Within this framework, workers are classified as well-matched in a proficiency domain if their proficiency score in that domain is between the minimum and maximum score observed for workers who are self-reported well-matched (in the same occupation and country and for a given proficiency domain). Workers are over-skilled in a domain if their score is higher than the maximum score of the self-reported well-matched while they are under-skilled if their score is lower than the minimum score of the self-reported well-matched.

Two additional features of the approach described in this paper are worth mentioning. First, alternative measures of the minimum and maximum skill requirements can be produced by comparing the extremes of the distributions of assessed competencies for the under- and over-skilled and the well-matched. Such comparison allows assessing the relevance of measurement error in the estimated requirements, an important advantage over most of the literature. Second, exploiting the rich background questionnaire of the PIAAC survey, it is possible to compare the utilization of skills in the workplace by similarly proficient workers who are well-matched or mismatched in their jobs, thus constructing indicators of the degree of under- and over-utilization of skills associated with mismatch.

In this paper the methodology is applied to the PIAAC survey to produce the most up-to-date and comprehensive evidence on skill mismatch across countries and skill domains. However, the technique is simple and general enough to be valid in a variety of other contexts and data.

The results of this analysis show that on average across the entire survey, approximately 86% of dependent employees are well-matched in the literacy domain, about 4% are under-skilled and 10% are over-skilled. The overlap between literacy and numeracy mismatch is

[1] Although this is the phrasing used in the OECD Survey of Adult Skills, very similar questions are asked in other surveys, such as IALS, ALL and numerous national skills surveys.

[2] Jobs are normally identified on the basis of standard occupational coding and, depending on the size of the sample and the quality of the data, the definition can be refined by intersecting occupations with industry categories and/or with grouping of firm size.

substantial: 94% of the workers who are well-matched in literacy are also well-matched in numeracy. Men are more likely to be over-skilled than women, whereas gender differences in under-skilling are minor. Tertiary graduates are substantially less likely to be under-skilled than less educated workers and they are also more likely to be over-skilled. Foreign workers are substantially more likely to be under-skilled and substantially less-likely to be over-skilled. Differences emerge also when looking across age groups.

A common problem of the entire empirical literature on skill or qualification mismatch is the lack of information about the demand side, so that job requirements need to be inferred on the basis of data regarding realized matches. Contrary to most of the literature, which takes a practical empirical approach to this problem, the methodology proposed in this paper is derived from a theory that, albeit being simple, spells out explicitly the assumptions used to define requirements and, thus, allows defining mismatch rigorously.

The term mismatch is often used to refer to rather different concepts in the literature, thus creating a certain confusion in an area that is attracting more and more policy attention and that, therefore, necessitates accurate definitions and measurement.

In its most general interpretation, mismatch refers to a mis-alignment between the composition of labor demand and labor supply. Besides the definition used in this paper, which is intrinsically micro, it is important to mention a more macro concept that is common to a rich strand of studies (Farber, 1999; Jovanovic, 1979; Robin, Meghir, and Lise, 2009) in the search & matching literature (Pissarides, 2000). In the context of models with heterogeneous jobs and workers, aggregate mismatch is defined as the existence of an allocation of workers to jobs that could improve productivity relative to the realized equilibrium. A popular variation of the same idea introduces a geographical dimension and refers to mismatch as the discrepancy between the geographical distribution of vacancies and unemployed workers (Şahin, Song, Topa, and Violante, 2012; Shimer, 2007).

This aggregate notion of mismatch is a feature of the joint distribution of workers and jobs and, as such, it is an intrinsically macro concept and there is no sense in which a single job-worker pair can be considered mismatched in isolation from the others. On the contrary, skill mismatch is a feature of each single pair and it is important to keep these two concepts separate to avoid confusion. For example, an economy may experience a very large fraction of workers who appear to be over- or under-skilled for their jobs and, at the same time, implement the optimal allocation of the existing workers to the existing jobs. In this case skill mismatch would be due to shortages (or oversupply) of skills in the economy. Similarly, an economy with very low or no skill mismatch may very well allocate workers to jobs inefficiently. For example, workers might still make use of all their skills but the most skilled workers may not be allocated to the jobs where their skills are the most productive.

Additionally, while aggregate mismatch can, in principle, be defined independently of the types of skills, skill mismatch can be specified in terms of specific abilities, such as literacy, numeracy or ITC competency. So, any given worker could be mismatched in terms of one skill but not others.

Defining and measuring mismatch rigorously is important especially because the policies that can address the different types of mismatch are potentially very different. For example, any intervention that improves the efficiency of the matching process, such as job search counseling or relocation assistance for either firms or workers, has the potential to reduce aggregate and/or geographical mismatch but may do very little for qualification mismatch. At the same time, when qualification or skill mismatch are due to shortages on either sides of the market improving the efficiency of the process that allocates workers to jobs may have little impact.

The rest of the paper is organized as follows. Section 2 lays out the proposed methodology to measure skill mismatch, starting from its theoretical underpinnings and including a discussion of the empirical implementation and of the impact of measurement error. Section 3 briefly describes the *Survey of Adult Skills (PIAAC)* and provides some descriptive statistics. Section 4 reports comparable estimates of skill mismatch and skill under- and over-utilization across the countries covered in PIAAC, for the entire population and for various subgroups. Section 5 concludes by highlighting the importance of this analysis for both academic research and policy making.

2 Deriving the *OECD measure of skill mismatch*

As already spelled out in the introduction, skill mismatch is a feature of the single job-worker pair and is associated with the question of whether the skills possessed by the worker are sufficient to carry out the tasks required by the job. A worker whose skills are below the level required by the job is labeled under-skilled, a worker whose skills are above those required by the job is labeled over-skilled.

The key difficulty in formalizing the notion of skill mismatch concerns the identification of the job requirements, as most of the times the data used for this type of analysis are collected through surveys of workers and do not contain direct information on the structure of the production process.

The same problem arises with qualification mismatch and most of the literature addresses it with a pragmatic empirical approach, by defining requirements either on the basis of titles of occupations or self-reported information about skill use by employed workers.

This paper takes a different approach and develops a simple theoretical framework that is helpful to define job requirements more formally and to spell out explicitly the assumptions imposed on the data to estimate them. One crucial feature of the theory is the treatment of skill use as an endogenous choice of the worker, similarly to the choice of effort in standard principal-agent models.

It is also worth emphasizing that the theoretical framework described in this section serves the important purpose of making explicit the assumptions underlying the proposed measure of skill mismatch. Other indicators of skill mismatch that have been used in the literature are obviously also based on a number of assumptions but they are rarely made explicit and are often more restrictive than the ones discussed here. For example, the assumption that jobs are homogeneous within occupations or that the production function is kinked are common to virtually all studies.

2.1 Theoretical foundations

For presentational ease, the model in this section rests on a number of simplifying assumptions, most of which can be simplified without affecting the qualitative implications of the theory in a major way, as discussed later on in subsection 2.5.

Building blocks. Consider an economy with heterogeneous workers and heterogeneous jobs. Workers, indexed by i, differ in their endowment of skills, labeled η_i, and they endogenously decide how much skills to deploy in their jobs. For simplicity, η_i is assumed to be a simple uni-dimensional skill and Section 2.5 discusses how this framework can be extended to multiple skills.

Deploying skills is costless within the limit of one's endowment and it is subject to a constant marginal cost for any skill level beyond one's endowment, as in Figure 1. In other words, workers are allowed to deploy a level of skills that goes beyond their endowments provided they pay a utility cost. This is necessary in order to rationalize the existence of under-skilled workers in the economy.

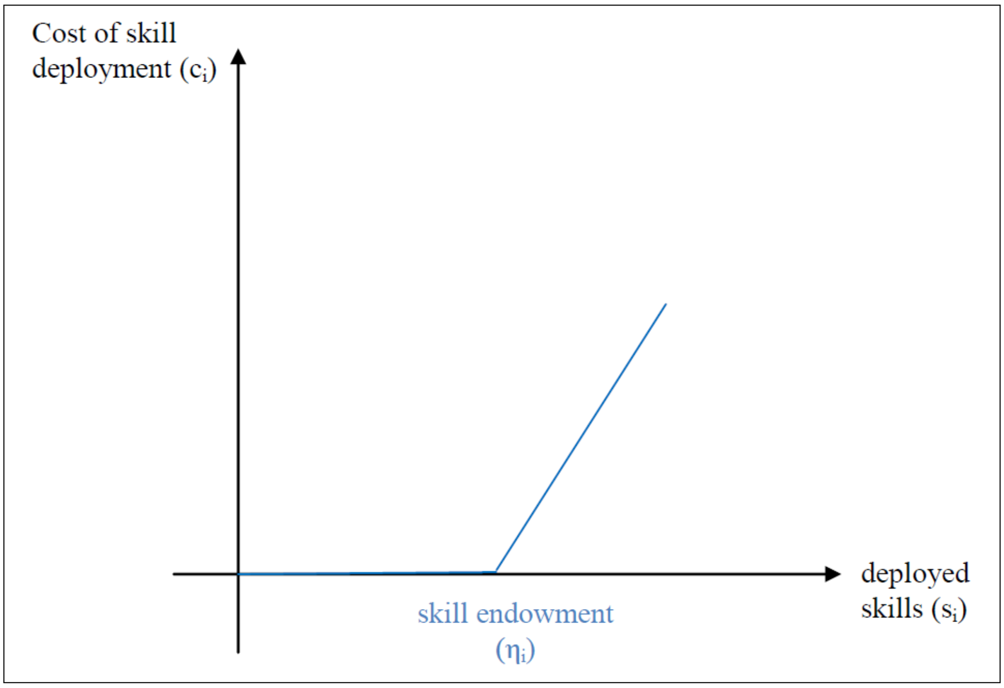

Figure 1: The cost of deploying skills

Jobs are defined as production functions, with skills being the only input. Each job employs one worker and is independent of other jobs (lack of complementarities). Different jobs have different production functions, which are characterized by three key features: (i) local linearity; (ii) fixed operational costs; (iii) discontinuously declining marginal productivity.

More specifically, assume that output y_{ij} of job j filled with worker i is a function of the amount of skills that the worker endogenously chooses to deploy on the job, s_i. Further assume that there are fixed costs k_j to operate the job and that the marginal product of deployed skills is locally constant and decreases above a certain threshold. For simplicity we will assume that the marginal product of skills is equal to zero beyond such threshold. Under this set of assumptions, the production function for a generic job looks as in Figure 2.

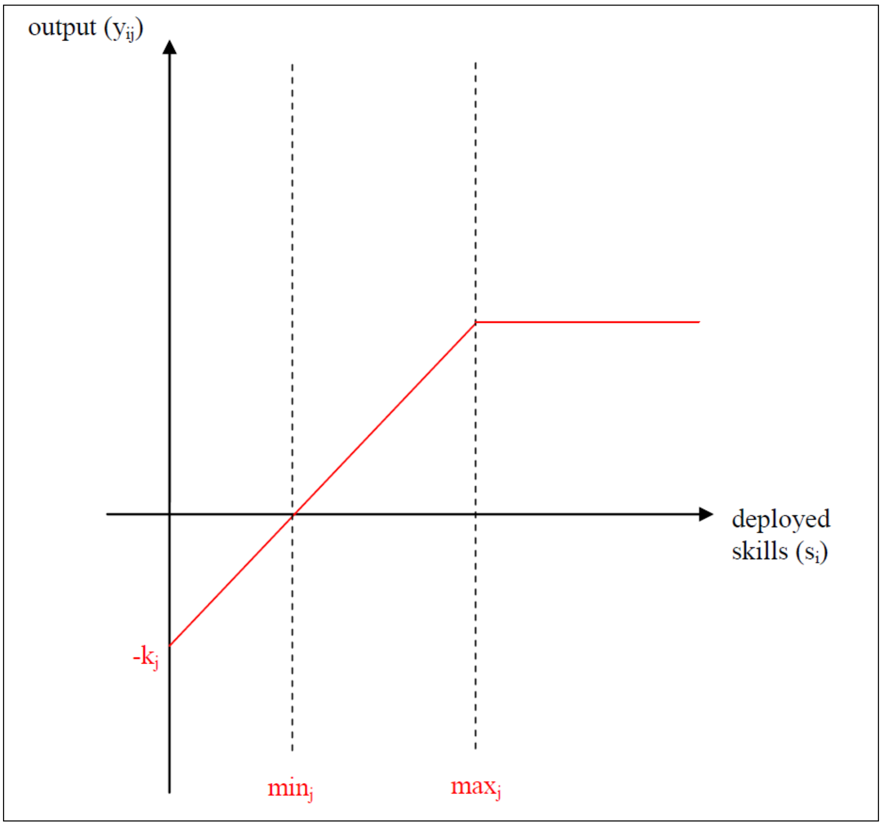

Figure 2: The production function

The combination of the fixed costs and the discontinuously declining marginal product generates two critical values in the distributions of skills that lead to a very natural definition of skill mismatch. Workers with skill endowments below min_j are under-skilled, workers with skill endowments between min_j and max_j are well matched, workers with skill endowments above max_j are over-skilled.

For simplicity, assume that workers are randomly assigned to jobs.[3] Conditional on the characteristics of their jobs, they choose how much of their skills to deploy in order to maximize the following utility function:

$$U_i = w_{ij} - 1(y_{ij} < 0)F - c_i(s_i) \tag{1}$$

where w_{ij} is the wage worker i is paid in job j, F is a utility cost associated with producing negative output (e.g. the cost of being fired and suffering a spell of unemployment) and $c_i(s_i)$ is the cost of deploying skills (Figure 1):[4]

$$c_i = \begin{cases} 0 & if \quad s \leq \eta_i \\ \delta s_i & if \quad s > \eta_i \end{cases} \tag{2}$$

with $\delta \geq 0$.

Assume wages are proportional to productivity:[5]

$$w_{ij} = \gamma_i y_i j \tag{3}$$

[3]This assumption is innocuous in the present context where the purpose of the analysis is merely the measurement of skill mismatch. Obviously, if one were to investigate the causes of mismatch, understanding the allocation process would be paramount.

[4]The subscript i to the function $c(\cdot)$ indicates that the function itself varies with η_i, which, in fact, determines the point where the slope of the function changes.

[5]This assumption can be easily justified in the context of search&matching models, that have become the standard view of the functioning of the labor market. In the standard version of such models the equilibrium wage is equal to a fraction of the job's output plus the outside option of the worker. Further assuming that the worker's outside option is itself a fraction of the wage (as in most unemployment insurance systems), leads precisely to an expression of the equilibrium wage as a fraction of productivity.

where for simplicity γ_i is allowed to vary only across workers and output is defined as:[6]

$$y_{ij} = \begin{cases} \beta_j s_i - k_j & if \quad s_i \leq min_j \\ \beta_j max_j - k_j & if \quad s_i > max_j \end{cases} \tag{4}$$

with $\beta_j \geq 0$ and $k_j \geq 0$ for all j.

Optimal skill deployment. Consider the following three cases.

1. Worker i is a good skill-match with job j, i.e. $min_j \leq \eta_i \leq max_j$. Given the above assumptions, workers in this condition would obviously find it optimal to deploy their entire endowment of skills on the job, $s_i^* = \eta_i$.

2. Worker i is under-skilled for job j, i.e. $\eta_i < min_j$. Assuming that that F is large enough to make the decision to deploy skills below min_j always suboptimal, under-skilled workers choose to deploy the minimum level of skills that allows them not to incur in the cost F: $s_i^* = min_j$.

3. Worker i is over-skilled for job j, i.e. $\eta_i > max_j$. Workers in this condition are indifferent between any level of skill deployment in the interval $[max_j, \eta_i]$.

It is now possible to look more formally at the meaning of skill mismatch. In order to do so, the optimal skill deployment of over and under-skilled workers should be compared to the counterfactual of their being well matched. Importantly, such comparison should be independent of other matches. In other words, the counterfactual should be viewed as a move of the mismatched worker to a previously vacant or even non-existent job or, equivalently, as a transformation of the production function of the job held by the mismatched worker. The alternative counterfactual, whereby the mismatched worker takes a job previously held by someone else, requires considering the effect of such a transition on the latter worker, thus making it impossible to define skill mismatch as a feature of the job-worker pair and bringing it nearer to the notion of aggregate mismatch.

[6]Allowing the sharing parameter γ to vary across jobs (or both across workers and jobs) is possible but it makes it less obvious to formalize a meaningful definition of skill mismatch.

In the simple theory spelled out in this section, jobs are characterized by three parameters: the operational costs (k_j), the returns to deployed skills (β_j) and the maximum skill level (max_j).[7] Hence, in order to become well-matched, any mismatched worker needs to move to a job with a different combination of these three parameters.

Consider the over-skilled first. In order to be well-matched they need to find a job h such that $max_h > max_j$ (j indicating their current jobs), where they would deploy more skills, as their optimal skill deployment increases from max_j to η_i. Unless the new job is also characterized by lower returns to skills ($\beta_h < \beta_j$), such a transition would also result into higher output.

As regards the under-skilled, in order to become well-matched they need to land a job h characterized either by lower operational costs ($k_h < k_j$) or by higher returns to skills ($\beta_h > \beta_j$) or both. In any event, were they well-matched they would deploy less skills but output would be unambiguously higher.

Hence, based on the definitions above over- and under-skilled workers are mismatched in the sense that their skills could be more productively used if the structural features of their jobs were different and such that they would be well-matched.

2.2 Empirical implementation

Having access to data that include observable measures of the skills possessed by employed workers, as in PIAAC, it is possible to identify and estimate the parameters min_j and max_j for each job, where jobs are defined as occupations or, depending on the size and quality of the data, as the combination of occupation and industry classes. In other words, all the jobs in the same class are assumed to be homogeneous, i.e. to be defined by the same production function.

The identification of job requirements rests on two questions that are asked to employed respondents in PIAAC but that are also common to other surveys, sometimes with variations (Allen and van der Velden, 2001; Green and McIntosh, 2007; Mavromaras, McGuinness, and

[7]The minimum skill requirement (min_j) can be easily and univocally derived from the triplet $[k_j, \beta_j, max_j]$, i.e. for each $[k_j, \beta_j, max_j]$ there exists one and only one min_j.

Wooden, 2007). The first question asks about whether one feels to have the skills to do a more demanding job. The exact phrasing is the following: *"Do you feel that you have the skills to cope with more demanding duties than those you are required to perform in your current job?"*. The second question is about the need of training and reads as follows: *"Do you feel that you need further training in order to cope well with your present duties?"*. Assuming that respondents who answer negatively to both questions are neither over-skilled nor under-skilled, it is possible to estimate min_j and max_j as the minimum and the maximum of their assessed skill:

- min_j = minimum level of assessed skills of workers who neither feel they could do a more demanding job nor feel the need of further training;
- max_j = maximum level of assessed skills of workers who neither feel they could do a more demanding job nor feel the need of further training.

For the moment, the assumption that selecting workers who answer negatively to both questions correctly identifies good matches, i.e. job-worker pairs such that $\eta_i \in [min_j, max_j]$, is maintained. The obvious concerns about misreporting in such questions are the object of the next section (Section 2.3).

Under such an assumption, simple application of the law of large numbers guarantees that these estimators are consistent. Then, it is possible to identify under-skilled workers as workers whose skill endowment is below min_j and, similarly, over-skilled workers are those whose skill endowment is above max_j. Well-matched workers are, by exclusion, those who are neither under- nor over-skilled.

Next, an optimal level of skill use can be defined for every worker in the economy as the skill use observed for workers with a similar level of skill endowments who are well-matched. Such a comparison is informative about the amount of skills that are under or over utilized.

2.3 Measurement error

The use of self-reported information about one's ability to perform one's current job and one's need for training may question the validity of the estimates of the job requirements. Despite not being immune to measurement error, the methodology described in Section 2.2 allows the derivation of alternative estimators of the job requirements and, by comparing such alternative estimators, it also allows producing evidence that is informative about the extent of mis-measurement.

Specifically, in addition to the estimators described in Section 2.2, min_j could alternatively be estimated as the maximum skill endowment of workers who report feeling the need of further training and not feeling able to do a more demanding job. Similarly, max_j could be estimated as the minimum skill endowment of workers who report feeling able to do a more demanding job and not feeling the need for further training.

It is useful to define these alternative estimators as follows:

- $\widehat{min}_j =$ minimum skill endowment of workers neither feeling able to do more demanding jobs nor feeling the need of training;
- $\widetilde{min}_j =$ maximum skill endowment of workers feeling the need of further training;
- $\widehat{max}_j =$ maximum skill endowment of workers neither feeling able to do more demanding jobs nor feeling the need of training;
- $\widetilde{max}_j =$ minimum skill endowment of workers feeling able to do more demanding jobs.

Figure 3 visually summarizes the intuition behind these estimators, each of which is affected differently by the most cumbersome sources of mis-measurement, namely over-confidence and the generalized need for training.

An important concern is that people might be overconfident and report being capable of doing more demanding jobs even when they are indeed well-matched or even under-skilled in their current employment. Interestingly, overconfidence is much more likely to bias $\widetilde{max}_j$ than $\widehat{max}_j$. In fact, a single (truly) well-matched worker who is overconfident and consequently

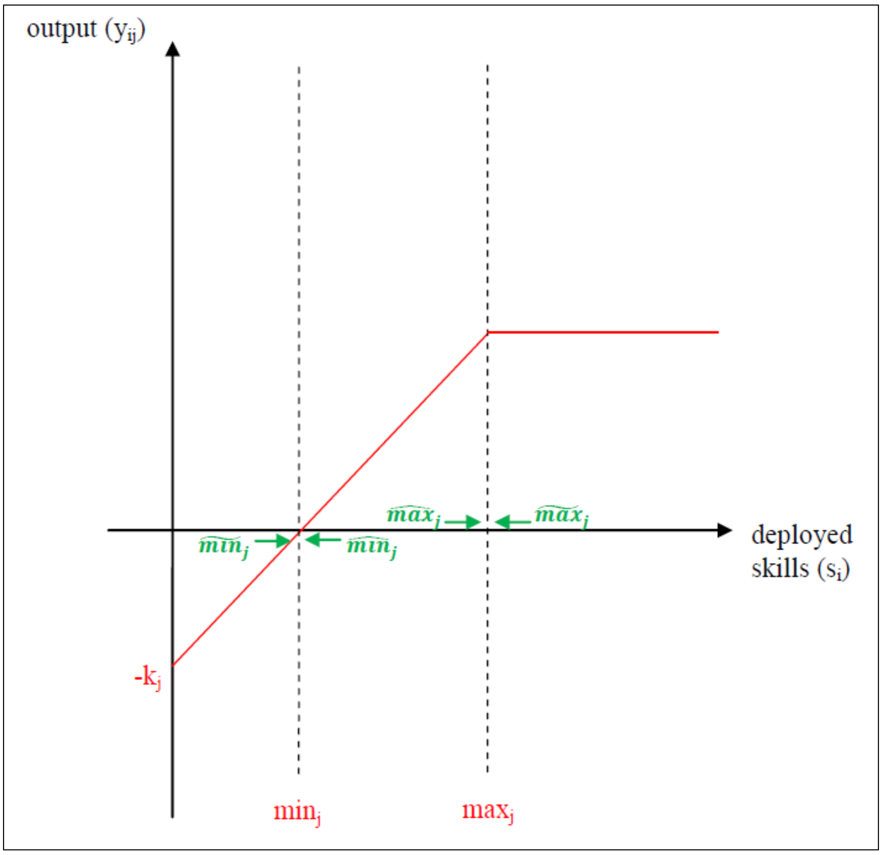

Figure 3: Alternative estimators of job requirements

reports being over-skilled crucially changes $\widetilde{max}_j$. On the other hand, only if the most skilled worker among the (truly) well-matched is over-confident $\widehat{max}_j$ changes. In practice, one can look at the magnitude of the difference between $\widehat{max}_j$ and $\widetilde{max}_j$ to assess the importance of overconfidence in the data.

Over-confidence does not affect the estimation of min_j, as the question about having the skills to cope with a more demanding job is not used for this purpose. However, another source of measurement error might affect the respondents' answers to the question about the need for training, which is the basis for estimating min_j. Such a question specifically asks whether the respondent feels the need of additional training to "cope well" with her present duties and people may attach different interpretations to the notion of "coping well", given that the quality of how task are performed can vary substantially. Hence, some people might answer that they do feel the need of additional training, under the assumption that with more training they could carry out their current tasks better (e.g. more rapidly, less expensively, et.) even though they already do so at an acceptable level or, in the terminology of our simple theory, they already deploy skills above min_j.

It seems reasonable to argue that the bias in $\widehat{min}_j$ is likely to be smaller than in $\widetilde{min}_j$. This is because any (truly) well-matched or over-skilled worker who misinterprets the question and reports needing training would crucially affect $\widetilde{min}_j$. On the other hand, $\widetilde{min}_j$ is biased only if the least skilled among the (truly) well-matched reports being in need of training.

An additional, although less worrisome, source of mis-measurement is the heterogeneity of jobs within occupations (or occupation-industry cells). In fact, despite the theoretical assumption that all jobs are identical within occupations, some heterogeneity necessarily exists in practice. Hence, in order to reduce its implications on the definition of the job requirements, it is useful to consider some bottom and top percentiles of the within-job distributions of workers' skills rather than the actual minimum and the maximum. In Section 4, the 95th and the 5th percentiles of the within-occupation distribution of skill endowments among workers who neither feel the need for further training nor feel capable of doing more demanding jobs are used as estimators of max_j and min_j, respectively.

2.4 Skill specific mismatch

So far the skill endowment of workers, η_i, has been assumed to be a simple uni-dimensional variable. However, one major advantage of PIAAC is the availability of measures of proficiency in three important skill domains, namely numeracy, literacy and problem solving. Hence, it allows producing measures of mismatch that are specific to each skill, as workers could use all their skills in some domains and be over-skilled or under-skilled along other dimensions.

In fact, the methodological framework presented in this section can be readily re-interpret in the context of multiple skills. Simply allow η_i to be a vector of several skills and, similarly, also the job requirements, min_j and max_j will be multidimensional vectors. Then, assume workers who report being over/under-skilled do so whenever any of their skills is above/below the corresponding minimum/maximum requirement, even if they are well-matched with regard to all the other skill dimensions. Under this additional assumption minimum and maximum requirements for each skill type can still be estimated as discussed in the section above and workers can be classified as under- or over-skilled by each skill domain.

Of course, the survey cannot cover the entire set of skills that are needed at work so that some individuals may still be mismatched along some dimensions that are not observed in the data.

2.5 Extensions

The theoretical framework described above clearly rests on a number of simplifying assumptions and, although most of them are crucial for the purpose of constructing measures of skill mismatch that can be implemented empirically, some serve the more modest purpose of easing the exposition of the arguments.

For example, in order to make sense of the notions of minimum and maximum requirements it is crucial to define production functions with either kinks or negative intercepts or both. Similarly, in order to conceptualize separately the endowment of skills and their deployment, one needs to introduce some costs of deploying one's endowment into the job.

However, the sharp assumptions about the return to skills dropping all the way to zero above max_j and the cost of skill deployment being exactly zero up to one's endowment can be relaxed. Specifically, the production and cost functions could very well look as in Figure 4 without compromising any of the implications that we derived from the model.

Provided the marginal cost of skill deployment increases above η_i and the returns to skills decline beyond max_j, nothing would change substantially in our framework. Only one additional assumption would be needed regarding the relative ratio of the returns to skills above and below max_j and the marginal costs above and below η_i to avoid unreasonable and uninteresting equilibria in which, for example, the under-skilled find it optimal to deploy skills above max_j.

Other assumptions that are worth mentioning here are the lack of complementarity of workers in the production process, the random assignment of workers to jobs and the limited variation in the sharing parameter γ which is constrained to be constant within workers across jobs.

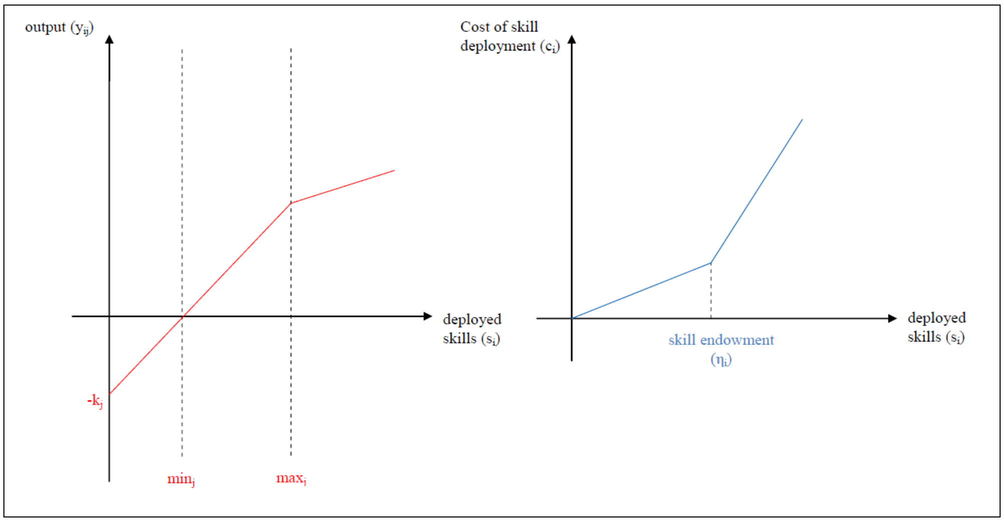

Figure 4: Alternative production and cost functions

Regarding complementarities, it is important to note that skill complementarity can be very easily incorporated in the model of Section 2.1. The linearity of output with respect to each specific skill is what makes the identification of job requirements particularly simple. However, it is still possible to allow the production function in Figure 2 to shift vertically in reaction to changing inputs of other skills. The model would still require some additional assumptions to avoid the minimum and maximum requirements for each skills to be affected by changes in the inputs of the others, a situation that would make the very definition of requirements extremely unclear. Hence, skill complementarity does not need to be totally ruled out but only some specific forms of complementarity can be incorporated in the model. In any event, incorporating them would necessarily complicate the model and make it empirically less tractable.

A similar argument can be made for complementarity across workers, which could be taken into account, provided it takes forms that still allow to define worker-job specific requirements. In the current ”one worker/one job” formulation, requirements indifferently refer to either the total input of skills in the production function or to the input provided by the single worker. With multiple workers contributing to the same production function these two notions of requirements do not coincide and they need to be defined separately.

The assumption that workers are randomly assigned to jobs is instrumental to the measurement of mismatch and relaxing it would profoundly change the meaning of the indicators of mismatch. By assuming random assignment, the model in Section 2.1 abstract from the reasons why workers end up in their jobs and simply tries to measure the quality of such matches from the point of view of production efficiency. It is of course interesting to understand the extent to which mismatch is due to structural frictions, imbalances of aggregate demand and supply or even preferences but this is irrelevant for the very narrow purpose of measuring mismatch.

Finally, allowing the sharing parameter, γ, to vary both across workers and across jobs is possible but it complicates the interpretation of mismatch. One convenient feature of the current formulation that would be lost if γ varied by job is the very sharp implications for optimal skill deployment. This is, in part, the result of having jobs and workers being defined by structural features that do not overlap with one another: workers are characterized by skill endowments (η_i) and jobs by the parameters of the production function (β_j, k_j and max_j). A sharing parameter that varies across both i and j would break this useful separation and make the derivation of both optimal deployment and the implications of mismatch much less clear.

3 The *Survey of Adult Skills (PIAAC)*

The *Survey of Adult Skills* is the main output of the *Programme for the International Assessment of Adult Competencies* run by the OECD in collaboration with national governments and a consortium of experts that assists the implementation of the survey and the preparation of the data.

The survey is a collection of country-specific household samples designed to be representative of the adult population aged between 16 and 65 years. The samples are constructed according to harmonized statistical procedures aimed at guaranteeing comparability across countries. The same background questionnaire is administered to all sampled individuals in all the countries, merely translated in the local language.[8]

There are currently 24 countries participating in the PIAAC project and the descriptive statistics of some key socio-economic variables are presented in Table 1.

[8]In a few countries the survey is administered in multiple languages.

Table 1: Descriptive statistics

Variable	Mean	St. dev.	Min	Max
Age	41.121	11.689	16	65
1=women	0.401	-	-	-
1=college[a]	0.438	-	-	-
1=foreign born	0.113			
	Total	Mean	Min	Max
Sample size[b]	57,698	2,748	1,613	11,102

[a] Individuals with some tertiary education.
[b] Total is the total size of the sample used for the analysis, across all countries. Mean is the mean sample size across countries. Min and max are the minimum and maximum sample size across countries.
Source: OECD Survey of Adult Skills (PIAAC).

One key element of PIAAC is the skill assessment exercise that all respondents are asked to take as part of the interview process. The exercise consists of a set of test questions organized into three domains: numeracy, literacy and problem solving. By default, all three tests are carried out on computers but literacy and numeracy can also be done on paper for those who prefer to do so and for those who lack basic IT literacy. Problem solving can only be taken on computers and those who refuse or cannot use a PC are simply routed out. As a consequence, the number of missing values in problem solving is relatively high in many countries (on average about 10% across all participating countries but up to over 20% in some). For this reason the analysis of problem solving skills is excluded from this paper.

As it is customary in the design of competency tests (OECD, 2012, 2013b), to reduce the length of the assessment and to maximize participation, not all respondents are administered all the questions and a purposely designed routing algorithm guides each respondent through a subset of the test items. Then, the entirety of the answers for all respondents in all countries is used to estimate a psychometric model based on *Item Response Theory (IRT)* that produces a skill proficiency measure for each participant in the survey with completed information from the background questionnaire (Ackerman, 2010; Jakubowski, 2013).

The purpose of the IRT model is the estimation of the unobservable respondents' ability in each domain (literacy, numeracy) using information about their observed performance in tasks that are associated to such domains. The number of tasks that could be associated with each

Table 2: Proficiency and use of literacy and numeracy

Variable	Mean	Median	Std. dev.
Proficiency in literacy	279.259	284.200	44.843
Proficiency in numeracy	273.145	277.872	49.970
Use of literacy	2.883	3.000	0.906
Use of numeracy	2.492	2.500	1.089

Source: OECD Survey of Adult Skills (PIAAC).

skill is potentially infinite and only a subset of them can be tested in practice. In the OECD SAS each respondent answers on average 20 questions in literacy and about the same in numeracy, taking approximately 1 minute for each item.

A number of arbitrary assumptions necessarily need to be made in this context. First, the association of tasks to skills is entirely discretionary and, while reading a text is clearly a literacy task and summing numbers is clearly a numeracy test, there are numerous examples of test items that could be associated with several skills.[9] Additionally, the theory does not provide guidance about the specific formulation of the IRT model in terms of both functional form and explanatory variables and the choice is usually made on the basis of computational convenience and data quality. PIAAC adopts a logistic model with two parameters, one reflecting the difficulty of the task and one measuring how well the task discriminates among respondents along the underlying skill. The resulting estimates are used to impute an indicator of skill proficiency for each respondent with completed information on the variables used in the IRT model.

For ease of use and interpretation, the skill indicators are transformed into a scale ranging from 0 to 500. The first two lines of Table 2 report some basic descriptive statistics for the indicators of proficiency in literacy and numeracy. The average proficiency is around 279 for literacy and slightly lower (273) for numeracy. In both cases the median is higher than the mean, suggesting that the distribution is skewed to the left due to a tail of individuals with very low scores. Additionally, the distribution of numeracy proficiency appears to be slightly more dispersed than that of literacy.

The background questionnaire of PIAAC also includes a very detailed section about the use

[9]Notice that the same item can be used to estimate more than one skill measure.

of skills at work. Participants are asked about the frequency with which they perform specific tasks, such as reading documents or making calculations, in the course of their work activities. This paper focuses on a limited set of such questions to construct indicators of the use of literacy and numeracy at work.[10]

The original frequency questions allow respondents to answer on a discrete scale of 5 values: never (one), less than once a month (two), less than once a week but at least once a month (three), at least once a week but not every day (four) and every day (five). The set of tasks considered to construct the indicator of literacy use includes reading and writing of a very wide set of documents.[11] The tasks considered for numeracy are also numerous and very detailed, including making various types of calculations and using calculators.[12]

This large number of questions are combined into skill use indicators for literacy and numeracy using Crombach alpha (Cronbach, 1951), which is essentially based on summing all the discrete frequency answers one on top of the other. This procedure has the advantage of reducing the dimensionality of the information gathered in the survey while at the same time maintaining a rather intuitive interpretation of the resulting scales, where a value of zero signifies that none of the tasks considered is ever performed and a value of 5 corresponds to performing each of the tasks every day. Basic descriptive statistics for such indicators are shown in the lower two lines of Table 2. The mean use of literacy is around 2.9, which is very close to the median (3.0). Numeracy tasks seem to be performed slightly less frequently, with a mean use around 2.5.

[10]OECD (2013a) analyses a larger set of skill use indicators.

[11]Directions, instructions, memos, letters, e-mails, articles (in newspapers, magazines, newsletters, professional and scholarly journals), books, manuals and reference materials, bills, invoices, financial statements, diagrams, maps and schematics.

[12]Calculating prices, costs or budgets; calculating fractions, decimals or percentages; using a calculator; preparing charts, graphs or tables; using algebra or formulas; using advanced mathematics (calculus), trigonometry, statistics, regression techniques.

4 Empirical results

The methodology described in Section 2 is applied to the PIAAC survey and the main results are in Tables 3 and 4 for literacy and numeracy, respectively. Jobs are defined separately for each country on the basis of 1-digit occupational codes (ISCO 1-digit).[13] The final working sample is restricted to dependent employees holding only one job.

The computation of the standard errors for the estimates presented in this section needs to take into account both the differences in the sampling frames across countries and the variation induced by the imputation of the ability scores. The appendix discusses in details how this is done.

Considering literacy proficiency, approximately 86% of dependent employees are classified as well-matched across all the countries covered by the survey, about 10% are over-skilled and 4% are under-skilled (Table 3). These average results mask a large heterogeneity across countries. For example, over-skilling can affect as many as 18% of workers in Austria and as few as 6.4% in Finland. Under-skilling is lowest in Austria (1.3%) and Germany (1.4%) and is highest in the United Kingdom (6.5%).

The results for numeracy (Table 4) are broadly similar to those for literacy and the ranking of countries is also similar. The Spearman rank correlation between the incidence of mismatch - i.e. the sum of the under- and over-skilled - in literacy and in numeracy is equal to 0.81.

In fact, Table 5 shows that 94% of the workers who are well-matched in literacy are also well-matched in numeracy.[14] The overlap is less strong but still very important among the under- and the over-skilled.

[13]Due to the small sample sizes, workers employed in occupations classified as ISCO codes 0 (armed forces) and 6 (skilled agricultural and fishery workers) have been dropped and those employed in ISCO codes 1 (managers) and 2 (professionals) have been grouped together. Moreover, occupations with fewer than 10 self-reported well-matched workers have also been dropped.

[14]For all the remaining of this analysis the results are presented only for the aggregate of all countries covered by the OECD Survey of Adult Skills. Results by country are available from the authors upon request.

Table 3: Skill mismatch by country - Literacy

Country	under-skilled	well-matched	over-skilled	Country	under-skilled	well-matched	over-skilled
Australia	0.028	0.881	0.091	Japan	0.031	0.871	0.098
	(0.017)	(0.058)	(0.045)		(0.08)	(0.019)	(0.020)
Austria	0.013	0.805	0.182	Korea	0.018	0.875	0.107
	(0.065)	(0.077)	(0.040)		(0.020)	(0.044)	(0.034)
Belgium[a]	0.039	0.883	0.079	Netherlands	0.027	0.905	0.068
	(0.014)	(0.021)	(0.016)		(0.013)	(0.025)	(0.017)
Canada	0.036	0.898	0.065	Norway	0.047	0.865	0.088
	(0.006)	(0.018)	(0.017)		(0.018)	(0.032)	(0.023)
Czech Republic	0.018	0.820	0.162	Poland	0.026	0.902	0.072
	(0.016)	(0.049)	(0.043)		(0.029)	(0.066)	(0.045)
Denmark	0.041	0.881	0.078	Slovak Republic	0.038	0.841	0.121
	(0.010)	(0.017)	(0.013)		(0.026)	(0.032)	(0.27)
Estonia	0.047	0.882	0.071	Spain	0.027	0.805	0.169
	(0.012)	(0.028)	(0.023)		(0.037)	(0.100)	(0.074)
Finland	0.037	0.899	0.064	Sweden	0.050	0.892	0.058
	(0.013)	(0.033)	(0.025)		(0.028)	(0.050)	(0.029)
Germany	0.014	0.841	0.145	United Kingdom[b]	0.065	0.854	0.081
	(0.046)	(0.093)	(0.059)		(0.013)	(0.029)	(0.024)
Ireland	0.045	0.804	0.151	United States	0.039	0.872	0.090
	(0.023)	(0.033)	(0.030)		(0.031)	(0.052)	(0.034)
Italy	0.060	0.823	0.117	Total	0.035	0.864	0.101
	(0.043)	(0.067)	(0.036)		(0.014)	(0.022)	(0.009)

Bootstrapped standard errors in parentheses. See Appendix for details on the bootstrap procedure.
[a] Only the Flanders region is included in the survey.
[b] Only England and Northern Ireland are included.
Source: OECD Survey of Adult Skills (PIAAC).

Table 4: Skill mismatch by country - Numeracy

Country	under-skilled	well-matched	over-skilled	Country	under-skilled	well-matched	over-skilled
Australia	0.025	0.880	0.095	Japan	0.037	0.884	0.079
	(0.018)	(0.049)	(0.036)		(0.007)	(0.017)	(0.015)
Austria	0.019	0.802	0.179	Korea	0.026	0.843	0.131
	(0.059)	(0.086)	(0.044)		(0.013)	(0.032)	(0.033)
Belgium[a]	0.041	0.891	0.067	Netherlands	0.030	0.919	0.051
	(0.013)	(0.024)	(0.018)		(0.011)	(0.024)	(0.017)
Canada	0.041	0.888	0.070	Norway	0.041	0.895	0.064
	(0.008)	(0.015)	(0.013)		(0.027)	(0.050)	(0.031)
Czech Republic	0.027	0.838	0.135	Poland	0.014	0.874	0.112
	(0.018)	(0.042)	(0.035)		(0.024)	(0.053)	(0.038)
Denmark	0.036	0.895	0.069	Slovak Republic	0.035	0.846	0.119
	(0.015)	(0.022)	(0.014)		(0.019)	(0.050)	(0.042)
Estonia	0.038	0.895	0.066	Spain	0.031	0.810	0.158
	(0.011)	(0.023)	(0.020)		(0.031)	(0.084)	(0.063)
Finland	0.035	0.896	0.070	Sweden	0.046	0.892	0.061
	(0.015)	(0.030)	(0.021)		(0.035)	(0.042)	(0.018)
Germany	0.018	0.829	0.153	United Kingdom[b]	0.069	0.865	0.066
	(0.038)	(0.072)	(0.046)		(0.016)	(0.044)	(0.039)
Ireland	0.045	0.825	0.130	United States	0.030	0.877	0.094
	(0.019)	(0.041)	(0.035)		(0.040)	(0.067)	(0.042)
Italy	0.075	0.800	0.126	Total	0.034	0.864	0.101
	(0.034)	(0.048)	(0.032)		(0.015)	(0.019)	(0.006)

[a] Only the Flanders region is included in the survey.
[b] Only England and Northern Ireland are included.
Source: OECD Survey of Adult Skills (PIAAC).

Table 5: Overlapping of skill mismatch in literacy and numeracy

		Numeracy		
		Under-skilled	Well-matched	Over-skilled
Literacy	Under-skilled	0.611	0.389	0.000
		(0.063)	(0.061)	(0.004)
	Well-matched	0.015	0.940	0.045
		(0.011)	(0.023)	(0.013)
	Over-skilled	0.000	0.378	0.622
		(0.002)	(0.091)	(0.092)

Shares of workers who are under-skilled, well-matched or over-skilled in numeracy by mismatch status in literacy. The numbers sum to 1 by rows. Bootstrapped standard errors in parentheses. See Appendix for details on the bootstrap procedure.
Source: OECD Survey of Adult Skills (PIAAC).

Table 6: Skill mismatch by socio-demographic groups

	Literacy		Numeracy	
	Under-skilled	Over-skilled	Under-skilled	Over-skilled
Men	0.037	0.121	0.034	0.133
	(0.011)	(0.006)	(0.009)	(0.006)
Women	0.032	0.079	0.035	0.066
	(0.018)	(0.016)	(0.021)	(0.012)
Non-graduates	0.046	0.084	0.046	0.082
	(0.013)	(0.015)	(0.013)	(0.017)
Graduates	0.017	0.126	0.016	0.131
	(0.017)	(0.008)	(0.018)	(0.016)
Age<40	0.023	0.129	0.026	0.123
	(0.012)	(0.013)	(0.012)	(0.009)
Age≥40	0.046	0.075	0.042	0.081
	(0.016)	(0.005)	(0.017)	(0.005)
Natives	0.025	0.108	0.028	0.107
	(0.015)	(0.008)	(0.013)	(0.007)
Foreigners	0.111	0.043	0.083	0.059
	(0.009)	(0.025)	(0.018)	(0.014)

Shares of under- and over-skilled workers.
Bootstrapped standard errors in parentheses. See Appendix for details on the bootstrap procedure.
Source: OECD Survey of Adult Skills (PIAAC).

Table 6 describes the incidence of under- and over-skilling across socio-demographic groups. Men appear to be affected by over-skilling more frequently than women, both with regard to literacy and numeracy, whereas gender differences in under-skilling are minor. This result is not obvious, as one may think that women, who often find employment more difficultly than men, might be more willing to take jobs that do not necessarily match their skills perfectly. On the other hand, OECD (2013a) shows that women use their skills less frequently than men, mostly because of the jobs in which they are occupied. Being in jobs where skills are not often used, they might also be less likely to be mismatched.

As one might expect and given the partial overlap of qualification and skill mismatch OECD (2013a); Quintini (2011a,b), graduate workers are less likely to be under-skilled than non-graduates. They are also more likely to be over-skilled. Literacy and numeracy follow similar patterns. All these differences are statistically significant at the 5% level.

Consistent with the notion that skills are accumulated not only in school but also over the course of one's career, older workers are more likely to be over-skilled, in both literacy and numeracy, than younger workers, whereas the opposite is true for under-skilling. This result may also suggest that under-skilling is merely a temporary phenomenon and it is absorbed as individuals progress through their working lives and either accumulate additional knowledge that allows them to carry out their jobs more appropriately or move across jobs in search of better matches (Topel and Ward, 1992). As for older workers, the higher incidence of over-skilling might be interpreted as an encouraging finding, especially for those countries facing rapidly ageing populations. According to the results in Table 6 approximately 8% of workers above the age of 40 are over-skilled, suggesting that improving the matching of older workers may help mitigate the impact of population aging on productivity.

Finally, foreign workers are 4 (3) times more likely than natives to be under-skilled in literacy (numeracy) and 60 (45)% less likely to be over-skilled. This result is easy to rationalize for literacy, given that in most cases the language of the destination country is different from migrants' mother tongues. For numeracy, the lower incidence of over-skilling contrasts with the common finding that immigrants often hold formal educational qualifications that are higher than those required by their jobs. The over-qualification of migrants is often attributed to the difficulties in having educational qualifications officially recognized across countries. However, the results in Table 6 seem to suggest that some of the over-qualified foreigners simply do not have the necessary skills to carry out their jobs satisfactorily, pointing to a large heterogeneity in the quality of schooling across countries.

The mismatch indicators are based on the minimum and maximum skill requirements by occupations, which are estimated as the minimum ($\widehat{min}_j$) and maximum ($\widehat{max}_j$) of the country-occupation distribution of proficiency for those workers who report neither feeling the need of training nor feeling to be able to do more demanding jobs.[15] As discussed in Section 2.3, the same requirements could also be estimated as the maximum proficiency level of workers who report feeling the need of training ($\widetilde{min}_j$) and the minimum proficiency of workers who feel they can do a more demanding job ($\widetilde{max}_j$). However, the first set of estimators ($\widehat{min}_j$ and $\widehat{max}_j$) is preferred because it is more robust to the most common sources of measurement error, such as respondents' overconfidence and the misinterpretation of the question about needing training. Comparing these alternative estimators can, therefore, provide an indication of the extent of measurement error.

Table 7 performs such a comparison for the pooled data of all the countries and occupations and separately for literacy and numeracy and it clearly shows that the two sets of estimates are massively different, thus emphasizing the importance of deriving indicators of mismatch that take measurement error into careful consideration. On average across all occupations and countries, $\widetilde{min}_j$ is approximately 1.8 times as large as $\widehat{min}_j$, whereas $\widehat{max}_j$ is approximately 1.6 times as big as $\widetilde{max}_j$. This implies that using the pure self-reported information to define skill-mismatch would lead to classify workers as over-skilled even if their assessed proficiency levels are very often below those of the self-reported well-matched or even under-skilled.

Using the proposed measure of skill mismatch it is also possible to compute indicators of skill under- and over-usage. The underlying idea is that over-skilled workers are not making

[15] In order to limit the impact of outliers, the 5th and the 95th percentiles are used instead of the actual minimum and maximum.

Table 7: Alternative estimates of the skill requirements

	$\widehat{min_j}^a$	$\widetilde{min_j}^b$	$\widehat{max_j}^c$	$\widetilde{max_j}^d$
Literacy	193.1	331.1	327.5	212.1
	(4.163)	(8.653)	(5.099)	(1.770)
Numeracy	175.9	333.1	325.5	199.1
	(10.722)	(11.377)	(9.776)	(2.579)

Bootstrapped standard errors in parentheses. See Appendix for details on the bootstrap procedure.
All figures are averages over occupational categories and countries.
[a] Fifth percentile of the proficiency distribution of workers not feeling able to do more demanding jobs nor feeling the need of training.
[b] Ninety-fifth percentile of the proficiency distribution of workers feeling the need of further training.
[c] Ninety-fifth percentile of the proficiency distribution of workers not feeling able to do more demanding jobs nor feeling the need of training.
[d] Fifth percentile of the proficiency distribution of workers feeling able to do more demanding jobs.
Source: OECD Survey of Adult Skills (PIAAC).

full use of all their skills and, similarly, under-skilled workers find themselves in the difficult position of having to over-use their skills in order to keep their jobs.

For each mismatched worker (either under- or over-skilled) it is possible to compare the use of skills with well-matched workers at their same level of proficiency and in the same country. Table 8 shows that, on average across countries, the indicator of literacy use at work for individuals who are under-skilled in literacy is about 25.9% higher than the corresponding indicator for similarly proficient workers who are well-matched, suggesting that they do actually over-use their skills. Consistent with the large overlap of mismatch across skill domains (see Table 5, literacy under-skilled workers also appear to over-use their numeracy at work (10.9% more than the well-matched). Notice that the over-usage of skills by the under-skilled is not necessarily an efficient outcome, since they could be more productive, while at the same time exerting less effort and being less stressed, if they were better matched.

Over-skilling is associated with a substantial waste of skills, as workers who are over skilled in literacy appear to use their skills at work substantially less than similarly proficient workers who are well-matched, namely 14.4% lower usage of literacy and 6.0% lower usage of numeracy. Looking at mismatch in numeracy shows very similar findings.

Table 8: Skill-mismatch and the use of skills at work

	Literacy mismatch		Numeracy mismatch	
	Over-use of literacy[a]	Over-use of numeracy[a]	Over-use of literacy[a]	Over-use of numeracy[a]
Under-skilled	0.259	0.109	0.163	0.170
	(0.031)	(0.071)	(0.013)	(0.128)
Well-matched	-	-0.002	-0.002	-
		(0.003)	(0.001)	
Over-skilled	-0.144	-0.060	-0.088	-0.097
	(0.012)	(0.008)	(0.010)	(0.017)
Total	-0.005	-0.004	-0.005	-0.004
	(0.004)	(0.007)	(0.004)	(0.007)

Bootstrapped standard errors in parentheses. See Appendix for details on the bootstrap procedure.
[a] Percentage difference between the average usage of the indicated category and the well-matched, conditional on proficiency.
Source: OECD Survey of Adult Skills (PIAAC).

5 Conclusions

This paper proposes a novel measure of skill mismatch, which allows classifying workers into under-skilled, well-matched and over-skilled along the skill domains of literacy and numeracy. The novelty lies mostly in the development of a theory-based procedure to identify jobs' requirements from data on workers in the absence of direct information about the production process.

The methodology is applied to the newly available data of the *Survey of Adult Skills (PIAAC)*, which contains direct assessments of skill proficiency in various domains for a large set of representative samples of individuals aged 16 to 65 in several countries. Employed workers are also asked a detailed battery of questions on skill use at work.

On average across the entire pooled sample, approximately 86% of dependent employees are well-matched in the literacy domain, about 4% are under-skilled and 10% are over-skilled. The overlap between literacy and numeracy mismatch is substantial: 94% of the workers who are well-matched in literacy are also well-matched in numeracy.

Men are more likely to be over-skilled than women, whereas gender differences in under-skilling are minor. Tertiary graduates are substantially less likely to be under-skilled than less educated workers and they are more likely to be over-skilled. Foreign workers are substantially more likely to be under-skilled and substantially less-likely to be over-skilled. Differences emerge also when looking across age groups.

Despite being mostly illustrative of the methodology, these findings have important implications for policy. A better match of the workers' skills to the requirements of their jobs can reduce the waste of skills among the over-skilled, improve the efficiency of the under-skilled while, at the same time, potentially reducing their levels of stress and, eventually, lead to important improvements of the overall productivity of the economy and the well-being of individuals.

References

Terry A. Ackerman. The theory and practice of item response theory by de ayala, r. j. *Journal of Educational Measurement*, 47(4):471–476, 2010.

Jim Allen and Rolf van der Velden. Educational mismatches versus skill mismatches: Effects on wages, job satisfaction, and on-the-job search. *Oxford Economic Papers*, 53(3):434–52, July 2001.

CEDEFOP. *The Skill Matching Challenge: Analysing Skill mismatch and Policy Implications.* Publications Office of the European Union, 2010.

Arnaud Chevalier. Measuring over-education. *Economica*, 70(279):509–531, 08 2003.

Arnaud Chevalier and Joanne Lindley. Overeducation and the skills of uk graduates. *Journal of the Royal Statistical Society Series A*, 172(2):307–337, 2009.

Lee J. Cronbach. Coefficient alpha and the interval structure of tests. *Psychometrika*, 16(3): 297–334, 1951.

Ayşegül Şahin, Joseph Song, Giorgio Topa, and Giovanni Violante. Mismatch unemployment. NBER Working Papers 18265, National Bureau of Economic Research, Inc, August 2012.

Richard Desjardins and Kjell Rubenson. An analysis of skill mismatch using direct measures of skills. OECD Education Working Papers 63, OECD Publishing, 2011.

Peter Dolton and Anna Vignoles. The incidence and effects of overeducation in the u.k. graduate labour market. *Economics of Education Review*, 19(2):179–198, April 2000.

Henry S. Farber. Mobility and stability: The dynamics of job change in labor markets. In O. Ashenfelter and D. Card, editors, *Handbook of Labor Economics*, volume 3 of *Handbook of Labor Economics*, chapter 37, pages 2439–2483. Elsevier, 1999.

Francis Green and Steven McIntosh. Is there a genuine under-utilization of skills amongst the over-qualified? *Applied Economics*, 39(4):427–439, 2007.

Wim Groot and Henriette Maassen van den Brink. Overeducation in the labor market: a meta-analysis. *Economics of Education Review*, 19(2):149–158, April 2000.

Maciej Jakubowski. Analysis of the predictive power of pisa test items. OECD Education Working Papers 87, OECD Publishing, 2013.

Boyan Jovanovic. Job matching and the theory of turnover. *Journal of Political Economy*, 87 (5):pp. 972–990, 1979.

Kostas Mavromaras, Seamus McGuinness, and Mark Wooden. Overskilling in the australian labour market. *Australian Economic Review*, 40(3):307–312, 09 2007.

Robert J. Mislevy. Should ”multiple imputations” be treated as ”multiple indicators”? *Psychometrika*, 58(1):79–85, 1993a.

Robert J. Mislevy. Some formulas for use with bayesian ability estimates. *Educational and Psychological Measurement*, 53(2):315–328, 1993b.

OECD. *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*. OECD Publishing, 2012.

OECD. *First International Report on PIAAC - Volume I*. OECD Publishing, 2013a.

OECD. *First International Report on PIAAC - Volume II.* OECD Publishing, 2013b.

Christopher A. Pissarides. *Equilibrium Unemployment Theory*. The MIT Press, second edition, 2000.

Glenda Quintini. Over-qualified or under-skilled: A review of existing literature. OECD Social, Employment and Migration Working Papers 121, OECD Publishing, 2011a.

Glenda Quintini. Right for the job: Over-qualified or under-skilled? OECD Social, Employment and Migration Working Papers 120, OECD Publishing, 2011b.

Jean-Marc Robin, Costas Meghir, and Jeremy Lise. Matching, sorting and wages. 2009 Meeting Papers 180, Society for Economic Dynamics, 2009.

Stephen Rubb. Overeducation: a short or long run phenomenon for individuals? *Economics of Education Review*, 22(4):389–394, August 2003.

Robert Shimer. Mismatch. *American Economic Review*, 97(4):1074–1101, September 2007.

Nachum Sicherman. Overeducation in the labor market. *Journal of Labor Economics*, 9(2): 101–22, April 1991.

P. J. Sloane, H. Battu, and P. T. Seaman. Overeducation, undereducation and the british labour market. *Applied Economics*, 31(11):1437–1453, 1999.

Robert H Topel and Michael P Ward. Job mobility and the careers of young men. *The Quarterly Journal of Economics*, 107(2):439–79, 1992.

Appendix. Inference

In order to make correct inference about the mismatch indicators reported in Section 4 it is necessary to take into proper account both the sample variability and the imputed nature of the skill measures (OECD, 2013b).

Taking proper account of the sampling variability is a particularly important issue in the OECD SAS, given the slightly different nature of the country samples. Although harmonized protocols guarantee the comparability of results, the OECD SAS remains a collection of country surveys, each of which have been constructed independently, although according to harmonized procedures that are meant to guarantee the comparability of the final data. Each country sample comes with a sampling weight that indicates the number of units in the target population represented by each sampled unit. Such a weight summarizes all the necessary information to obtain point estimates that are representative of the target population.[16]

In order compute asymptotically valid standard errors around the estimates presented in the main text the following procedure has been adopted. First, each sampled unit is identically replicated a number of times equal to its sampling weight (rounded to the closest integer), so as to generate a sample that replicates the full target population.[17] Then, such expanded datasets are fully representative of the target population and can be used to extract sequences of S bootstrapped samples of the same size of the original country samples. The entire analysis is then repeated on each bootstrapped sample resulting in a sequence of S estimates for each of the statistics presented in Section 4. The empirical distribution of the such statistics is then used to compute standard errors that are asymptotically valid by construction.[18]

[16]The OECD SAS also provides a sequence of *replicate weights* that can be used to assess the sampling variability (OECD, 2013b). However, it is not obvious how to use them with complex estimation procedures such as the derivation of the skill mismatch indicators and the related statistics. Moreover, additional adjustments would still be needed to take proper account of the imputation of the skill measurements.

[17]To reduce the size of the resulting datasets, all sampling weights have been divided by the minimum weight in the country so that each sampled unit is represented at least once and, at the same time, all relative weights remain unchanged.

[18]Performing correct bootstrapping without expanding the sample would require knowledge of the details of the sampling process in each county, namely stratification units, primary and secondary units, et. Unfortunately, this information is not provided in the OECD SAS (and

The above procedure can be easily adjusted to take into proper account the additional variability induced by the imputed nature of the skill measurements. As it is now common practice with IRT-derived measures of psychometric traits, a series of *plausible values* for each trait is provided. In the specific case of the OECD SAS, 10 *plausible values* for each of the three skills considered (literacy, numeracy and problem solving) are available. Each of them is an equally good proxy of the underlying unobservable psychometric construct, however each of them is a noisy proxy and the dispersion across the *plausible values* reflect measurement error (Mislevy, 1993a,b; OECD, 2013b).

All the point estimates presented in the main text are constructed using the means of the 10 *plausible values* for literacy and for numeracy as proxies of the corresponding underlying skill.[19]

In the bootstrapping procedure described above, at each replication one randomly selected plausible value is used to proxy skills (one plausible value for literacy and one for numeracy), so as to incorporate in the resulting sequence of estimates the additional variability induced by the imputed nature of the measurements.

the replicate weights are meant to replace it) for two sets of reasons. First, since the sampling structures of the country samples are sometimes quite different. For example, in some cases the original sampling frame is a standard population register whereas in other instances data are originally drawn from administrative archives. As a consequence , providing complete information about the sampling structure in a compact and comparable format across all countries is problematic. The second reason is related to the various confidentiality norms present in each participating country, many of which would be breached by the full disclosure of all the sampling information (OECD, 2013b).

[19]Alternatively, any plausible value could have been used and the resulting point estimates would have the exact same asymptotic properties.