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Research Article

Cooperative Advertising in a Supply Chain with Horizontal Competition

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Cooperative advertising programs are usually provided by manufacturers to stimulate retailers investing more in local advertising to increase the sales of their products or services. While previous literature on cooperative advertising mainly focuses on a "single-manufacturer single-retailer" framework, the decision-making framework with "multiple-manufacturer single-retailer" becomes more realistic because of the increasing power of retailers as well as the increased competition among the manufacturers. In view of this, in this paper we investigate the cooperative advertising program in a "two-manufacturer single-retailer" supply chain in three different scenarios; that is, (i) each channel member makes decisions independently; (ii) the retailer is vertically integrated with one manufacturer; (iii) two manufacturers are horizontally integrated. Utilizing differential game theory, the open-loop equilibrium-advertising strategies of each channel member are obtained and compared. Also, we investigate the effects of competitive intensity on the firm's profit in three different scenarios by using the numerical analysis.

1. Introduction

To increase the sales of their products or services, some manufacturers or service providers utilize cooperative advertising programs, through which they share a part of the retailer's advertising cost, to stimulate retailers advertising more on their products or services. Generally, advertising can be divided into national advertising and local advertising. The former focuses on building brand image about the products or services. The latter is often price oriented to stimulate consumer to purchase the products or services at once. Supported by subsidies from a manufacturer's cooperative advertising program, retailers would always increase their local advertising expenditures and thus improve their profits [1].

Surveys showed that, for many manufacturers or service providers such as General Electric, their advertising budgets to retailers via cooperative advertising programs are more three times of that they spent on national advertising [2]. Further, Dant and Berger found that 25–40% of local advertisements are cooperatively funded [3]. Total expenditures on cooperative advertising in 2000 were estimated at $15 billion, compared to $900 million in 1970, nearly a four-fold increase in real terms [4]. In 2010, about $50 billion was spent on cooperative advertising programs [5].

The tendency toward increased spending on cooperative advertising has received significant attention from researchers. The cooperative advertising models under study can be divided into two categories: static models [1, 6–12] and dynamic models [13–19]. However, these studies mainly focus on a "single-manufacturer single-retailer" framework.

Retailers in today's market are increasingly more powerful than manufacturers. Useem found that sales through Wal-Mart accounted for 17% of P&G's total sales in 2002, 39% of Tandy's, and over 10% for many other large manufacturers [20]. Tesco is the largest grocer in the United Kingdom, accounting for almost 30% of the supermarket sales [21]. Home Depot and Lowe have more than 50% of the home improvement market [22]. As retailers become more dominant, manufacturers face fierce competition among themselves. Thus, it is necessary to take the competition among manufacturers into account when studying the cooperative advertising model.

The significant contribution of this paper is that it generalizes existing cooperative advertising work on "single-manufacturer single-retailer" framework to the "two-manufacturer single-retailer" framework. This generalization has provided new analytical results about how the competition affects the advertising efforts and profit for channel member. In detail, we study the open-loop equilibrium advertising strategies of each channel member in three different scenarios, including that (i) each channel member makes decisions independently; (ii) the retailer is vertically integrated with one manufacturer; (iii) two manufacturers are horizontally integrated. Specifically, the following research questions are addressed in this paper. (i) For each scenario, what are the equilibrium advertising efforts for each channel member and what is the manufacturer's optimal participation rate for the retailer's local advertising expenditures? (ii) When the retailer integrates with one manufacturer, does the manufacturer change its decisions about national advertising expenditures and participation rates? (iii) How does the horizontal integration of two manufacturers affect the decisions of each channel member?

To answer the above questions, we focus on the cooperative advertising problem in a "two-manufacturer single-retailer" framework. The dynamic advertising models are proposed based on the Nerlove-Arrow model. Utilizing differential game theory, the open-loop equilibrium advertising strategies of each channel member are obtained and compared in three different scenarios.

The remainder of the paper is structured as follows. Previous literature related to our topic is reviewed in Section 2. Section 3 develops the proposed models, and then the equilibrium advertising efforts and participation rates in three different scenarios are discussed. Section 4 offers a numerical analysis. Conclusions and suggestions for future research are in Section 5. Proofs for all propositions in the paper are given in Appendices.

2. Literature Review

Our work is related to several research streams. First is the stream of literature that focuses on cooperative advertising, which can be divided into two main categories: static models and dynamic models. A primary static model was proposed by Berger [6], who was the first to analyze cooperative advertising. Bergen and John developed two formal models to study the effects of the participation rate offered by manufacturers [7]. By dividing advertising into national and local, Huang et al. were able to study co-op advertising models in a static supply chain framework and discuss the channel members' advertising decisions for different relationship configurations between the channel members [1, 8]. Based on the work of Huang and Li [1], Yue et al. studied the co-op advertising problem by considering a price discount in demand elasticity market circumstance [9]. Xie and Neyret proposed a more general model, including co-op advertising and pricing [10]. Further, Seyed Esfahani et al. considered vertical co-op advertising along with pricing decisions in a supply chain and proved that both the manufacturer and the retailer reach the highest profits level when they follow a cooperation strategy [12]. For the dynamic advertising models, Chintagunta and Jain extended the work of Nerlove and Arrow [23] to consider the interaction effects of manufacturer and retailer goodwill on channel sales and developed a dynamic model to study the equilibrium advertising strategies in a two-member marketing channel [13]. Jørgensen et al. provided a dynamic model for a cooperative advertising framework, which allows both channel members to make long- and short-term advertising efforts to enhance sales and consumer goodwill [14]. Further, Jørgensen et al. introduced decreasing marginal returns to goodwill and adopted a more flexible functional form for the sales dynamics [15]. Jørgensen et al. studied the cooperative advertising program in the case where the retailer's promotions can damage the brand image [16]. Extending the work of Jørgensen et al. [15], Karray and Zaccour considered a differential game model for a marketing channel formed by one manufacturer and one retailer and concluded that a cooperative advertising program can help the manufacturer mitigate the competitive impact of the private label [18]. He et al. provided a theoretical analysis of cooperative advertising plans in a dynamic stochastic supply chain [19].

The above literature is mainly focused on a "single-manufacturer single-retailer" framework. Few studies address a "multiple-manufacturer single-retailer" framework or any other framework. Kurtuluş and Toktay considered a model including two competing manufacturers and one retailer; the result revealed that the retailer can use the form of category management and the category shelf space to control the intensity of competition between manufacturers to his benefit [24]. Adida and DeMiguel studied competition in a supply chain where multiple manufacturers compete in quantities to supply a set of products to multiple risk-averse retailers who compete in quantities to satisfy the uncertain consumer demand [25]. Cachon and Kök also studied a supply chain system with competing manufacturers and a single retailer; the results showed that the properties a contractual form exhibits in a one-manufacturer supply chain may not carry over to the realistic setting in which multiple manufacturers must compete to sell their goods through the same retailer [26]. Further, Lu et al. highlighted the importance of service from manufacturers in the interactions between two competing manufacturers and their common retailer, and their result showed that as the market base of one product increases, the second manufacturer also benefits but at a lesser amount than the first manufacturer [27]. However, the abovementioned works with multiple manufacturers do not consider cooperative advertising. There are some cooperative advertising works that focus on a "multiple-retailer" framework. For example, He et al. [28, 29] extended He et al. [19] by considering the competing retailers, and their results showed that the manufacturer's support for its retailer is higher under competition than in its absence.

To our best knowledge, research relating to cooperative advertising focused on a "multiple-manufacturer single-retailer" framework in the supply chain has not been explored in literature. In this study, we investigate a cooperative advertising model using the "two-manufacturer single-retailer" framework.

3. Model Description

As shown in Figure 1, we consider a supply chain system consisting of two competing manufacturers and one retailer. The two manufacturers produce similar products with different brands which are denoted as i, $i \in \{1,2\}$ that the retailer sells simultaneously. The competition is based primarily on the use of nonprice competitive strategies, namely, the two manufacturers each advertise their products, and the retailer advertises two products simultaneously.

We introduce the additional notation in this paper (see Table 1).

As our goodwill-based model is based upon the model of Nerlove-Arrow, the changing of the stock of goodwill of product i is given by

$$\dot{G}_i(t) = U_{Mi} - \theta U_{M(3-i)} - \delta G_i, \quad i \in \{1,2\}, \tag{1}$$

where $0 < \theta < 1$ is a constant which represents the rival advertising's negative effect on goodwill, as seen in previous literature [30]. Next, $\delta > 0$ is the diminishing rate of goodwill, which captures the idea that consumers may forget the brand to some extent. National advertising mainly focuses on firm's long-term objectives such as brand awareness, image, and credibility [31]. Therefore, we only take the effect of national advertising into the stock of goodwill here. Further, the initial goodwill of the two products is denoted as

$$G_1(0) = G_{10} \geq 0, \qquad G_2(0) = G_{20} \geq 0, \tag{2}$$

and the sales $S_i(t)$ of the two brands along time t satisfy

$$S_i(t) = \max\{0, \alpha G_i + \lambda U_{Ri} - \chi U_{R(3-i)}\}, \quad i \in \{1,2\}. \tag{3}$$

In (3), α, λ, χ are all positive constants. For the sake of simplicity, we suppose that the influence coefficient is identical for these two products. In (3), the item αG_i represents the long-term effect of national advertising on sales, and the item λU_{Ri} represents the effect of the retailer's local advertising on product i. As in previous research such as Jørgensen et al. [14], we only take the promotion effect of local advertising into the functions of sales here without the stock of goodwill. The item $-\chi U_{R(3-i)}$ illustrates the rival local advertising's negative effect on sales. Next, $\lambda > \chi > 0$, which implies that the effects of rival advertising are generally smaller than the effects of one's own advertising effect, which is a fairly common assumption in the relevant literature [30].

The advertising cost functions are quadratic with respect to marketing efforts, namely,

$$C(U_{Mi}(t)) = \frac{U_{Mi}^2(t)}{2}, \quad C(U_{Ri}) = \frac{U_{Ri}^2(t)}{2}, \quad i \in \{1,2\}. \tag{4}$$

This assumption about the advertising cost function is commonly found in literature [32]. The convex cost function implies increasing marginal cost of effort.

Without considering advertising expenditures, the marginal profit of manufacturer i is assumed as $\rho_{Mi} \geq 0$, and the marginal profit of the retailer selling the product i is

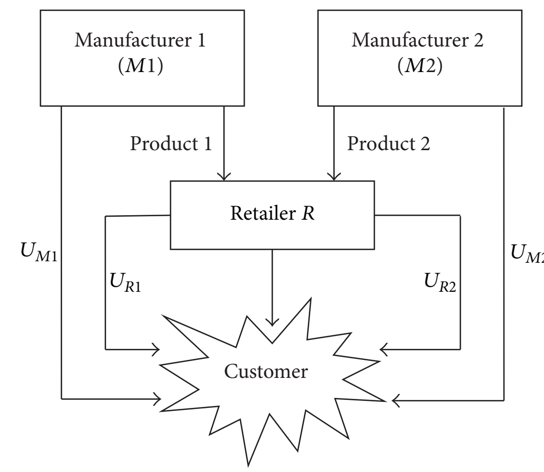

Figure 1

Table 1: Notation.

t	Time t, $t \geq 0$
$G_i(t)$	Goodwill of the product i at time t, $i \in \{1,2\}$
$U_{Mi}(t)$	Manufacturer i's national advertising efforts at time t
$U_{Ri}(t)$	Retailer's local advertising efforts for product i at time t
$S_i(t)$	Sales of product i along time t
$\theta \in [0,1]$	The national advertising's competition coefficient
$\chi \in [0,1]$	The local advertising's competition coefficient
$\phi_i \in [0,1]$	Manufacturer i's participation rate for the retailer's advertising cost
$\rho_{Mi} \geq 0$	Marginal profit of manufacturer i
$\rho_{Ri} \geq 0$	Marginal profit of the retailer sells the product i
$\delta > 0$	Diminishing rate of goodwill
$r > 0$	Discount rate of the manufacturers and the retailer
π_{Mi}, π_R	Profit functions for Mi and R, respectively
J_{Mi}, J_R	Current value of profit functions for Mi and R, respectively

$\rho_{Ri} \geq 0$. The profit functions of the two manufacturers are then

$$\pi_{Mi}(t) = \rho_{Mi} S_i(t) - \frac{1}{2} U_{Mi}^2(t) - \frac{1}{2} \phi_i U_{Ri}^2(t), \quad i \in \{1,2\}, \tag{5}$$

and the profit function of the retailer is

$$\pi_R(t) = \sum_{i=1}^{2} \left(\rho_{Ri} S_i(t) - \frac{1}{2}(1-\phi_i) U_{Ri}^2(t) \right). \tag{6}$$

In this paper, we assume the participation rate is a constant over time for the following reasons. (i) Although much more literature assumes that the participation rate changes along time [19], a changing participation rate is so complex that there are no cooperative advertising programs in practice. In the empirical studies of Nagler [4], all the 1470 plans explicitly listed a single fixed participation rate. If a firm provides a cooperative advertising program with a changing participation rate, the manufacturer would have to know the retailer's daily advertising cost exactly, which is much more difficult than learning the whole advertising cost over a certain period of time. (ii) Even, in previous studies which model the participation rate as a function of time, the final optimal decision for participation rates were all constant over time [14, 18, 19, 28].

Please note that U_{Mi} and ϕ_i are manufacturer's decision variables, and U_{Ri} is retailer's decision variable. Then we consider a two-stage game in this paper. The manufacturers offer their participation rates for the retailer's local advertising expenditure at stage 1, and then the manufacturers and retailer determine their advertising efforts along time t simultaneously at stage 2. We firstly keep the participation rates ϕ_i $(i = 1, 2)$ as fixed, calculate the advertising efforts of the manufacturers and retailer utilizing differential game theory, and then decide the manufacturers' optimal participation rates.

3.1. Each Channel Member Makes Decisions Independently. In this scenario, each channel member makes decisions independently, and the profit functions of all channel members are given by (5) and (6). Note the profits for all the channel members changes along with time t. Each channel member, then, strives to maximize the current values of its profit. With a common discount rate $r > 0$ and for the sales $S_i \geq 0$, we have

$$\max_{U_{Mi}\geq 0, 1\geq\phi_i\geq 0} J_{Mi} = \int_0^{+\infty} e^{-rt}\pi_{Mi}(t)\,dt, \quad i \in \{1, 2\}, \tag{7}$$

and, for the retailer, we have

$$\max_{U_{R1}\geq 0, U_{R2}\geq 0} J_R = \int_0^{+\infty} e^{-rt}\pi_R(t)\,dt. \tag{8}$$

Taking (1) into account, we get the current value Hamiltonian of two manufacturers as

$$\begin{aligned} H_{M1} &= \pi_{M1} + \mu_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \mu_{12}(U_{M2} - \theta U_{M1} - \delta G_2), \\ H_{M2} &= \pi_{M2} + \mu_{21}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \mu_{22}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \tag{9}$$

Similarly, we get the retailer's current value Hamiltonian as

$$\begin{aligned} H_R &= \pi_R + \mu_{31}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \mu_{32}(U_{M2} - \theta U_{M1} - \delta G_2), \end{aligned} \tag{10}$$

where μ_{i1} and μ_{i2} $(i = 1, 2, 3)$ represent the costate variables in the firm's problem corresponding to the firm's goodwill levels.

Then using the necessary conditions for equilibrium, we obtain the following results.

Proposition 1. *When each channel member makes decisions independently and the participation rates ϕ_i $(i = 1, 2)$ are fixed, the equilibrium advertising efforts for two manufacturers on their products along time t are all constants; that is,*

$$U_{Mi}^{(1)} = \frac{\alpha\rho_{Mi}}{(r+\delta)}, \quad i \in \{1, 2\}. \tag{11}$$

Proposition 1 illustrates the following facts. (i) Whatever the participation rate that the manufacturer undertakes for the retailer's advertising cost, the manufacturer's equilibrium national advertising efforts are kept the same and are just linear with its own marginal profit. The larger the marginal profit, the more the manufacturer would spend on national advertising. (ii) The manufacturer's equilibrium advertising efforts are determined by the item $\alpha\rho_{Mi}/(r+\delta)$, which is aimed at maintaining the long-term effect of advertising. Specifically, this item decreases sharply when the diminishing rate of consumer goodwill becomes very large or the decision makers are more short sighted. Therefore, when the decision makers do not feel confident in future, or the customer's goodwill diminishes quickly, the advertising efforts would drop.

Further, we obtain the retailer's equilibrium advertising efforts for two brands as follows.

Proposition 2. *When each channel member makes decisions independently and the participation rates ϕ_i $(i = 1, 2)$ are fixed, the retailer's equilibrium advertising efforts for the two brands along time t are all constants:*

$$U_{Ri}^{(1)} = \begin{cases} \dfrac{\lambda\rho_{Ri} - \chi\rho_{R(3-i)}}{1-\phi_i} & if\ \lambda\rho_{Ri} - \chi\rho_{R(3-i)} \geq 0, \\ 0 & else \end{cases} \qquad i \in \{1, 2\}. \tag{12}$$

Proposition 2 holds the following managerial implications. (i) When condition $\lambda\rho_{Ri} - \chi\rho_{R,(3-i)} \geq 0$, $i \in \{1, 2\}$ is satisfied, a higher participation rate leads the retailer to spend more on local advertising but the advertising efforts are independent of the participation rate which the other manufacturer provides to the retailer. Therefore, the manufacturer can use the participation rate to guide the retailer's advertising efforts for his product. (ii) The equilibrium local advertising efforts on product i are increased by the retailer's marginal profit of product i.

Furthermore, when condition $\lambda > \chi > 0$ is satisfied, we get the following results by (12).

(i) If condition $\lambda\rho_{R2} - \chi\rho_{R1} < 0$ holds, we have $U_{R1}^{(1)} = (\lambda\rho_{R1} - \chi\rho_{R2})/(1-\phi_1)$ and $U_{R2}^{(1)} = 0$. Because the marginal profit which the retailer obtains from product 2 is extremely small, the benefit of $U_{R2}^{(1)}$ from product 2 does not offset the loss from product 1. Thus, the retailer would not advertise product 2. In other words, whatever participation rate manufacturer 2 offers, the retailer would never advertise product 2. Under this situation, manufacturer 2 only can change the situation of no-local-advertising efforts on his product by offering the retailer a higher margin.

(ii) If conditions $\lambda\rho_{R1}-\chi\rho_{R2}\geq 0$ and $\lambda\rho_{R2}-\chi\rho_{R1}\geq 0$ hold, we can get $U_{R1}^{(1)}=(\lambda\rho_{R1}-\chi\rho_{R2})/(1-\phi_1)$ and $U_{R2}^{(1)}=(\lambda\rho_{R2}-\chi\rho_{R1})/(1-\phi_2)$. In this situation, advertising for two brands would lead to so much gain for the retailer that the retailer would advertise both products at a certain level.

(iii) If condition $\lambda\rho_{R1}-\chi\rho_{R2}<0$ holds, we have $U_{R1}^{(1)}=0$ and $U_{R2}^{(1)}=(\lambda\rho_{R2}-\chi\rho_{R1})/(1-\phi_2)$. This situation is similar to the first situation; the retailer would advertise product 2, but not product 1.

When the two manufacturers' participation rates are fixed, the equilibrium advertising efforts for all channel members are given by Propositions 1 and 2. Based on these results, we can work out the stock of goodwill for the two products as well as for the current value of profits for all channel members, which is given by Proposition 3.

Proposition 3. *When each channel member makes decisions independently and their advertising efforts are kept as constants, that is,* $U_{Mi}(t)=U_{Mi}^{(1)}$ *and* $U_{Ri}(t)=U_{Ri}^{(1)}$, $i=1,2$, *then the accumulated goodwill of two products along time t is*

$$G_i(t)=D_i e^{-\delta t}+G_{iSS}^{(1)},\quad i\in\{1,2\},\tag{13}$$

where $D_i=G_{i0}-(U_{Mi}^{(1)}-\theta U_{M,(3-i)}^{(1)})/\delta$ *and* $G_{iSS}^{(1)}=(U_{Mi}^{(1)}-\theta U_{M,(3-i)}^{(1)})/\delta$, $i=1,2$. $G_{iSS}^{(1)}$ *is the steady-state goodwill for product i when* $t\to\infty$.

From (13), we obtain the following facts: (i) the steady-state goodwill for product i increases with manufacturer 1's national advertising efforts; (ii) the steady-state goodwill for product i decreases with the rival manufacturer's national advertising efforts because of the competitive effect; (iii) steady-state goodwill is only affected by the manufacturer's advertising efforts because local advertising has only an instant promotion effect that has no impact on the stock of goodwill; (iv) when the diminishing rate of goodwill becomes very large, steady-state goodwill decreases.

Substituting (11)–(13) into (7) and (8) and with the participation rates ϕ_i ($i=1,2$) being fixed, we get the current value of manufacturer 1's profit under the equilibrium condition as follows:

$$\begin{aligned}J_{Mi}^{(1)}=&\frac{D_i\alpha\rho_{Mi}}{r+\delta}+\frac{\alpha\rho_{Mi}\left(U_{Mi}^{(1)}-\theta U_{M,(3-i)}^{(1)}\right)}{r\delta}\\&+\frac{\rho_{Mi}\left(\lambda U_{Ri}^{(1)}-\chi U_{R,(3-i)}^{(1)}\right)}{r}\\&-\frac{\left(U_{Mi}^{(1)}\right)^2+\phi_i\left(U_{Ri}^{(1)}\right)^2}{2r},\quad i\in\{1,2\}.\end{aligned}\tag{14}$$

The current value of the retailer's profit is

$$\begin{aligned}J_R^{(1)}=&\frac{D_1\alpha\rho_{R1}+D_2\alpha\rho_{R2}}{r+\delta}\\&+\frac{\alpha\rho_{R1}\left(U_{M1}^{(1)}-\theta U_{M2}^{(1)}\right)+\alpha\rho_{R2}\left(U_{M2}^{(1)}-\theta U_{M1}^{(1)}\right)}{r\delta}\\&-\frac{1-\phi_1}{2}\left(U_{R1}^{(1)}\right)^2+\frac{\rho_{R1}\left(\lambda U_{R1}^{(1)}-\chi U_{R2}^{(1)}\right)}{r}\\&+\frac{\rho_{R2}\left(\lambda U_{R2}^{(1)}-\chi U_{R1}^{(1)}\right)}{r}-\frac{1-\phi_2}{2}\left(U_{R2}^{(1)}\right)^2,\end{aligned}\tag{15}$$

where $D_i=G_{i0}-(U_{Mi}^{(1)}-\theta U_{M,(3-i)}^{(1)})/\delta$, $i\in\{1,2\}$.

Differentiating $J_{Mi}^{(1)}$ with the participation rate ϕ_i, we get optimal participation rates, from are given by Proposition 4.

Proposition 4. *When all the channel members make decisions independently, the optimal participation rates that the two manufacturers provide to the retailer under the equilibrium condition are*

$$\phi_i^{(1)}=\begin{cases}\dfrac{\lambda(2\rho_{Mi}-\rho_{Ri})+\chi\rho_{R(3-i)}}{\lambda(2\rho_{Mi}+\rho_{Ri})-\chi\rho_{R(3-i)}} & \textit{if}\ \rho_{Mi}\geq\dfrac{(\lambda\rho_{Ri}-\chi\rho_{R(3-i)})}{2\lambda},\\ 0 & \textit{else}\end{cases}\quad i\in\{1,2\}.\tag{16}$$

For (16), the restraining condition $\rho_{M1}\geq(\lambda\rho_{R1}-\chi\rho_{R2})/2\lambda$ implies that manufacturer 1 is willing to provide the participation rate with the retailer only when he can obtain a large enough marginal profit. Differentiating $\phi_1^{(1)}$ from ρ_{M1}, ρ_{R1}, and ρ_{R2}, and knowing that $\rho_{M1}\geq(\lambda\rho_{R1}-\chi\rho_{R2})/2\lambda$, we find that $\partial\phi_1^{(1)}/\partial\rho_{M1}>0$; $\partial\phi_1^{(1)}/\partial\rho_{R1}<0$; $\partial\phi_1^{(1)}/\partial\rho_{R2}>0$. The above expressions show that (i) when the manufacturer's marginal profit increases, he would offer a high participation rate to the retailer; (ii) when a high marginal profit would be obtained by the retailer, the manufacturer has less incentive to offer a high participation rate for the cooperative program; (iii) manufacturer 1 would offer a high participation rate if the retailer obtains a larger marginal profit from product 2.

Furthermore, substituting the optimal participation rates into (12), we find that the retailer's equilibrium local advertising efforts on the two brands are all constants, that is,

$$U_{Ri}^{(1)}=\begin{cases}\lambda\rho_{Mi}+\dfrac{1}{2}\lambda\rho_{Ri}-\dfrac{1}{2}\chi\rho_{R(3-i)}, & \text{if } 0<\dfrac{(\lambda\rho_{Ri}-\chi\rho_{R(3-i)})}{2\lambda}\leq\rho_{Mi},\\ \lambda\rho_{Ri}-\chi\rho_{R(3-i)}, & \text{if } \rho_{Mi}<\dfrac{(\lambda\rho_{Ri}-\chi\rho_{R(3-i)})}{2\lambda},\\ 0 & \text{else}\end{cases}\quad i\in\{1,2\}.\tag{17}$$

Equation (17) shows that the retailer's equilibrium advertising level $U_{Ri}^{(1)}$ for a product is not only linear with his own marginal profit ρ_{Ri}, but also linear with the manufacturer's marginal profit ρ_{Mi} if the conditions $0 < (\lambda\rho_{Ri} - \chi\rho_{R(3-i)})/2\lambda \leq \rho_{Mi}$ hold. Supposing that $\rho_{Mi} + \rho_{Ri} = \rho_i$ is the channel marginal profit of one product, the equilibrium advertising level $U_{Ri}^{(1)}$ can be rewritten as $U_{Ri}^{(1)} = \lambda\rho_{Mi}/2 + \lambda\rho_i/2 - \chi\rho_{R,(3-i)}/2$ if and only if $0 < \lambda\rho_{Ri} - \chi\rho_{R,(3-i)} \leq 2\lambda\rho_{Mi}$, $i = 1, 2$ hold. The channel marginal profit of product i is not changed in the short term; therefore, the above equations imply that the retailer's equilibrium advertising efforts are independent of the marginal profit which the retailer obtains from product i.

3.2. Retailer Integrates with a Manufacturer. In second scenario, the retailer integrates with one of the manufacturers. We assume this manufacturer is $M1$. Then, the profit function of the integration system is $\pi_{M1,R} = \pi_{M1} + \pi_R$, and the objective of integration system is

$$\max_{U_{M1}\geq 0, U_{R1}\geq 0, U_{R2}\geq 0} J_{M1,R} = \int_0^{+\infty} e^{-rt}\left(\pi_{M1} + \pi_R\right) dt. \tag{18}$$

Further, the objective of manufacturer 2 is

$$\max_{1\geq\phi_2\geq 0, U_{M2}\geq 0} J_{M2} = \int_0^{+\infty} e^{-rt}\pi_{M2} dt. \tag{19}$$

Taking state equation (1) into account, the current value Hamiltonian of the vertical integration system ($M1$ and R) is

$$\begin{aligned} H_{M1,R} &= \pi_{M1} + \pi_R + \gamma_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \gamma_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right), \end{aligned} \tag{20}$$

and that of manufacturer 2 is

$$\begin{aligned} H_{M2} &= \pi_{M2} + \gamma_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &\quad + \gamma_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right), \end{aligned} \tag{21}$$

where γ_{i1} and γ_{i2} $(i = 1, 2)$ represent the costate variables in the channel member's problem corresponding to the firm's goodwill level.

Then using (20) and (21), we obtain the following results.

Proposition 5. *When the retailer integrates with manufacturer 1 and the participation rate ϕ_2 provided by manufacturer 2 is fixed, the equilibrium advertising efforts of manufacturer 2 are constant, that is,*

$$U_{M2}^{(2)} = \frac{\alpha\rho_{M2}}{(r+\delta)}. \tag{22}$$

Compared to Proposition 1, we find that manufacturer 2's equilibrium advertising level is the same, which implies that manufacturer 2's advertising level does not depend on the integration between manufacturer 1 and retailer.

Proposition 6. *When the retailer integrates with manufacturer 1 and the participation rate ϕ_2 provided by manufacturer 2 is fixed, manufacturer 1's equilibrium advertising efforts are constant, that is,*

$$U_{M1}^{(2)} = \begin{cases} \dfrac{\alpha\rho_1 - \theta\alpha\rho_{R2}}{r+\delta} & \text{if } \rho_1 \geq \theta\rho_{R2}, \\ 0 & \text{else}, \end{cases} \tag{23}$$

and the retailer's equilibrium advertising efforts for the two brands along time t are all constants, that is,

$$U_{R1}^{(2)} = \begin{cases} \lambda\rho_1 - \chi\rho_{R2} & \text{if } \lambda\rho_1 - \chi\rho_{R2} \geq 0, \\ 0 & \text{else}, \end{cases} \tag{24}$$

$$U_{R2}^{(2)} = \begin{cases} \dfrac{\lambda\rho_{R2} - \chi\rho_1}{1-\phi_2} & \text{if } \lambda\rho_{R2} - \chi\rho_1 \geq 0, \\ 0 & \text{else}, \end{cases} \tag{25}$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

Proposition 6 shows the following trends. If the condition $\rho_1 > \theta\rho_{R2}$ holds, the national advertising efforts $U_{M1}^{(2)}$ are affected by the item $(\alpha\rho_1 - \theta\alpha\rho_{R2})/(r+\delta)$. From this item, we know that the larger the channel marginal profit of product 1 (ρ_1), the more the integration system would spend on national advertising for product 1. As opposed to the first scenario, in this scenario the national advertising efforts are also affected by the rival product's marginal profit ρ_{R2}. When ρ_{R2} is increased, the integration system would decrease national advertising efforts for product 1 and thus decrease product 1's adverse influence on product 2. Further, if the channel marginal profit of product 1 is too small, the integration system would not advertise product 1.

If we subtract (17) from (24), we get

$$\Delta U_{R1} = \begin{cases} \lambda\rho_{M1} & \text{if } 2\lambda\rho_{M1} < \lambda\rho_{R1} - \chi\rho_{R2}, \\ \dfrac{\lambda\rho_{R1} - \chi\rho_{R2}}{2} & \text{if } 0 < \lambda\rho_{R1} - \chi\rho_{R2} \leq 2\lambda\rho_{M1}, \\ \lambda\rho_1 - \chi\rho_{R2} & \text{if } -\lambda\rho_{M1} < \lambda\rho_{R1} - \chi\rho_{R2} \leq 0, \\ 0 & \text{else}, \end{cases} \tag{26}$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

It is easy to prove that ΔU_{R1} given by (26) is nonnegative, which means that the integration between the retailer and the manufacturer would increase the retailer's equilibrium local advertising efforts for product 1.

Furthermore, combining (24) and (25), we obtain similar managerial implications as the results of Proposition 2, but we also find some differences.

(i) When $\lambda\rho_{R2} - \chi\rho_1 < 0$ holds, we have $U_{R2}^{(2)} = 0$ and $U_{R1}^{(2)} = \lambda\rho_1 - \chi\rho_{R2}$. Note that $\rho_1 = \rho_{M1} + \rho_{R1} > \rho_{R1}$, which implies that the integration between the retailer and manufacturer 1 would lead to the increase in the retailer's local advertising threshold for product 2 and would also increase the retailer's equilibrium advertising efforts for product 1.

(ii) If conditions $\lambda\rho_1 - \chi\rho_{R2} \geq 0$ and $\lambda\rho_{R2} - \chi\rho_1 \geq 0$ hold, $U_{R2}^{(2)} = \lambda\rho_1 - \chi\rho_{R2}$ and $U_{R2}^{(2)} = (\lambda\rho_{R2} - \chi\rho_1)/(1 - \phi_2)$. In this situation, the retailer increases the equilibrium advertising efforts for product 1 and decreases efforts for product 2.

(iii) When $\lambda\rho_1 - \chi\rho_{R2} < 0$ holds, $U_{R1}^{(2)} = 0$ and $U_{R2}^{(2)} = (\lambda\rho_{R2} - \chi\rho_1)/(1 - \phi_2)$. Note that $\rho_1 > \rho_{R1}$, which implies that the integration between the retailer and manufacturer 1 reduces the retailer's local advertising threshold for product 1 and also decreases the retailer's equilibrium advertising efforts for product 2.

We can calculate the stock of goodwill for the two products and the current value of profits for all channel members, which are given by Proposition 7.

Proposition 7. *When the retailer integrates with manufacturer M1, and their advertising efforts are kept constant, that is, $U_{Mi}(t) = U_{Mi}^{(2)}$ and $U_{Ri}(t) = U_{Ri}^{(2)}$, $i = 1, 2$, then the accumulated goodwill of the two products along time t is*

$$G_i(t) = E_i e^{-\delta t} + G_{iSS}^{(2)}, \quad i \in \{1, 2\}, \tag{27}$$

where $G_{iSS}^{(2)} = (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $E_i = G_{i0} - (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1, 2$. $G_{iSS}^{(2)}$ is the steady-state goodwill for product i when $t \to \infty$.

Substituting (22) through (25) and (27) into (18) and (19), and assuming that the participation rates ϕ_i $(i = 1, 2)$ are fixed, we get the current value of the integration system's profit as follows:

$$\begin{aligned} J_{M1,R}^{(2)} = {} & \frac{E_1\alpha\rho_1 + E_2\alpha\rho_{R2}}{r+\delta} \\ & + \frac{\alpha\rho_1\left(U_{M1}^{(2)} - \theta U_{M2}^{(2)}\right) + \alpha\rho_{R2}\left(U_{M2}^{(2)} - \theta U_{M1}^{(2)}\right)}{r\delta} \\ & - \frac{\left(U_{M1}^{(2)}\right)^2 + \left(U_{R1}^{(2)}\right)^2}{2r} + \frac{\rho_1\left(\lambda U_{R1}^{(2)} - \chi U_{R2}^{(2)}\right)}{r} \\ & + \frac{\rho_{R2}\left(\lambda U_{R2}^{(2)} - \chi U_{R1}^{(2)}\right)}{r} - \frac{(1-\phi_2)\left(U_{R2}^{(2)}\right)^2}{2r}, \end{aligned} \tag{28}$$

and the profit for manufacturer 2 is

$$\begin{aligned} J_{M2}^{(2)} = {} & \frac{E_2\alpha\rho_{M2}}{r+\delta} + \frac{\alpha\rho_{M2}\left(U_{M2}^{(2)} - \theta U_{M1}^{(2)}\right)}{r\delta} \\ & + \frac{\rho_{M2}\left(\lambda U_{R2}^{(2)} - \chi U_{R1}^{(2)}\right)}{r} - \frac{\left(U_{M2}^{(2)}\right)^2 - \phi_2\left(U_{R2}^{(2)}\right)^2}{2r}, \end{aligned} \tag{29}$$

where $E_i = G_{i0} - (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1, 2$.

Differentiating $J_{M2}^{(2)}$ from the participation rate ϕ_2, we get the optimal participation rate, which is given by Proposition 8.

Proposition 8. *When the retailer integrates with manufacturer 1, and the advertising levels are kept as constants, that is, $U_{Mi}(t) = U_{Mi}^{(2)}, U_{Ri}(t) = U_{Ri}^{(2)}$, $i = 1, 2$, the optimal participation rate which manufacturer 2 provides is*

$$\phi_2^{(2)} = \begin{cases} \dfrac{\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1}{\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_1} & if\ \rho_{M2} \geq \dfrac{(\lambda\rho_{R2} - \chi\rho_1)}{2\lambda}, \\ 0 & else, \end{cases} \tag{30}$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

Subtracting (16) from (30), we have

$$\Delta\phi_2 = \begin{cases} \dfrac{4\lambda\chi\rho_{M1}\rho_{M2}}{L_1 L_2} \\ \qquad \text{if } \dfrac{(\lambda\rho_{R2} - \chi\rho_{R1})}{2\lambda} \leq \rho_{M2}, \\ \dfrac{\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1}{\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_1} \\ \qquad \text{if } \rho_{M2} < \dfrac{(\lambda\rho_{R2} - \chi\rho_{R1})}{2\lambda} \leq \rho_{M2} + \dfrac{\chi\rho_{M1}}{2\lambda}, \\ 0 \qquad \text{else}, \end{cases} \tag{31}$$

where $L_1 = 2\lambda\rho_{M2} + \lambda\rho_{R2} - \chi\rho_1$ and $L_2 = 2\lambda\rho_{M2} + \lambda\rho_{R2} - \chi\rho_{R1}$.

We can prove that (31) is nonnegative, which implies that manufacturer 2 would increase his participation rate to the retailer when the retailer integrates with manufacturer 1.

Further, substituting the optimal participation rate into (25), we see that the retailer's equilibrium local advertising level for product 2 is constant, that is,

$$U_{R2}^{(2)} = \begin{cases} \lambda\rho_{R2} - \chi\rho_1 & \text{if } \rho_{M2} < \dfrac{(\lambda\rho_{R2} - \chi\rho_1)}{2\lambda}, \\ \lambda\rho_{M2} + \dfrac{1}{2}\lambda\rho_{R2} - \dfrac{1}{2}\chi\rho_1 & \text{if } 0 < \dfrac{(\lambda\rho_{R2} - \chi\rho_1)}{2\lambda} \leq \rho_{M2}, \\ 0 & \text{else}, \end{cases} \tag{32}$$

where $\rho_1 = \rho_{M1} + \rho_{R1}$.

Subtracting (17) from (32), we have

$$\Delta U_{R2}=\begin{cases}-\chi\rho_{M1} & \text{if } 2\lambda\rho_{M2}+\chi\rho_{M1}<\lambda\rho_{R2}-\chi\rho_{R1}\\ \dfrac{2\lambda\rho_{M2}-\left(\lambda\rho_{R2}-\chi\rho_{R1}\right)-\chi\rho_{M1}}{2} & \text{if } 2\lambda\rho_{M2}<\lambda\rho_{R2}-\chi\rho_{R1}\leq 2\lambda\rho_{M2}+\chi\rho_{M1}\\ -\dfrac{\chi\rho_{M1}}{2} & \text{if } \chi\rho_{M1}<\lambda\rho_{R2}-\chi\rho_{R1}\leq 2\lambda\rho_{M2}\\ -\lambda\rho_{M2}-\dfrac{1}{2}\lambda\rho_{R2}+\dfrac{1}{2}\chi\rho_{R1} & \text{if } 0<\lambda\rho_{R2}-\chi\rho_{R1}\leq\chi\rho_{M1}\\ 0 & \text{else.}\end{cases} \tag{33}$$

Note that the result of (33) is less than zero; the intuition behind this can be explained as follows. When the retailer integrates with manufacturer 1, the retailer would always reduce the local advertising efforts for product 2 to decrease the competitive influence on product 1.

3.3. The Two Manufacturers Are Horizontally Integrated. When the two manufacturers integrated, it can be seen as a single firm with two different brands in the same product category. Examples in practice include Lenovo. IBM's personal computing division was acquired by Lenovo in 2004, and the PC of IBM became a subbrand of Lenovo Group named "Thinkpad." This is a historical precedent of two manufacturers behaving as a single player, yet, as far as we know, previous researches on dynamic cooperative advertising programs have never studied such scenario, a single manufacturer with two different brands. Most previous research investigated a "single-manufacturer single-retailer" supply chain with a single brand/product. When the manufacturer advertises two different brands, the result does change; therefore, the third scenario must be considered. In this scenario, the integration system's profit function is $\pi_{M1,M2} = \pi_{M1} + \pi_{M2}$, so the objective is

$$\max_{\substack{U_{M1}\geq 0, U_{M2}\geq 0\\ 1\geq\phi_i\geq 0}} J_{M1,M2} = \int_0^{+\infty} e^{-rt}\left(\pi_{M1}+\pi_{M2}\right)dt, \tag{34}$$

and the retailer's objective is

$$\max_{U_{R1}\geq 0, U_{R2}\geq 0} J_R = \int_0^{+\infty} e^{-rt}\pi_R dt. \tag{35}$$

The current value Hamiltonian of the integration system ($M1$ and $M2$) is

$$\begin{aligned}H_{M1,M2} &= \pi_{M1}+\pi_{M2}+\nu_{11}\left(U_{M1}-\theta U_{M2}-\delta G_1\right)\\ &\quad+\nu_{12}\left(U_{M2}-\theta U_{M1}-\delta G_2\right),\end{aligned} \tag{36}$$

and that of the retailer is

$$\begin{aligned}H_R &= \pi_R+\nu_{21}\left(U_{M1}-\theta U_{M2}-\delta G_1\right)\\ &\quad+\nu_{22}\left(U_{M2}-\theta U_{M1}-\delta G_2\right),\end{aligned} \tag{37}$$

where ν_{i1} and ν_{i2} $(i = 1, 2)$ are the costate variables to the firm's goodwill levels.

Using the necessary conditions for equilibrium, we get the following results.

Proposition 9. *When the two manufacturers are horizontally integrated and the participation rate ϕ_i $(i = 1, 2)$ is kept fixed, the equilibrium national advertising efforts for the two manufacturers are all constants, that is,*

$$U_{Mi}^{(3)}=\begin{cases}\dfrac{\alpha\left(\rho_{Mi}-\theta\rho_{M(3-i)}\right)}{r+\delta} & \textit{if } \rho_{Mi}\geq\theta\rho_{M(3-i)},\\ 0 & \textit{else}\end{cases}\quad i\in\{1,2\}, \tag{38}$$

and the retailer's equilibrium local advertising efforts for the two products are all constants, that is,

$$U_{Ri}^{(3)}=\begin{cases}\dfrac{\lambda\rho_{Ri}-\chi\rho_{R(3-i)}}{1-\phi_i} & \textit{if } \lambda\rho_{Ri}-\chi\rho_{R(3-i)}\geq 0,\\ 0 & \textit{else}\end{cases}\quad i\in\{1,2\}. \tag{39}$$

Note that the retailer's equilibrium local advertising efforts given by (39) are just the same as Proposition 2. This result implies that whether the two manufacturers integrate with each other or not, the retailer always keeps the same local advertising efforts for products 1 and 2 only if the participation rates are not changed.

In addition, comparing (38) with the results of Proposition 1, we have

$$\Delta U_{Mi}=\begin{cases}-\dfrac{\alpha\theta\rho_{M3-i}}{r+\delta} & \text{if } \rho_{Mi}\geq\theta\rho_{M,3-i},\\ -\dfrac{\alpha\rho_{M1}}{r+\delta} & \text{else}\end{cases}\quad i\in\{1,2\}. \tag{40}$$

Equation (40) illustrates the following fact. When the two manufacturers integrate as a horizontal alliance, they would decrease their equilibrium advertising efforts to avoid internal conflict. Specifically, combing the conditions of (38) we have $U_{Mi} = 0$ and $U_{M,(3-i)} = (\alpha\rho_{M3-i} - \alpha\theta\rho_{Mi})/(r + \delta)$ if the condition $\rho_{Mi} < \theta\rho_{M,3-i}$ holds. This implies that when the marginal profit of one product for the manufacturer is rather low, the horizontal integration system would invest in national advertising only for the other product.

Proposition 10. *When the two manufacturers are horizontally integrated and all channel members' advertising efforts are kept as constants, that is, $U_{Mi} = U_{Mi}^{(3)}$ and $U_{Ri} = U_{Ri}^{(3)}$, then the accumulated goodwill for the two products along time t is*

$$G_i(t) = F_i e^{-\delta t} + G_{iSS}^{(3)},\quad i\in\{1,2\}, \tag{41}$$

where $G_{iSS}^{(3)} = (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $F_i = G_{i0} - (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1, 2$. $G_{iSS}^{(3)}$ *is the steady-state goodwill for product* i *when* $t \to \infty$.

Substituting (38), (39), and (41) into (34) and (35) and with the participation rates ϕ_i $(i = 1, 2)$ fixed, we get that the current value of profit for the horizontal integration system is

$$J_{M1,M2}^{(3)} = \frac{F_1\alpha\rho_{M1} + F_2\alpha\rho_{M2}}{r+\delta} + \frac{\alpha\rho_{M1}\left(U_{M1}^{(3)} - \theta U_{M2}^{(3)}\right) + \alpha\rho_{M2}\left(U_{M2}^{(3)} - \theta U_{M1}^{(3)}\right)}{r\delta} + \frac{\rho_{M1}\left(\lambda U_{R1}^{(3)} - \chi U_{R2}^{(3)}\right)}{r} + \frac{\rho_{M2}\left(\lambda U_{R2}^{(3)} - \chi U_{R1}^{(3)}\right)}{r} - \frac{\left(U_{M1}^{(3)}\right)^2 + \left(U_{M2}^{(3)}\right)^2 + \phi_1\left(U_{R1}^{(3)}\right)^2 + \phi_2\left(U_{R2}^{(3)}\right)^2}{2r}, \quad (42)$$

and the current value of the profit for the retailer is

$$J_{R}^{(3)} = \frac{F_1\alpha\rho_{R1} + F_2\alpha\rho_{R2}}{r+\delta} + \frac{\alpha\rho_{R1}\left(U_{M1}^{(3)} - \theta U_{M2}^{(3)}\right) + \alpha\rho_{R2}\left(U_{M2}^{(3)} - \theta U_{M1}^{(3)}\right)}{r\delta} - \frac{1-\phi_1}{2}\left(U_{R1}^{(3)}\right)^2 + \frac{\rho_{R1}\left(\lambda U_{R1}^{(3)} - \chi U_{R2}^{(3)}\right)}{r} + \frac{\rho_{R2}\left(\lambda U_{R2}^{(3)} - \chi U_{R1}^{(3)}\right)}{r} - \frac{1-\phi_2}{2}\left(U_{R2}^{(3)}\right)^2, \quad (43)$$

where $F_i = G_{i0} - (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1, 2$.

Differentiating $J_{M1,M2}^{(3)}$ with the participation rate ϕ_1 and ϕ_2, we get the optimal participation rates:

$$\phi_i^{(3)} = \begin{cases} \dfrac{\lambda\left(2\rho_{Mi} - \rho_{Ri}\right) + \chi\rho_{R(3-i)}}{\lambda\left(2\rho_{Mi} + \rho_{Ri}\right) - \chi\rho_{R(3-i)}} & \text{if } \rho_{Mi} \geq \dfrac{\left(\lambda\rho_{Ri} - \chi\rho_{R(3-i)}\right)}{2\lambda} \\ 0 & \text{else,} \end{cases} \quad i \in \{1, 2\}. \quad (44)$$

Note that the above expressions of participation rates are identical with the results of Proposition 4. Together with the results of Proposition 9, we find that the equilibrium local advertising efforts for the two products are identical no matter whether two manufacturers are horizontally integrated or not.

In this scenario, the equilibrium advertising efforts for the two manufacturers become lower, but the equilibrium local advertising efforts for the two products are not changed. This could lead to the phenomenon that the retailer has so much power from advertising the two products that the retailer has incentive to prevent the horizontal alliance between the two manufacturers. That is why a successful manufacturer's horizontal integration in a dominant retailer market is very rare in actual practice.

4. Numerical Analysis

In this section, we use numerical analysis to further illustrate the impact of local advertising competition on the profits for all channel members and supplement insights from these theoretical results. In our numerical analysis, we use the following values to establish ranges for model parameters: $\alpha = 12$, $r = 0.3$, $\delta = 0.2$, $\theta = 0.2$, $\lambda = 10$, $\chi = 4$, $\rho_{M1} = 7$, $\rho_{M2} = 8$, $\rho_{R1} = 5$, $\rho_{R2} = 4$, $G_{10} = 300$, and $G_{20} = 320$.

To obtain qualitative insight regarding how the current value of each channel member's profit varies as competition coefficients θ and χ vary in scenario 1, we keep other parameters fixed and draw their relationships in Figure 2.

Figure 2 suggests that, in scenario 1, the profit for each channel member decreases as competition coefficients θ and χ increase. From Figure 2, if competition coefficients θ and χ equal zero, advertising for one product would not adversely influence the sales of the other product. In this situation, all channel members would obtain the maximum profits. As the competition effects of advertising become intense, the advertising effect on sales would scale down, and the profits for all channel members would decrease.

Figure 3 illustrates the effect of the competition coefficients θ and χ on the current value of the retailer's profit in scenario 1 and scenario 3, keeping other parameters fixed.

From Figure 3, we obtain the following facts. (i) When two manufacturers are horizontally integrated, the retailer's profit would decrease compared with his profit in scenario 1. Because the retailer would obtain a larger impact on the sales of products in scenario 3, the retailer would have incentive to prevent the horizontal alliance between manufacturers. (ii) Similarly to Figure 2, as competition coefficients θ and χ increase, the retailer's profit would decrease whether the two manufacturers integrate or not.

Finally, Figure 4 illustrates the impacts of the competition coefficients θ and χ on the current value of the profit of manufacturer 2. It suggests the following tendencies: (i) similarly to Figure 2, with competition coefficients θ and χ increasing, the profit for manufacturer 2 would decrease whether manufacturer 1 integrates with the retailer or not and (ii) when manufacturer 1 integrates with the retailer, the profit for manufacturer 2 would decrease compared to his profit in scenario 1. From Figures 3 and 4, we find that regardless of which two firms (i.e., $M1$ and retailer, $M2$ and retailer, or $M1$ and $M2$) integrate their efforts, the third firm would suffer.

5. Conclusion

Previous research primarily focused on a "single-manufacturer single-retailer" framework, whereas few studies address a "multiple-manufacturer single-retailer" framework. To fill this gap, this paper investigates the advertising strategies for a "two-manufacturer single-retailer" supply chain in

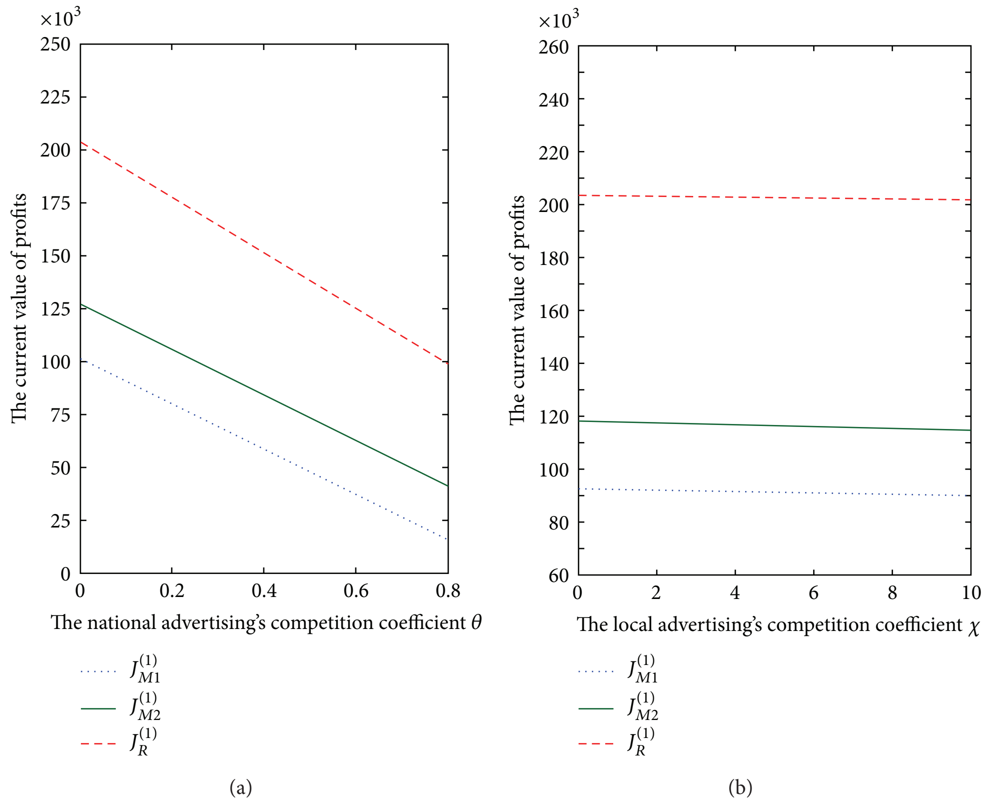

FIGURE 2: Relationships between profits and competition coefficients θ and χ.

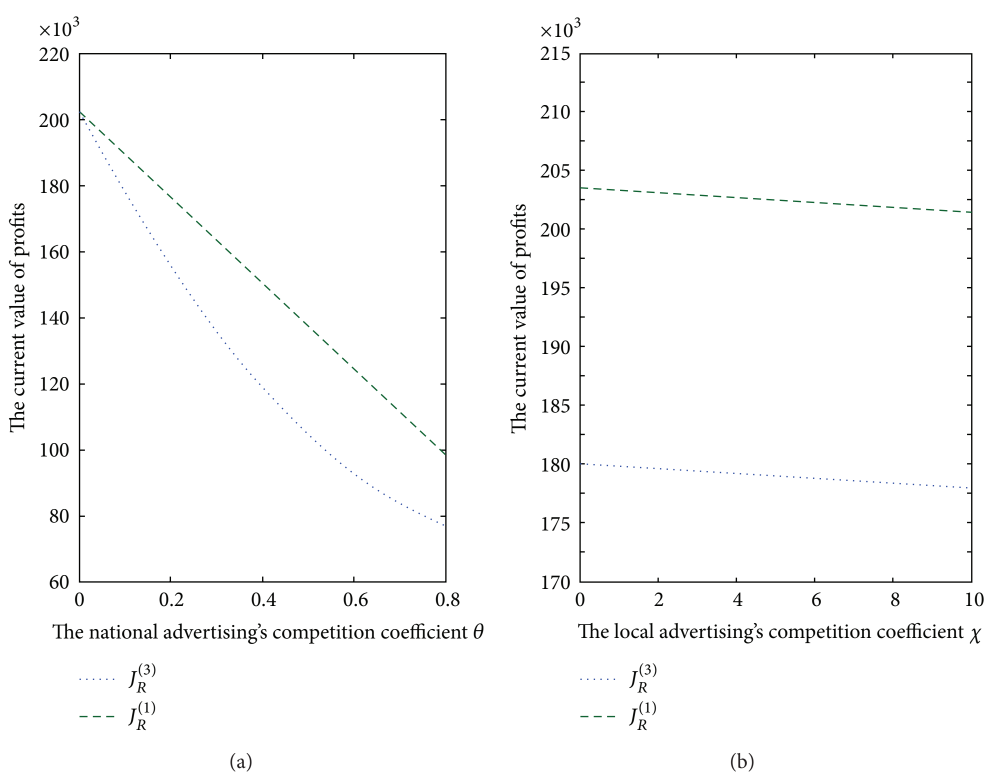

FIGURE 3: Relationships between the retailer's profit and the competition coefficients θ and χ.

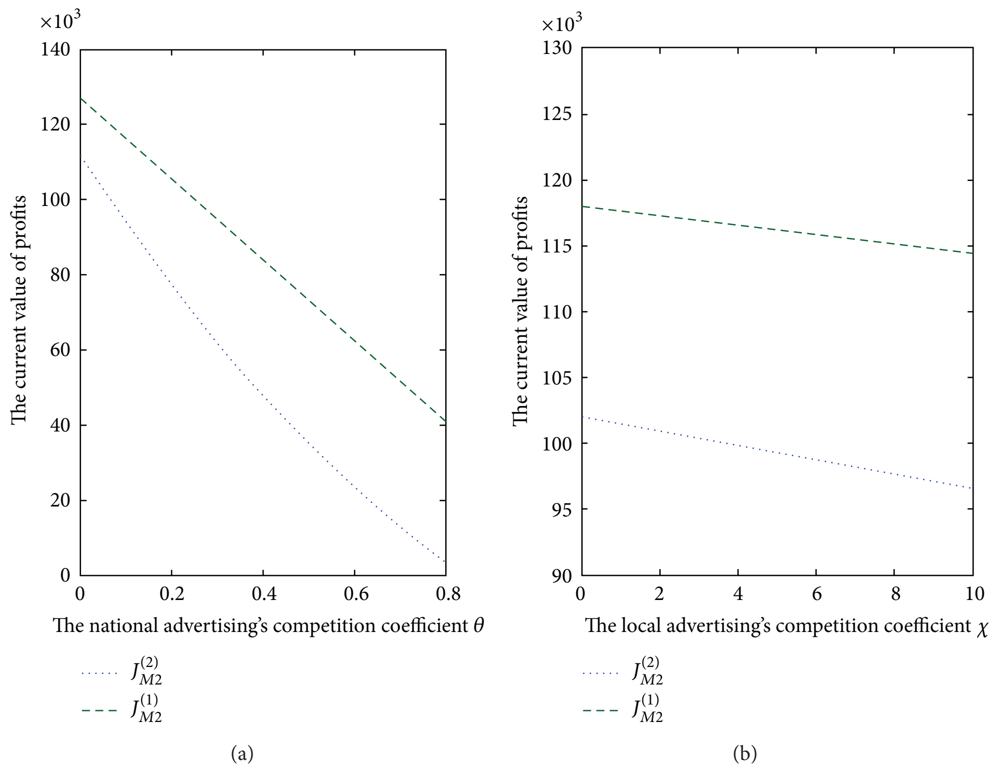

FIGURE 4: Relationships between the profit of $M2$ and the competition coefficients θ and χ.

three different scenarios: (i) each channel member makes decisions independently; (ii) the retailer integrates with one of the manufacturers; (iii) two manufacturers are horizontally integrated.

Based on the results of the three scenarios, we find the following results. (i) The manufacturer's equilibrium advertising efforts are independent of the participation rates that the two manufacturers offer to the retailer in all three scenarios. (ii) When the retailer integrates with one manufacturer, the other manufacturer's equilibrium advertising efforts would not be changed. The retailer would enhance the local advertising efforts for the integrated manufacturer and reduce the local advertising efforts for the other manufacturer. In response, the other manufacturer would offer a higher (compared to scenario 1) advertising cost participation rate to the retailer. (iii) When the two manufacturers are horizontally integrated, they would reduce the national advertising efforts to avoid internal conflict. They also offer the same advertising cost participation rate to the retailer as in scenario 1. (iv) If any two firms (i.e., $M1$ and retailer, $M2$ and retailer, or $M1$ and $M2$) are integrated, the profit of the third firm would decrease. All these insights provide important implications and guidelines for cooperative advertising program design in supply chain practice.

It should be noted that our models only consider the effects of advertising, but this situation may not always hold. In addition, it may be more interesting if we introduce the factors of pricing and quality to the cooperative advertising model. Additionally, our work on the "two-manufacturer single-retailer" framework can be extended into a "multiple-manufacturer single-retailer" framework.

Appendices

A. Each Channel Member Makes Decisions Independently

Proof of Propositions 1 and 3. When each channel member makes decisions independently, the current value Hamiltonian of manufacturer 1 is

$$\begin{aligned} H_{M1} = \pi_{M1} &+ \mu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &+ \mu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right). \end{aligned} \tag{A.1}$$

Then we form the Lagrangian:

$$\begin{aligned} L_{M1} = \pi_{M1} &+ \mu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ &+ \mu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right) \\ &+ \eta_{11}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ &+ \eta_{12}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right), \end{aligned} \tag{A.2}$$

the necessary conditions for equilibrium are given by

$$\frac{\partial L_{M1}}{\partial U_{M1}} = 0, \tag{A.3}$$

$$\dot{\mu}_{11} = r\mu_{11} - \frac{\partial L_{M1}}{\partial G_1}, \tag{A.4}$$

$$\dot{\mu}_{12} = r\mu_{12} - \frac{\partial L_{M1}}{\partial G_2}, \tag{A.5}$$

$$\frac{\partial L_{M1}}{\partial \eta_{1i}} > 0, \quad \eta_{1i} \geq 0, \quad \eta_{1i}\frac{\partial L_{M1}}{\partial \eta_{1i}} = 0, \quad i = 1, 2. \tag{A.6}$$

Equation (A.3) implies

$$U_{M1} = \mu_{11} - \theta\mu_{12}. \tag{A.7}$$

Solving (A.4)–(A.6), we get

$$\begin{aligned} \dot{\mu}_{11} &= (r + \delta)\,\mu_{11} - \alpha\rho_{M1} - \alpha\eta_1, \\ \dot{\mu}_{12} &= (r + \delta)\,\mu_{12} - \alpha\eta_2. \end{aligned} \tag{A.8}$$

Equation (A.6) implies: $\eta_{1i} = 0$, then substituting $\eta_{1i} = 0$ into (A.8), we get

$$\begin{aligned} \dot{\mu}_{11} &= (r + \delta)\,\mu_{11} - \alpha\rho_{M1}, \\ \dot{\mu}_{12} &= (r + \delta)\,\mu_{12}. \end{aligned} \tag{A.9}$$

Differentiating (A.7) with respect to time and substituting for μ_{11}, μ_{12} and their time derivative in (A.9), we get

$$\dot{U}_{M1} = (r + \delta)\,U_{M1} - \alpha\rho_{M1}. \tag{A.10}$$

Solving (A.10) to get the time paths of U_{M1}, we find

$$U_{M1}(t) = C_1 e^{(r+\delta)t} + \frac{\alpha\rho_{M1}}{(r + \delta)}. \tag{A.11}$$

Because there is no constraint at $t \to \infty$, U_{M1} should satisfy the free-boundary condition:

$$\lim_{t\to\infty} U_{M1}(t) < \infty. \tag{A.12}$$

Condition (A.12) implies that $C_1 = 0$. Therefore, we obtain the equilibrium advertising effort for manufacturer 1 as follows:

$$U_{M1}^{(1)} = \frac{\alpha\rho_{M1}}{r + \delta}. \tag{A.13}$$

Similarly considering manufacturer 2's profit maximizing problem, we obtain the equilibrium advertising level for manufacturer 1 as follows:

$$U_{M2}^{(1)} = \frac{\alpha\rho_{M2}}{r + \delta}. \tag{A.14}$$

For (1), we can get the general solutions of (1) as

$$G_i(t) = D_i e^{-\delta t} + G_{iSS}, \quad i \in \{1, 2\}, \tag{A.15}$$

where $G_{iSS} = (U_{Mi} - \theta U_{M,(3-i)})/\delta$, $i = 1, 2$.

D_i is an arbitrary constant. Letting $t = 0$ in (A.15) and utilizing the initial conditions of (2), we get $D_i = G_{i0} - (U_{Mi} - \theta U_{M,(3-i)})/\delta$, $i = 1, 2$.

Substituting (A.13) and (A.14) into (A.15), we find that

$$G_i(t) = D_i e^{-\delta t} + G_{iSS}^{(1)}, \quad i \in \{1, 2\}, \tag{A.16}$$

where $D_i = G_{i0} - (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$, $i = 1, 2$ and $G_{iSS}^{(1)} = (U_{Mi}^{(1)} - \theta U_{M,(3-i)}^{(1)})/\delta$, $i = 1, 2$, when condition $0 \leq \theta \leq \min\{\rho_{M1}/\rho_{M2}, \rho_{M2}/\rho_{M1}\}$ holds, the steady-state goodwill G_{iSS} is nonnegative. □

Proof of Proposition 2. When each channel member makes decisions independently, the current value Hamiltonian of the retailer is:

$$\begin{aligned} H_R = {} & \rho_{R1}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \rho_{R2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) \\ & - \frac{1}{2}(1 - \phi_1)\,U_{R1}^2 - \frac{1}{2}(1 - \phi_2)\,U_{R2}^2 \\ & + \mu_{31}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \mu_{32}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \tag{A.17}$$

Then we form the Lagrangian

$$\begin{aligned} L_R = {} & \rho_{R1}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \rho_{R2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) \\ & - \frac{1}{2}(1 - \phi_1)\,U_{R1}^2 - \frac{1}{2}(1 - \phi_2)\,U_{R2}^2 \\ & + \mu_{31}(U_{M1} - \theta U_{M2} - \delta G_1) \\ & + \mu_{32}(U_{M2} - \theta U_{M1} - \delta G_2) \\ & + \eta_{31}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ & + \eta_{32}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}). \end{aligned} \tag{A.18}$$

The necessary conditions for equilibrium are given by

$$\frac{\partial L_R}{\partial U_{R1}} = 0, \tag{A.19}$$

$$\frac{\partial L_R}{\partial U_{R2}} = 0, \tag{A.20}$$

$$\dot{\mu}_{31} = r\mu_{31} - \frac{\partial L_R}{\partial G_1}, \tag{A.21}$$

$$\dot{\mu}_{32} = r\mu_{32} - \frac{\partial L_R}{\partial G_2}, \tag{A.22}$$

$$\frac{\partial L_R}{\partial \eta_{3i}} > 0, \quad \eta_{3i} \geq 0, \quad \eta_{3i}\frac{\partial L_R}{\partial \eta_{3i}} = 0, \quad i = 1, 2. \tag{A.23}$$

Because U_{Ri} is constrained to be nonnegative, (A.19) implies that

$$U_{R1}(t) = \max\left\{0, \frac{(\lambda\rho_{R1} - \chi\rho_{R2})}{(1 - \phi_1)}\right\}. \tag{A.24}$$

Through a similar proof, we find that

$$U_{R2}(t) = \max\left\{0, \frac{(\lambda\rho_{R2} - \chi\rho_{R1})}{(1-\phi_2)}\right\}. \quad (A.25)$$

Therefore we can obtain the following results:

$$U_{Ri}^{(1)} = \begin{cases} \dfrac{\lambda\rho_{Ri} - \chi\rho_{R(3-i)}}{1-\phi_i} & \text{if } \lambda\rho_{Ri} - \chi\rho_{R(3-i)} \geq 0, \\ 0 & \text{else,} \end{cases} \quad (A.26)$$

$$i \in \{1, 2\}.$$

□

Proof of Proposition 4. There are no relationships between participation rate ϕ_1 and $U_{R2}^{(1)}$ or the two manufacturer's national advertising efforts. Thus in differentiating $J_{M1}^{(1)}$ from the participation rate ϕ_1 we only consider two situations. Situation (1) when $\lambda\rho_{R1} - \chi\rho_{R2} \geq 0$ holds, $U_{R1}^{(1)} = (\lambda\rho_{R1} - \chi\rho_{R2})/(1-\phi_1)$, substituting the above expression into (20) and differentiating J_{M1} with the participation rate ϕ_1, we get manufacturer 1's optimal participation rate: $\phi_1 = [\lambda(2\rho_{M1} - \rho_{R1}) + \chi\rho_{R2}]/[\lambda(2\rho_{M1} + \rho_{R1}) - \chi\rho_{R2}]$. Since $0 \leq \phi_1 \leq 1$, the condition $2\lambda\rho_{M1} \geq (\lambda\rho_{R1} - \chi\rho_{R2})$ is required. Situation (2) if condition $(\lambda\rho_{R1} - \chi\rho_{R2}) < 0$ holds, we get $U_{R1}^{(1)} = 0$. In this situation, whatever participation rate manufacturer 1 offers, the retailer would never advertise product 1. The participation rate ϕ_1 is an arbitrary constant. We suppose $\phi_1 = [\lambda(2\rho_{M1} - \rho_{R1}) + \chi\rho_{R2}]/[\lambda(2\rho_{M1} + \rho_{R1}) - \chi\rho_{R2}]$. The participation rate ϕ_1 is useless in this situation; therefore, the participation rate could be negative.

In conclusion, we get the following results:

$$\phi_1^{(1)} = \begin{cases} \dfrac{\lambda(2\rho_{M1} - \rho_{R1}) + \chi\rho_{R2}}{\lambda(2\rho_{M1} + \rho_{R1}) - \chi\rho_{R2}} & \text{if } 2\lambda\rho_{M1} \geq \lambda\rho_{R1} - \chi\rho_{R2} \\ 0 & \text{else.} \end{cases} \quad (A.27)$$

Through a similar proof, we get manufacturer 2's optimal share rate is

$$\phi_2^{(1)} = \begin{cases} \dfrac{\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_{R1}}{\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_{R1}} & \text{if } 2\lambda\rho_{M2} \geq \lambda\rho_{R2} - \chi\rho_{R1} \\ 0 & \text{else.} \end{cases} \quad (A.28)$$

□

B. The Retailer Is Vertically Integrated with One Manufacturer

Proof of Proposition 5. When the retailer integrates with a manufacturer, the current value Hamiltonian of manufacturer 2 is

$$\begin{aligned} H_{M2} &= \rho_{M2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) \\ &\quad - \frac{1}{2}U_{M2}^2 - \frac{1}{2}\phi_2 U_{R2}^2 \\ &\quad + \gamma_{21}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \gamma_{22}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \quad (B.1)$$

Then we form the Lagrangian:

$$\begin{aligned} L_{M2} &= \rho_{M2}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}) - \frac{1}{2}U_{M2}^2 - \frac{1}{2}\phi_2 U_{R2}^2 \\ &\quad + \gamma_{21}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \gamma_{22}(U_{M2} - \theta U_{M1} - \delta G_2) \\ &\quad + \xi_{21}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ &\quad + \xi_{22}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}). \end{aligned} \quad (B.2)$$

At optimality, the necessary conditions are

$$\begin{aligned} &\frac{\partial L_{M2}}{\partial U_{M2}} = 0, \\ &\dot{\gamma}_{21} = r\gamma_{21} - \frac{\partial L_{M1}}{\partial G_1}, \\ &\dot{\gamma}_{22} = r\gamma_{22} - \frac{\partial L_{M1}}{\partial G_2}, \\ &\frac{\partial L_{M2}}{\partial \xi_{2i}} > 0, \quad \xi_{2i} \geq 0, \quad \xi_{2i}\frac{\partial L_{M2}}{\partial \xi_{2i}} = 0, \quad i = 1, 2. \end{aligned} \quad (B.3)$$

Proceeding as in the proof for Proposition 1, we get

$$U_{M2}^{(2)} = \frac{\alpha\rho_{M2}}{(r+\delta)}. \quad (B.4)$$

□

Proof of Propositions 6 and 7. When the retailer integrates with a manufacturer, the current value Hamiltonian for integration system is given by

$$\begin{aligned} H_{M1,R} &= \pi_{M1} + \pi_R + \gamma_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \gamma_{12}(U_{M2} - \theta U_{M1} - \delta G_2). \end{aligned} \quad (B.5)$$

Then we form the Lagrangian:

$$\begin{aligned} L_{M1,R} &= \pi_{M1} + \pi_R + \gamma_{11}(U_{M1} - \theta U_{M2} - \delta G_1) \\ &\quad + \gamma_{12}(U_{M2} - \theta U_{M1} - \delta G_2) \\ &\quad + \xi_{11}(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}) \\ &\quad + \xi_{12}(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}). \end{aligned} \quad (B.6)$$

Proceeding as in the proof for Proposition 1, and constraining,as in most cases, the advertising efforts $U(t)$ to be nonnegative, we get the following results:

$$U_{R1}^{(2)} = \max\left\{0, \lambda\left(\rho_{M1} + \rho_{R1}\right) - \chi\rho_{R2}\right\},$$
$$U_{R2}^{(2)} = \max\left\{0, \frac{\lambda\rho_{R2} - \chi\left(\rho_{R1} + \rho_{M1}\right)}{1 - \phi_2}\right\}. \tag{B.7}$$

Thus, the equilibrium advertising levels for manufacturer 2 are as follows:

$$U_{M1}^{(2)} = \begin{cases} \dfrac{\alpha\left(\rho_{M1} + \rho_{R1}\right)}{r + \delta} - \dfrac{\theta\alpha\rho_{R2}}{r + \delta} & \text{if } \left(\rho_{M1} + \rho_{R1}\right) \geq \theta\rho_{R2} \\ 0 & \text{else.} \end{cases} \tag{B.8}$$

Also, we obtain the equilibrium local advertising levels for the two products:

$$U_{R1}^{(2)} = \begin{cases} \lambda\left(\rho_{M1} + \rho_{R1}\right) - \chi\rho_{R2} & \text{if } \lambda\rho_1 - \chi\rho_{R2} \geq 0 \\ 0 & \text{else,} \end{cases}$$
$$U_{R2}^{(2)} = \begin{cases} \dfrac{\lambda\rho_{R2} - \chi\left(\rho_{R1} + \rho_{M1}\right)}{1 - \phi_2} & \text{if } \lambda\rho_{R2} - \chi\rho_1 \geq 0 \\ 0 & \text{else.} \end{cases} \tag{B.9}$$

Substituting (B.4) and (B.8) into (A.15), we find that

$$G_i\left(t\right) = E_i e^{-\delta t} + G_{iSS}^{(2)}, \quad i \in \{1, 2\}, \tag{B.10}$$

where $E_i = G_{i0} - (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1, 2$ and $G_{iSS}^{(2)} = (U_{Mi}^{(2)} - \theta U_{M,(3-i)}^{(2)})/\delta$, $i = 1, 2$.

When condition $0 \leq \theta \leq \min\{\rho_1/\rho_2, (\rho_1 - \sqrt{\rho_1^2 - 4\rho_{M2}\rho_{R2}})/2\rho_{R2}\}$ holds, the steady-state goodwill $G_{iSS}^{(2)}$ is nonnegative. □

Proof of Proposition 8. There are no relationships between participation rate ϕ_2 and $U_{R2}^{(2)}$ or two manufacturer's national advertising efforts. Thus in differentiating $J_{M2}^{(2)}$ from the participation rate ϕ_2 we only consider two situations. (1) When $\lambda\rho_{R2} - \chi\rho_1 \geq 0$ holds, we get $U_{R2}^{(2)} = [\lambda\rho_{R2} - \chi(\rho_{R1} + \rho_{M1})]/(1 - \phi_2)$. Substituting the above expression into (38), and differentiating J_{M2} from the participation rate ϕ_2, we get manufacturer 2's optimal participation rate. $\phi_2 = [\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1]/[\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_1]$. Since $0 \leq \phi_2 \leq 1$, the assumption $\rho_{M2} \geq (\lambda\rho_{R2} - \chi\rho_1)/2\lambda$ is required. (2) When condition $(\lambda\rho_{R2} - \chi\rho_{R1}) < 0$ holds, we get $U_{R2}^{(2)} = 0$. In this situation, whatever participation rate manufacturer 2 offers, the retailer would never advertise product 2. Thus, ϕ_2 is an arbitrary constant. Here we suppose: $\phi_2 = [\lambda(2\rho_{M2} - \rho_{R2}) + \chi\rho_1]/[\lambda(2\rho_{M2} + \rho_{R2}) - \chi\rho_1]$. The participation rate ϕ_1 is useless in this situation; therefore the participation rate could be negative.

In conclusion, we get the following results:

$$\phi_2^{(2)} = \begin{cases} \dfrac{\lambda\left(2\rho_{M2} - \rho_{R2}\right) + \chi\rho_1}{\lambda\left(2\rho_{M2} + \rho_{R2}\right) - \chi\rho_1} & \text{if } \rho_{M2} \geq \dfrac{\left(\lambda\rho_{R2} - \chi\rho_1\right)}{2\lambda} \\ 0 & \text{else.} \end{cases} \tag{B.11}$$

□

C. Two Manufacturers Are Horizontally Integrated

Proof of Proposition 9. When the two manufacturers are horizontally integrated, the current value Hamiltonian for the integration system is given by

$$\begin{aligned} H_{M1,M2} = {} & \rho_{M1}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \rho_{M2}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right) \\ & - \frac{1}{2}U_{M1}^2 - \frac{1}{2}\left(1 - \phi_1\right)U_{R1}^2 \\ & - \frac{1}{2}U_{M2}^2 - \frac{1}{2}\left(1 - \phi_2\right)U_{R2}^2 \\ & + \nu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ & + \nu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right). \end{aligned} \tag{C.1}$$

Then we form the Lagrangian

$$\begin{aligned} L_{M1,M2} = {} & \rho_{M1}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \rho_{M2}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right) \\ & - \frac{1}{2}U_{M1}^2 - \frac{1}{2}\left(1 - \phi_1\right)U_{R1}^2 \\ & - \frac{1}{2}U_{M2}^2 - \frac{1}{2}\left(1 - \phi_2\right)U_{R2}^2 \\ & + \nu_{11}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ & + \nu_{12}\left(U_{M2} - \theta U_{M1} - \delta G_2\right) \\ & + \vartheta_{11}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \vartheta_{12}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right). \end{aligned} \tag{C.2}$$

At optimality, the necessary conditions are

$$\frac{\partial L_{M1,M2}}{\partial U_{M1}} = 0, \tag{C.3}$$

$$\dot{\nu}_{11} = r\nu_{11} - \frac{\partial L_{M1,M2}}{\partial G_1}, \tag{C.4}$$

$$\dot{\nu}_{12} = r\nu_{12} - \frac{\partial L_{M1,M2}}{\partial G_2}, \tag{C.5}$$

$$\frac{\partial L_{M1,M2}}{\partial \vartheta_{1i}} > 0, \quad \vartheta_{1i} \geq 0, \quad \vartheta_{1i}\frac{\partial L_{M1,M2}}{\partial \vartheta_{1i}} = 0, \quad i = 1, 2. \tag{C.6}$$

In most case the advertising effort U_{M1} is constrained to be nonnegative. Thus, (C.3) implies

$$U_{M1} = \max\left\{0, \beta\rho_{M1} + \nu_{11} - \theta\nu_{12}\right\}. \tag{C.7}$$

Solving (C.4)-(C.5), we get

$$\begin{aligned} \dot{\nu}_{11} &= \left(r + \delta\right)\nu_{11} - \alpha\rho_{M1} - \alpha\vartheta_{11}, \\ \dot{\nu}_{12} &= \left(r + \delta\right)\nu_{12} - \alpha\rho_{M2} - \alpha\vartheta_{12}. \end{aligned} \tag{C.8}$$

Equation (C.6) implies, $\vartheta_{1i} = 0$; then substituting $\vartheta_{1i} = 0$ into (C.8), we get

$$\dot{\nu}_{11} = (r+\delta)\,\nu_{11} - \alpha\rho_{M1}, \tag{C.9}$$

$$\dot{\nu}_{12} = (r+\delta)\,\nu_{12} - \alpha\rho_{M2}. \tag{C.10}$$

Differentiating (C.7) with respect to time and substituting for ν_{11}, ν_{12} and their time derivative in (C.9)-(C.10), we get

$$\dot{U}_{M1} = (r+\delta)\,U_{M1} - \alpha\rho_{M1} + \theta\alpha\rho_{M2}. \tag{C.11}$$

Proceeding as in the proof of Proposition 1, we get

$$\begin{aligned} U_{M1}^{(3)} &= \max\left\{0, \frac{\alpha\rho_{M1}}{(r+\delta)} - \frac{\theta\alpha\rho_{M2}}{(r+\delta)}\right\}, \\ U_{M2}^{(3)} &= \max\left\{0, \frac{\alpha\rho_{M2}}{(r+\delta)} - \frac{\theta\alpha\rho_{M1}}{(r+\delta)}\right\}. \end{aligned} \tag{C.12}$$

Thus, the equilibrium advertising levels for two manufacturers are as follows:

$$U_{Mi}^{(3)} = \begin{cases} \dfrac{\alpha\left(\rho_{Mi} - \theta\rho_{M(3-i)}\right)}{r+\delta} & \text{if } \rho_{Mi} \geq \theta\rho_{M,3-i}, \\ 0 & \text{else} \end{cases} \qquad i \in \{1,2\}. \tag{C.13}$$

Substituting (C.13) into (A.15), we find that

$$G_i(t) = F_i e^{-\delta t} + G_{iSS}^{(3)}, \quad i \in \{1,2\}, \tag{C.14}$$

where $F_i = G_{i0} - (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1,2$ and $G_{iSS}^{(3)} = (U_{Mi}^{(3)} - \theta U_{M,(3-i)}^{(3)})/\delta$, $i = 1,2$. When condition $0 \leq 2\theta/(1+\theta^2) \leq \min\{\rho_{M1}/\rho_{M2}, \rho_{M2}/\rho_{M1}\}$ holds, the steady-state goodwill $G_{iSS}^{(3)}$ is nonnegative.

The current value Hamiltonian for the retailer is given by

$$\begin{aligned} H_R = {} & \rho_{R1}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \rho_{R2}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right) - \frac{1}{2}\left(1-\phi_1\right)U_{R1}^2 \\ & - \frac{1}{2}\left(1-\phi_2\right)U_{R2}^2 + \nu_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ & + \nu_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right). \end{aligned} \tag{C.15}$$

Then we form the Lagrangian:

$$\begin{aligned} L_R = {} & \rho_{R1}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \rho_{R2}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right) \\ & - \frac{1}{2}\left(1-\phi_1\right)U_{R1}^2 - \frac{1}{2}\left(1-\phi_2\right)U_{R2}^2 \\ & + \nu_{21}\left(U_{M1} - \theta U_{M2} - \delta G_1\right) \\ & + \nu_{22}\left(U_{M2} - \theta U_{M1} - \delta G_2\right) \\ & + \vartheta_{21}\left(\alpha G_1 + \lambda U_{R1} - \chi U_{R2}\right) \\ & + \vartheta_{22}\left(\alpha G_2 + \lambda U_{R2} - \chi U_{R1}\right). \end{aligned} \tag{C.16}$$

Proceeding as in the proof of Proposition 2, we get

$$\begin{aligned} U_{R1}^{(3)} &= \max\left\{0, \frac{\left(\lambda\rho_{R1} - \chi\rho_{R2}\right)}{\left(1-\phi_1\right)}\right\}, \\ U_{R2}^{(3)} &= \max\left\{0, \frac{\left(\lambda\rho_{R2} - \chi\rho_{R1}\right)}{\left(1-\phi_2\right)}\right\}. \end{aligned} \tag{C.17}$$

Then we get the following results:

$$U_{Ri}^{(3)} = \begin{cases} \dfrac{\lambda\rho_{Ri} - \chi\rho_{R(3-i)}}{1-\phi_i} & \text{if } \lambda\rho_{Ri} - \chi\rho_{R(3-i)} \geq 0 \\ 0 & \text{else,} \end{cases} \qquad i \in \{1,2\}. \tag{C.18}$$

□

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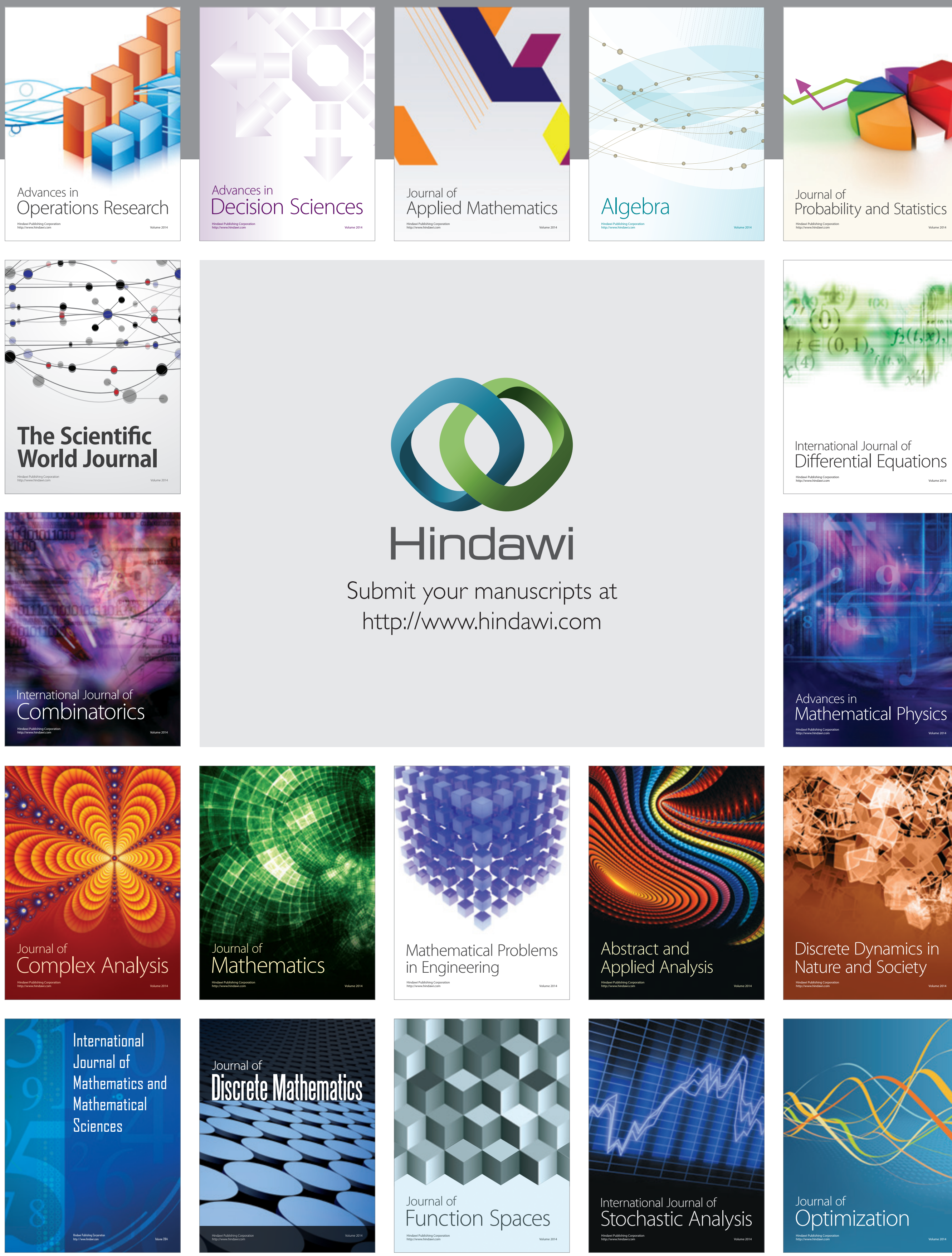

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