# Synthetic Stellar Populations Guy Worthey 

## From

Encyclopedia of Astronomy \& Astrophysics
P. Murdin
© IOP Publishing Ltd 2006

ISBN: 0333750888

## Synthetic Stellar Populations

Modern telescopes are built with ever-better angular resolution, but for galaxies and clusters outside the local group we generally cannot hope to study the stars in them one by one. Instead, we see the sum of the light from all of the stars. To take this integrated starlight and extract information on the ages or heavy-element abundances requires a model; a synthetic version of how many and what kind of stars are present. To guide the models, we rely on nearby stars and star clusters where stars can be studied one by one and also on theoretical models of stellar evolution and stellar atmospheres.

A stellar population is a group of stars born at the same time and sharing the same initial composition. A STAR CLUSTER is an example of a simple stellar population, characterized by a single age and a single abundance mixture. The miLKY way is an example of a complicated mixture of many stellar populations; old and young, metal poor and metal rich.

In the Milky Way, there is a striking concordance of age-metallicity with spatial extent and kinematics. The galactic metal-poor halo is thought to have formed in the first third of the Galaxy's history and is composed of stars and clusters on randomly oriented orbits so that the net effect is a large halo of roughly spherical shape. On the other hand, if one finds all the stars in the Galaxy younger than a billion years, virtually all of them are in the very thin Galactic disk. They have orderly, near-circular orbits with all stars rotating clockwise as seen from above the Galactic north pole (see Galactic thin disk). This dichotomy gave rise to the notion of two distinct stellar populations in the Milky Way around 1944 (W Baade is credited with this idea). By the 1960s it became evident that there were several, rather than two, distinct populations. For instance, old but metal-rich stars exist both in the disk and in the bulge. Today, we have given up counting the number of stellar populations, and instead speak of the age and abundance distributions within kinematic subgroups.

From distant galaxies and clusters, what information can one hope to derive from integrated star light? Ultimately, one could hope to find the complete star formation history of the galaxy, and at each past epoch of star formation the population's dynamic state, a distribution of heavy elements, and an initial mass function (abbreviated IMF; the number of stars born at each mass). Such a detailed picture is out of reach for the present. Today, we have clues about each of these quantities, but we are far from being able to wrestle such a detailed picture from integrated light observations.

## Several types of synthetic stellar population models

Figure 1 shows galaxy spectra covering the range of spectral shapes that are seen along with a sampling of some of the stellar types out of which the galaxies are composed. From the figure one can glimpse similarities between the galaxies and the underlying stellar types.


Figure 1. Example galaxy (top) and star (bottom) spectra are shown, arbitrarily scaled $F_{v}$ versus wavelength in $\AA$. The strongest emission lines in the starburst spectrum were truncated for clarity, and a few important spectral features are marked on the stellar spectra. This is the synthesis problem: adding the star spectra together in different combinations and with different multiplicative weights, how closely can the galaxy spectrum be matched? By eye, one can see that the starburst galaxy must have a strong $O$ star component, and the spiral and elliptical galaxies must have many G stars. However, TiO features appear in the spiral and elliptical spectra, so KM stars must also be present.

Extreme starburst galaxies emit most of their light in the ultraviolet. Their spectra are matched by OB-type stars with strongly muted evidence of other spectral types. SPiral and elliptical galaxies have minority populations of hot stars, as seen by their small ultraviolet fluxes, and obviously have a strong G-star component. However, the strong, broad TiO features characteristic of M-type stars are seen in the red portion of the spectrum, so very cool stars are present as well.

The first type of model is this kind of star-by-star
addition problem. Early workers such as Spinrad and Taylor would observe, spectrophotometrically, a collection of stars and a few galaxies. Then, by method of trial and error, they would construct a 'model' consisting of the relative proportions of the stars that best matched the galaxy spectrum. That is, given a galaxy spectrum $L_{\lambda}$ and a collection of stellar spectra $l_{i, \lambda}$, the model consists of the weights $w_{i}$ by which the stars are multiplied:

$$
\begin{equation*}
L_{\lambda}=\sum_{i} w_{i} l_{i, \lambda} . \tag{1}
\end{equation*}
$$

The trial-and-error part comes in because the same set of weights must work for all $\lambda$. A useful improvement came with the work of Faber in the early 1970s. Through a careful statement of the problem and judicious use of technique (quadratic programming or linear programming) the trial-and-error portion of assigning weights was replaced by numerical optimization.

This form of population modeling is often termed empirical population synthesis. Empirical population synthesis has a couple of drawbacks. First, solutions are not unique. For instance, the spectrum of a G dwarf closely resembles that of a G giant, and replacement of one by the other probably will not noticeably affect the goodness of fit. Second, information on galaxy age or metal abundance requires interpretation. Suppose the synthesis says that G0 dwarfs are numerous, but F8 dwarfs are rare. The interpretation would be that the main sequence turnoff is at spectral type G0. One would then refer to observed star clusters or stellar evolutionary isochrones to find out the age at which the main sequence turnoff is spectral type G0.

The connection to age and metal abundance can be made more direct by considering, not stars, but star clusters as the basic building block. The cluster will already have the 'correct' proportions of the different kinds of stars for the cluster's age and abundance, and finding the best match between target (galaxy) and template cluster should, in principle, yield a mean age and metal abundance for the galaxy. In practice, two problems arise. First, in the Milky Way and vicinity, open clusters are so diffuse that obtaining an integrated spectrum is observationally impossible. One is then left with the globular clusters and the clusters in the LMC and SMC for possible spectral templates. E Bica has made a study of such cluster spectra. While these clusters span a considerable range of age and abundance, not all possibilities are covered. In particular, the high metallicities typical of elliptical galaxies cannot be matched by local template clusters. The second problem is age-metallicity degeneracy, which is the observation that changes in age and metal abundance have very similar, almost indistinguishable, effects on the integrated spectrum, so that an old, metal-poor population may very closely resemble a young, metal-rich population. This topic is revisited below.

The final variant of model was developed by B M Tinsley starting in the late 1960s and today is the


Figure 2. Isochrone age effects are shown. The isochrones plotted are for solar composition, ages 5 million and 15 billion years. Approximate spectral type is indicated on the top axis, and $\log T_{\text {eff }}$ (in kelvins) on the bottom. The vertical scale is the logarithm of the bolometric (total) luminosity in units of the solar luminosity. The masses of main sequence stars are marked. The 'main sequence turnoff' is the point where the isochrone drifts brighter than the zero-age main sequence. Stars in this phase often dominate the integrated light and are also crucial for age estimation. Post-main-sequence evolution happens on much quicker timescales than main sequence lifetimes. Massive stars undergo 'blue loops' in response to internal structural changes, and may also spend some time as M-type supergiants before they explode in supernovae explosions. Low-mass stars have entirely different post-main-sequence evolution. They evolve up a 'red giant branch' as they fuse hydrogen in a shell around the helium core. At $\log L \approx 3.4$, the helium reaches ignition temperature and the star immediately adjusts its structure to land in the 'clump'. In a metal-poor population, the clump may spread over a large range of temperature, and in this case is called the 'horizontal branch'. After the clump, the star climbs the asymptotic giant branch (AGB) before shedding its outer layers, undergoing a planetary nebula phase, and finally becoming a white dwarf.
most widely used variant. It is usually referred to as evolutionary population synthesis or isochrone synthesis because it uses theoretical stellar evolutionary isochrones as its basic unit rather than observed clusters. A stellar evolutionary isochrone (isochrone for short) is made from a collection of evolutionary tracks for different masses of stars. A track describes the luminosity and effective temperature of a star of a given mass with time. An isochrone shows the locus of luminosities and temperatures at one instant in time for stars of all masses, and thus is built to mimic a star cluster or a single-age stellar population.

Sample isochrones together with some terminology

[^0]

Figure 3. The four panels in this figure give a visual impression of which phases of evolution dominate the integrated light at different wavelengths. Isochrones of 15 billion year age are shown for $[\mathrm{Fe} / \mathrm{H}]=-0.5,0.0$ and 0.5 dex. The effect of changing the abundance is to shift the isochrones in temperature. The $[\mathrm{Fe} / \mathrm{H}]=0.0$ isochrone has a 'horizontal branch', an 'asymptotic giant branch' and a 'post-asymptotic giant branch' (marked PAGB) showing evolution in and around the planetary nebula phase for two postulated white dwarf masses. For clarity, these late stages are not illustrated for the other two isochrones. The horizontal branch is included for illustration purposes only; the usual morphology of this phase for a metal-rich population is a clump. Each bin along the isochrone is represented by a circle, which changes size according to how much energy is spent at that location. Therefore, most of the light comes from where the largest circles are located. The four panels show bolometric (all-wavelength) light in which the light contributions are split between main sequence turnoff and giant branch, the ultraviolet U filter in which only the warmer stars contribute, the visual V filter which resembles the bolometric panel and the infrared $K$ filter in which the brightest giants, all but invisible in the other panels, completely dominate the light.
are shown in figure 2. There are two essential points about isochrones. First, more massive stars have much shorter main sequence lifetimes, so the main sequence turnoff point will move steadily dimmer and cooler with increasing age. Second, the post-main-sequence phases of evolution are much shorter than the main sequence lifetimes. In most situations stars near the main sequence turnoff region will dominate the integrated light. One case where this is not true is for an old population where one is restricted to looking in the red or infrared. In that case, the cool red giant stars dominate the light. The isochrones in figure 2 are of solar abundance, but model builders also consider other abundances. We use the standard bracket notation $[\mathrm{Fe} / \mathrm{H}]$ or $[\mathrm{M} / \mathrm{H}]$ to mean the logarithmic heavy-
element abundance in units of the solar heavy-element abundance:

$$
\begin{equation*}
[\mathrm{Fe} / \mathrm{H}]=\log [n(\mathrm{Fe}) / n(\mathrm{H})]_{\text {stars }}-\log [n(\mathrm{Fe}) / n(\mathrm{H})]_{\odot} . \tag{2}
\end{equation*}
$$

In addition to the isochrone, a collection of stellar spectra is required. These spectra can be empirical, theoretical or any combination. The spectra may even consist solely of broad-band colors, if that is the only information desired of the model at the end. The final model spectrum is a weighted sum of the stellar spectra mathematically identical to equation (1):

$$
\begin{equation*}
L_{\lambda}=\sum_{i} N_{i}(M, \delta M) l_{\lambda}\left(T_{\mathrm{eff}}, g, Z\right) \tag{3}
\end{equation*}
$$

[^1]where $L_{\lambda}$ is the model spectrum, $i$ is the index over points along the isochrone, $N_{i}(M, \delta M)$ is the number of stars associated with each point along the isochrone, and $l_{\lambda}\left(T_{\text {eff }}, g, Z\right)$ is the stellar spectrum, which is typically interpolated from a grid of star fluxes indexed by effective temperature, surface gravity and abundance. The number of stars comes from the IMF, which gives the number of stars per interval of mass. A simple power-law IMF is still in common use today, for example
\[

$$
\begin{equation*}
N_{i}(M, \delta M)=C M^{-x} \delta M \tag{4}
\end{equation*}
$$

\]

where $C$ is a constant and $x=2.35$ is the 1955 Salpeter estimate. Masses for each isochrone point are tabulated in the isochrones together with temperature and luminosity.

Examples of how the addition goes are shown in figure 3, in which isochrones are plotted point by point. The sizes of the points are varied so that the area of each is proportional to $N \times l$, the total luminosity coming from that isochrone point. Therefore, one can obtain a visual impression of what evolutionary phases are important by looking for the larger symbols. In total energy output $\left(L_{\mathrm{bol}}\right)$ it is evident that the bright red giants are dominant, with main sequence turnoff stars secondary. In the $U$ filter, the giants contribute almost nothing, with the main sequence turnoff stars dominant. The horizontal branch, if present, is a significant source of light. In the $V$ filter stars of many different kinds contribute, but in the nearinfrared $K$ filter nothing but the brightest KM giants contribute significantly. For populations younger than about half a billion years, the giant branch does not develop as illustrated, and the main sequence stars are more important.

## Synthesis model behavior and uncertainties

The basic behavior of a stellar population is (1) the population becomes dimmer with age, (2) the population becomes dimmer with increasing abundance in passbands bluer than about $1 \mu \mathrm{~m}$, brighter in passbands redder than that, (3) the population becomes redder with age or increasing abundance and (4) neutral metallic lines increase in strength and, after about $10^{8} \mathrm{yr}$, hydrogen Balmer lines decrease in strength with increasing age or abundance. The age caveat in point (4) arises because the hot $O B$ stars present in very young populations have relatively weak Balmer lines, and when those stars dominate the Balmer lines fade somewhat. The Balmer lines peak in strength for turnoff stars at about A0 spectral type ( $\sim 7 \times 10^{8}$ yr isochrone age) and fade for cooler stars. Metal lines increase monotonically with decreasing star temperature. The behavior is shown in figure 4, which shows dimming of a population in the $V$ band, and reddening of the $B-V$ color.

The rate of dimming with age is controlled by the IMF, which controls how many stars exist at each mass. If there are relatively more massive stars in a population, it will be brighter when young. The IMF does not strongly affect the color evolution and is almost negligible in its effects


Figure 4. Population fading in the V band is shown as a function of age. The V magnitude of a population of $10^{6} M_{\odot}$ is shown by the solid curve, and the left-hand axis gives the scale. Color reddening is also shown where $B-V$ color is shown by the dotted curve and the right-hand axis gives the scale.
after a substantial red giant branch develops, a bit shy of a billion years of age. This is because almost all the light comes from stars near the main sequence turnoff or from post-main-sequence stars. All of these stars have nearly the same mass (because post-main-sequence lifetimes are relatively short) so there is little mass differential for the IMF to work on.

Since we do not know how much the IMF varies from environment to environment there is corresponding uncertainty in the synthesis models. This is an astrophysical uncertainty, but there are technical uncertainties as well since most of the ingredients in an isochrone synthesis model are suspect at some level. The isochrones themselves appear to be the dominant source of error. Stellar lifetimes and luminosities are somewhat uncertain, but the temperatures predicted by the isochrones are the most critical component and one of the hardest to obtain accurately. Temperature errors lead directly to errors in derived ages and abundances. The stellar flux libraries have problems also. Theoretical fluxes do not match real stars, especially in the ultraviolet and infrared where the atomic and molecular line lists are incomplete. Observed libraries have observational error, limited wavelength coverage, uncertain flux calibration and often inappropriate resolution. The combined effect of these technical problems is estimated to render derived ages uncertain by about $35 \%$ for populations older than a billion years. The uncertainty is larger for younger populations.

There are sometimes problems in the observational material that make it difficult to apply a model based solely on star light. Dust may be present in the observed galaxy,
possibly with a variety of geometries. This will affect colors owing to scattering and absorption of starlight. If stars are currently forming in a galaxy, nebular emission lines from H II regions are present. Together with other species, the hydrogen Balmer lines are present in emission, filling in the stellar absorption lines. This can make the Balmer absorptions, which are important age diagnostics, very hard or impossible to measure.

Another significant hurdle for all types of population models is age-metallicity degeneracy, in which age effects look similar to metallicity effects in integrated light. To illustrate the problem figure 5 shows isochrones and fluxes for a trio of populations. The 5 billion year old, $[\mathrm{Fe} / \mathrm{H}]=-0.1$ dex population is the reference population. The second population is three times older than the reference, the third twice as metal rich, but their spectra fall virtually on top of one another. The null-change slope is approximately $\Delta \log ($ age $)=(-3 / 2) \Delta \log Z$.

Is age hopelessly intertwined with abundance? Fortunately, it is not. A careful look at the model spectra reveals several features that do not participate in the $-3 / 2$ slope. Some metallic absorption features are relatively more sensitive to metals. Hydrogen Balmer lines track the main sequence turnoff temperature closely without much regard for the giant branch temperature and hence are relatively age sensitive. Ultraviolet-to-red colors behave the same way (but in practice are hampered by the presence of a minority of hot horizontal branch or post-AGB stars and by dust, if present). The presence of luminous red AGB stars has been proposed as another age indicator, since the AGB is observed to be very strong in clusters less than about 2 billion years of age, but the stellar evolutionary details of this are difficult to work out. Additional age indicators are being sought by today's researchers. For the moment, it appears that arraying a Balmer feature against a sensitive metal feature is the most reliable way to estimate simultaneously a mean age and a mean metal abundance. Example diagnostic diagrams are shown in figure 6.

Another astrophysical uncertainty is the mixture of elements that are used for the 'abundance'. A scaledsolar abundance pattern seems to apply approximately throughout the local disk, but metal-poor halo stars have light elements that are more abundant relative to Fe-peak elements. A similar abundance pattern applies in the galactic bulge and also in large elliptical galaxies (see below). The impact of changing the heavy-element ratios is primarily on the temperature structure of the isochrone and the appearance of the stellar spectra, with secondary effects on stellar lifetimes and luminosities. However, the modeling that is required to calibrate these changes is only now starting to be carried out, so the ultimate impact of differing elemental mixtures is difficult to estimate. As it stands now, the local template stars have spectra which differ in line strength pattern from the spectra of some galaxies and clusters, so a perfect model-to-galaxy match is not possible.


Figure 5. (a) Matched isochrones and (b) model spectra that illustrate age-metal degeneracy are shown. The base population is age 5 billion years, with slightly less than solar metallicity. Of the other two populations, one has age tripled, the other has metallicity doubled. Despite differing in abundance by a factor of 2 and differing in age by a factor of 3 both spectra lie nearly on top of one another.

A final astrophysical uncertainty is the difficulty of composite populations, systems that are composed of many different ages and abundances. In particular, there is a strong masking effect when a young population is present together with an underlying old population.


Figure 6. While most index-index diagrams are age-metal degenerate, as in the top panel showing an Na D index versus an index measuring mostly molecular carbon, some show fairly good ability to separate the two effects, as in the bottom panel showing $\mathrm{H} \beta$. Note that an 'index' could be a color, an absorption feature index or even an SBF magnitude. The choice of indices presented here is arbitrary. Metallicity values and ages in billions of years are marked on the grid lines that represent simple-population models.

Since young populations are very much brighter, they will dominate the integrated light even when the young population represents only a few per cent of the total mass. When 'mean ages' are derived for galaxies, the youngest population present gets far more weight than the older ones.

## Applications of synthesis models

Over the years we have learned a great deal about distant globular clusters and galaxies from the study of their integrated starlight as interpreted through synthesis models. The following examples illustrate some of the ways that this class of models has been applied

Globular cluster systems are attractive targets because all but the very largest of them appear to be not only co-eval but also monometallic so that they can be characterized by a model of a single age and metal abundance. The first globular cluster system to be studied was the Milky Way's. Direct stellar abundance estimates are available for these nearby systems, so the strong connection between integrated color (or metal line strength) and abundance is obvious. The Milky Way globular clusters are believed to have formed at various times during the first third of the Galaxy's life so they are uniformly ancient. They remain crucial benchmarks for testing synthesis models.

Colors of globular clusters around other galaxies are now becoming available out to $\sim 8000 \mathrm{~km} \mathrm{~s}^{-1}$ and lowresolution spectra can be obtained out to $\sim 1500 \mathrm{~km} \mathrm{~s}^{-1}$ with 8 m telescopes. Clusters around elliptical galaxies are the most common target. Metal abundance can be derived by assuming old ages and using synthesis models to go from observed color to abundance. Note that the $-3 / 2$ age-metal slope works in our favor when metal abundance is sought: a $30 \%$ age uncertainty corresponds to a $20 \%$ abundance uncertainty, which is less than 0.1 dex. The typical globular cluster system has both metal-poor and metal-rich clusters, some of which have apparently greater than solar abundance. The Milky Way is missing the metal-rich component seen in most other galaxies. The metal-rich component often appears as a separate hump in the abundance histogram and is seen to be more centrally concentrated than the metal-poor clusters.

Elliptical galaxies show a color-magnitude relation in which brighter galaxies are redder. Since it is unlikely that smaller galaxies are all younger than their larger brethren, and since synthesis models predict reddening with increasing abundance, a higher abundance in the larger ellipticals is inferred.

Mean ages of early-type galaxies without emission lines can be estimated from Balmer absorption strength arrayed against metallic feature strengths. The results for local galaxies show a strong clump of old-appearing ages, with a significant trail of galaxies toward young ages. This indicates a degree of star formation activity inconsistent with a universal primordial formation for all elliptical and S0 galaxies.

Light/heavy element ratios in elliptical galaxies are not constant. When absorption features rather than colors are examined, Fe or Ca features are nearly constant regardless of galaxy size, but features sensitive to $\mathrm{Mg}, \mathrm{Na}$ and N abundance are stronger in larger galaxies. This was discovered when synthesis models were compared with absorption line data. On the positive side, this tells us much about the types of supernovae responsible for the chemical enrichment in galaxies of different sizes. On the negative side, this introduces more parameters to be encompassed by the synthesis models, increasing uncertainty.

Surface brightness fluctuations are pixel-to-pixel variations about the mean brightness of an image of a galaxy due to almost-resolved stars. The further away a galaxy is, the smoother its appearance because more stars fit into each pixel, suppressing the 'noise'. Tonry and Schechter quantified this phenomenon and exploited it as a distance indicator with potentially excellent accuracy. The size of the observed surface brightness fluctuations depends only on galaxy distance and the luminosity function of the brighter stars present in the galaxy (see also Luminosity function of galaxies). The luminosity function can be predicted from population models, which show a slope with color and predict an absolute zero point for the distance scale.

Post-starburst objects can be reliably found with variations on the technique of using Balmer lines in conjunction with features more sensitive to abundance. Large numbers of galaxies that look quiescent but in fact have had a large burst of star formation in the recent past have been discovered in nearby galaxy clusters, supporting a view in which galaxy clusters are in the midst of forming even today.

Cosmological studies of galaxies at high redshift almost always require evolutionary synthesis models to interpret the observed magnitudes and colors. The Doppler shift moves the spectrum to longer wavelengths, affecting the colors. Furthermore, at large look-back times the stars seen will be younger than their local counterparts, and therefore brighter and bluer. The synthesis models allow an accounting of these effects.

Starburst galaxies are bravely examined, despite dust and nebular emission, with the hope of deriving a highmass IMF, among other things. The key observables are the C IV and Si IV resonance lines in the vacuum ultraviolet. Massive stars have winds that generate P Cygni profiles in these lines. More massive stars have faster and denser winds, generating stronger P Cygni profiles with larger blueshift. This technique offers a sensitive IMF test for stars with $M>30 M_{\odot}$. To date, the evidence suggests that the IMF looks about the same in distant starburst galaxies as it does in local group star forming regions.

The $G$ dwarf problem is the observation that the solar neighborhood is deficient in stars of abundance less than $1 / 10$ th of solar abundance compared with the simple closed-box model of chemical evolution. Synthesis models have been used to estimate the metal-poor population fraction present in early-type galaxies by looking at the amount of main sequence turnoff light contributed in the near-UV and at spectral features sensitive to the presence of A-type metal-poor horizontal branch stars. The conclusion is that the G dwarf problem is universal. Other galaxies, even the nuclei of elliptical galaxies, do not contain as many metal-poor stars as the simple model predicts.

Chemo-dynamical models attempt to start with a cloud of gas and numerically turn it into a galaxy, with correct dynamics and chemistry along the way. These complex computer models can also include a population synthesis portion so that the surface brightnesses and colors of the model galaxies can be simulated as well. Today's models are approximate and underconstrained, but this is a promising avenue for future work.

## Bibliography

For references to recent literature on virtually all of the topics touched upon in this article see the conference review

Leitherer C, Fritze-von Alvensleben U and Huchra J (ed) 1996 From Stars to Galaxies: The Impact of Stellar Physics
on Galaxy Evolution (ASP Conf. Ser. 98) (San Francisco, CA: Astronomical Society of the Pacific)


[^0]:    Copyright © Nature Publishing Group 2001
    Brunel Road, Houndmills, Basingstoke, Hampshire, RG21 6XS, UK Registered No. 785998
    and Institute of Physics Publishing 2001
    Dirac House, Temple Back, Bristol, BS1 6BE, UK

[^1]:    Copyright © Nature Publishing Group 2001
    Brunel Road, Houndmills, Basingstoke, Hampshire, RG21 6XS, UK Registered No. 785998
    and Institute of Physics Publishing 2001
    Dirac House, Temple Back, Bristol, BS1 6BE, UK

