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THERMAL BENDING OF SHEAR-DEFORMABLE ORTHOTROPIC CYLINDRICAL SHELLS REINFORCED BY CYLINDRICALLY-ORTHOTROPIC RINGS

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Abstract—Bending of shear-deformable orthotropic cylindrical shells, reinforced by ring stiffeners, manufactured from a cylindrically-orthotropic material, in a steady-state thermal field, is considered. The difference between the temperatures outside and inside the shell remains constant. The material properties of the shell and the stiffeners can depend on the temperature. Closed-form solutions are obtained in a number of important particular cases. Numerical examples illustrate that even moderate differences between the external and internal temperatures can result in significant stresses and deformations of the shell.

INTRODUCTION

Thermoelastic problems of composite material structures represent significant interest for aerospace, shipbuilding and pressure-vessel industries. In particular, composite cylindrical shells reinforced in the axial and/or circumferential directions are an important structural element. Static and dynamic problems of reinforced composite cylindrical shells and panels have been studied by Thielemann (1960), Block (1968), Bogdanovich and Koshkina (1983, 1984), Bogdanovich (1986), Bushnell *et al.* (1988), Birman (1988, 1990a–c) and Birman and Bert (1990). Thermal effects have been considered in two of the papers listed above (Birman, 1990b; Birman and Bert, 1990). However, material properties were assumed to be independent of temperature in these papers.

In the present study, an axisymmetric thermoelastic problem of ring-reinforced composite cylindrical shells with material properties affected by temperature is considered. A steady-state thermal field is due to a constant difference between the temperatures outside and inside the shell. The shell material is shear deformable and specially orthotropic so that the fibers' directions coincide with the shell axis. The ring stiffeners are cylindrically orthotropic and equally spaced.

The closed-form solutions are obtained for deformations (and stresses) of the shell between the stiffeners and for compliances of the stiffeners manufactured from materials with temperature-independent properties. If the properties of the stiffeners depend on temperature, an approximate solution can be obtained by the collocation method as shown in this paper.

ANALYSIS

Consider a specially-orthotropic circular cylindrical shear-deformable shell reinforced by equally-spaced ring stiffeners, Fig. 1. The fibers are oriented along the axis so that the shell material is transversely isotropic. The movements of the end cross-sections in the axial direction are not restricted. A steady-state thermal field is characterized by constant external and internal temperatures, denoted T_e and T_i respectively. In addition, a uniform pressure p is applied to the external surface of the shell.

The stress analysis is carried out as follows. First, deflections within a bay between two adjacent stiffeners are considered. The boundary conditions necessary to specify these deflections are determined from the analysis of the radial deformations of the ring stiffeners. Finally, the stresses in the shell and in the stiffeners are calculated. A similar approach was

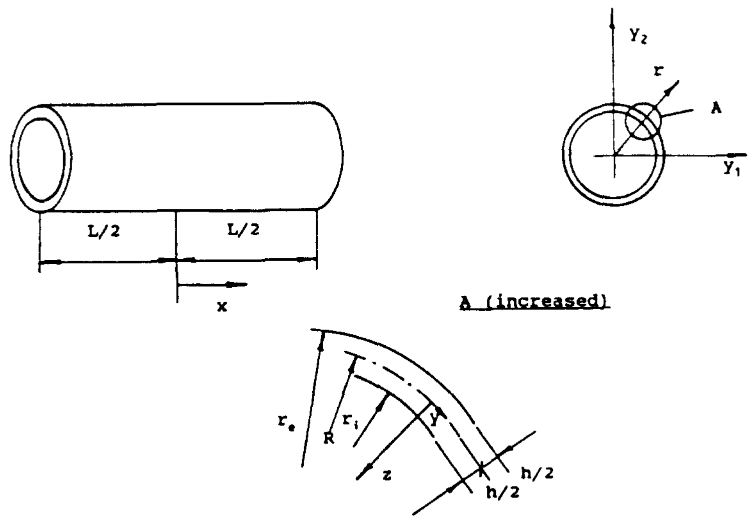

Fig. 1. Cylindrical shell between two adjacent ring stiffeners and coordinate systems used in the analysis.

applied to the study of the bending of ring-reinforced isotropic cylindrical shells without thermal effects by Papkovitch (1947).

(1) *Axisymmetric heat transfer problem for a cylindrical shell with thermal conductivity dependent on temperature*

A bay between two adjacent rings represents a cylindrical shell as shown in Fig. 1. The thermal field being independent of the axial (x) coordinate, the heat conduction equation reads

$$(k_{y1}T_{,y1})_{,y1}+(k_{y2}T_{,y2})_{,y2}=0 \tag{1}$$

where k_{y1} and k_{y2} are thermal conductivities. In the problem considered here, the thermal conductivities k_{y1} and k_{y2} are equal at any point, i.e. $k_{y1}=k_{y2}=k(T)$.

Introducing cylindrical coordinates one can transform the heat conduction equation:

$$k\left(T_{,rr}+\frac{1}{r}T_{,r}\right)+k_{,r}T_{,r}=0. \tag{2}$$

Now the relationship $k=k(r)$ has to be specified. In this paper it is assumed that the thermal conductivity in the direction perpendicular to the fibers is a linear function of the temperature. Even if this assumption is not applicable, the analysis shown below can be used. In this case the shell must be subdivided into a number of cylindrical sublayers of very small thickness so that within each sublayer the relationship $k(T)$ can be replaced by a best-fit linear function. The analysis can be carried out for each sublayer while temperatures at the boundaries of adjacent sublayers should be determined from the continuity conditions and from the boundary conditions $T(r_e)=T_e$, $T(r_i)=T_i$.

If

$$k=k_1T+k_0 \tag{3}$$

where k_1 and k_0 are constants, eqn (2) reads

$$(k_1T+k_0)\left(T_{,rr}+\frac{1}{r}T_{,r}\right)+k_1(T_{,r})^2=0. \tag{4}$$

Now a new variable is introduced:

$$Y=\left(T+\frac{k_0}{k_1}\right)^2 \tag{5}$$

so that eqn (4) can be transformed to

$$\bar{r}Y_{,\bar{r}}=\text{const} \tag{6}$$

where

$$\bar{r}=r/r_i. \tag{7}$$

The solution of (6) yields

$$T=-\frac{k_0}{k_1}+\sqrt{A\ln\bar{r}+B} \tag{8}$$

where

$$A=\frac{(T_e-T_i)\left(T_e+T_i+2\frac{k_0}{k_1}\right)}{\sqrt{\ln r_e/r_i}},\qquad B=\left(T_i+\frac{k_0}{k_1}\right)^2. \tag{9}$$

Note that if thermal conductivity is independent of temperature, the relationship (8) should be replaced by the well-known result

$$T=A_1\ln\bar{r}+B_1$$

$$A_1=\frac{T_e-T_i}{\ln r_e/r_i}\qquad B_1=T_i. \tag{10}$$

(2) *Axisymmetric stress problem for a shear-deformable cylindrical shell*

The strain-displacement relationships for such a shell read

$$\varepsilon_x=u_{,x}+z\psi_{,x}\qquad \varepsilon_z=\gamma_{xy}=\gamma_{yz}=0$$

$$\varepsilon_y=-\frac{w}{R}\qquad \gamma_{xz}=\psi+w_{,x} \tag{11}$$

where u and w are the axial and radial displacements, respectively, the coordinate z is positive in the inward direction and ψ is the bending slope.

The constitutive equations for a specially-orthotropic shell undergoing axisymmetric deformations are

$$\sigma_x = Q_{11}\left(\varepsilon_x - \int_0^T \alpha_x\,dT\right) + Q_{12}\left(\varepsilon_y - \int_0^T \alpha_y\,dT\right)$$
$$\sigma_y = Q_{12}\left(\varepsilon_x - \int_0^T \alpha_x\,dT\right) + Q_{22}\left(\varepsilon_y - \int_0^T \alpha_y\,dT\right)$$
$$\tau_{xz} = Q_{55}\gamma_{xz}$$
$$\sigma_z = \tau_{xy} = \tau_{yz} = 0 \tag{12}$$

where Q_{ij} are the reduced stiffnesses and α_i are the coefficients of thermal expansion which are, in general, functions of temperature.

The equilibrium equations are

$$N_{x,x} = 0 \qquad Q_{x,x} + \frac{N_y}{R} + N_x w_{,xx} + p = 0 \qquad Q_x = M_{x,x} \tag{13}$$

where N_x and N_y are in-surface stress resultants, Q_x is a transverse shear stress resultant and M_x is a stress couple. These stress resultants and the stress couple can be obtained in a form resembling the expressions used if the material properties are independent of temperature:

$$N_x = A_{11}u_{,x} - A_{12}\frac{w}{R} + B_{11}\psi_{,x} - N_x^T$$
$$N_y = A_{12}u_{,x} - A_{22}\frac{w}{R} + B_{12}\psi_{,x} - N_y^T$$
$$Q_x = A_{55}(\psi + w_{,x})$$
$$M_x = B_{11}u_{,x} + D_{11}\psi_{,x} - B_{12}\frac{w}{R} - M_x^T \tag{14}$$

where

$$\{A_{11}, A_{12}, A_{55}\} = \int_{-h/2}^{h/2} \{Q_{11}(z), Q_{12}(z), Q_{55}(z)\}\,dz$$
$$\{B_{11}, B_{12}\} = \int_{-h/2}^{h/2} \{Q_{11}(z), Q_{12}(z)\}\,z\,dz$$
$$D_{11} = \int_{-h/2}^{h/2} Q_{11}(z)z^2\,dz$$
$$[N_x^T, M_x^T] = \int_{-h/2}^{h/2} \left\{Q_{11}(z)\int_0^{T(z)} \alpha_x\,dT + Q_{12}(z)\int_0^{T(z)} \alpha_y\,dT\right\}[1, z]\,dz$$
$$N_y^T = \int_{-h/2}^{h/2} \left\{Q_{12}(z)\int_0^{T(z)} \alpha_x\,dT + Q_{22}(z)\int_0^{T(z)} \alpha_y\,dT\right\}dz, \tag{15}$$

h being the thickness of the shell.

The integrals in (15) depend on the particular relationships $T(z)$, $\alpha_i(T)$ and $Q_{ij}(T)$. If analytical expressions for the coefficients of thermal expansion and the moduli of elasticity are known, and the temperature-radius formulae (8) or (10) are used, these integrals can be evaluated numerically or, sometimes, analytically. Note that to perform the integration the radius r in (8) and (10) should be related to the radial coordinate by $z = R - r$.

The integration of the equations of equilibrium is carried out as follows. The stress resultant N_x is constant according to the first eqn (13), i.e. $N_x = N_x^0$ where N_x^0 is the stress resultant of the external axial loads applied to the ends of the shell. Then the first two eqns (14) yield

$$N_y = \frac{A_{12}}{A_{11}}(N_x^0+N_x^T)-N_y^T+\left(\frac{A_{12}^2}{A_{11}}-A_{22}\right)\frac{w}{R}+\left(B_{12}-\frac{A_{12}B_{11}}{A_{11}}\right)\psi_{,x}. \tag{16}$$

Substitution of N_y given by (16) and Q_x given by (14) into the second eqn (13) results in

$$\psi_{,x} = S_1+S_2w_{,xx}+S_3w \tag{17}$$

where

$$S_1 = -\left[p+\frac{A_{12}}{A_{11}R}(N_x^0+N_x^T)-\frac{N_y^T}{R}\right]\bigg/S$$

$$S_2 = -(A_{55}+N_x^0)/S$$

$$S_3 = -\left(\frac{A_{12}^2}{A_{11}}-A_{22}\right)\bigg/SR^2$$

$$S = A_{55}+\left(B_{12}-\frac{A_{12}}{A_{11}}B_{11}\right)\frac{1}{R}. \tag{18}$$

Now the third eqn (13) can be written in terms of w only:

$$\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)S_2w_{,xxxx}+\left[\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)S_3+\frac{B_{11}A_{12}}{A_{11}R}-\frac{B_{12}}{R}-A_{55}(1+S_2)\right]w_{,xx}-A_{55}S_3w = A_{55}S_1. \tag{19}$$

The solution of (19) is

$$w = \sum_{i=1}^{4} K_i\,e^{t_ix}-\frac{S_1}{S_3} \tag{20}$$

where the K_i are constants of integration and the t_i are the non-zero roots of the corresponding characteristic equation.

The values of four constants of integration can be evaluated from the symmetry conditions:

$$w_{,x}\left(\frac{L}{2}\right) = w_{,x}\left(-\frac{L}{2}\right) = 0 \tag{21}$$

and from the conditions

$$w\left(\pm\frac{L}{2}\right) = gQ_x\left(\pm\frac{L}{2}\right) \tag{22}$$

where g is the compliance of the ring stiffener calculated in the following sections. The transverse shear stress resultant Q_x can be evaluated in terms of w from the third eqn (13) where the stress couple has to be represented as $M_x(w)$ using eqns (14) and (17).

If shear deformability can be neglected, $\psi = -w_{,x}$ and expressions (14) are modified accordingly. The equilibrium equations become

$$N_{,x} = 0 \qquad M_{x,xx} + N_x^0 w_{,xx} + \frac{N_y}{R} + p = 0. \tag{23}$$

Transformations yield

$$u_{,x} = \left(N_x^0 + A_{12}\frac{w}{R} + B_{11}w_{,xx} + N_x^T\right)\Big/A_{11} \tag{24}$$

$$\left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)w_{,xxxx} + \left[2\left(B_{12} - \frac{B_{11}A_{12}}{A_{11}}\right)\frac{1}{R} - N_x^0\right]w_{,xx} + \frac{1}{R^2}\left(-\frac{A_{12}^2}{A_{11}} + A_{22}\right)w$$
$$= p + \frac{A_{12}}{A_{11}R}(N_x^0 + N_x^T) - \frac{N_y^T}{R}. \tag{25}$$

The solution of eqn (25) is similar to that of (19). This equation is simplified even more if material properties can be assumed insensitive to temperature and $N_x^0 = 0$. In the latter case the coefficient at $w_{,xx}$ is equal to zero and standard solutions can be used.

(3) *Axisymmetric heat transfer problem for cylindrically-orthotropic ring stiffeners*

Two possibilities are considered here, i.e. the case of very thin rings attached to the internal surface of the shell and the case of thicker rings which are either attached to a very thin shell or extend to the external surface. Obviously, in the first case it is safe to assume that the temperature is constant throughout the ring, i.e. $T = T_i$. In the latter case a distribution of temperature in the ring can be obtained from the heat conduction equation. This equation is written here by the assumption that the thermal conductivities in the radial and thickness directions are independent of temperature:

$$k_r T_{,rr} + k_{\bar z} T_{,\bar z\bar z} = 0 \tag{26}$$

where k_r and $k_{\bar z}$ are the thermal conductivities and the coordinate $\bar z$ is introduced according to Fig. 2. The solution of (26) must satisfy the following boundary conditions:

$$\bar z = 0, t: \quad T = T_i$$
$$r = \tilde r_i: \quad T = T_i$$
$$r = \tilde r_e: \quad T = T_e. \tag{27}$$

Conditions (27) imply that the thickness of the shell is negligible compared to the depth of the stiffener. The solution is sought in the form

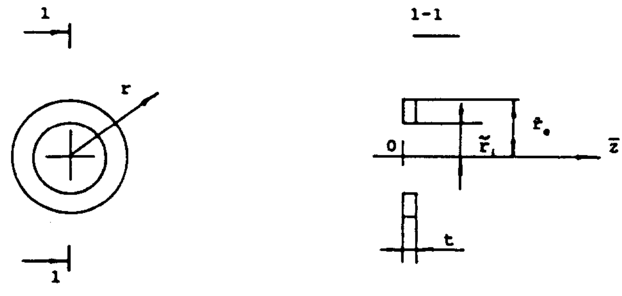

Fig. 2. Geometry of the ring stiffener.

$$T = T_R(r)T_z(z).\tag{28}$$

Substitution of (28) into (26) and some transformations yield

$$T_R = C_R \sinh\frac{\lambda}{\sqrt{k_r}}r + \cosh\frac{\lambda}{\sqrt{k_r}}r \qquad T_z = A_z \sin\frac{\lambda}{\sqrt{k_z}}z + B_z \cos\frac{\lambda}{\sqrt{k_z}}z\tag{29}$$

where A_z, B_z, C_R and λ are constants which can be found from the boundary conditions.

Consider, for example, the case where $T_i = 0$. Then

$$T = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4T_e}{n\pi f_n(\bar{r}_e)} f_n(r)\sin\frac{n\pi\bar{z}}{t}\tag{30}$$

where

$$f_n(r) = \sinh\left(\frac{\pi n}{t}\sqrt{\frac{k_z}{k_r}}r\right) - \tanh\left(\frac{\pi n}{t}\sqrt{\frac{k_z}{k_r}}\bar{r}_i\right)\cosh\left(\frac{\pi n}{t}\sqrt{\frac{k_z}{k_r}}r\right).\tag{31}$$

(4) *Compliances of cylindrically-orthotropic heated annular plates*

Consider now the problem of axisymmetric deformations of cylindrically-orthotropic annular plates subject to a uniform compressive loading of intensity q at the outer boundary. The strain–displacement relationships are

$$\varepsilon_r = \tilde{u}_{,r} \qquad \varepsilon_\theta = \frac{\tilde{u}}{r},\tag{32}$$

$\tilde{u}$ being a radial displacement.

The constitutive relationships read

$$\sigma_r = \left\{\varepsilon_r + \nu_{\theta r}\varepsilon_\theta - \int_0^T [\alpha_r(T) + \nu_{\theta r}\alpha_\theta(T)]\,\mathrm{d}T\right\}\frac{E_r}{1-\nu_{r\theta}\nu_{\theta r}}$$

$$\sigma_\theta = \left\{\varepsilon_\theta + \nu_{r\theta}\varepsilon_r - \int_0^T [\alpha_\theta(T) + \nu_{r\theta}\alpha_r(T)]\,\mathrm{d}T\right\}\frac{E_\theta}{1-\nu_{r\theta}\nu_{\theta r}}\tag{33}$$

where Poisson's ratios $\nu_{\theta r}$ and $\nu_{r\theta}$ can be assumed to be independent of the temperature. Substitution of (32) into (33) and subsequent integration throughout the plate thickness yield the stress resultants:

$$N_r = A_{rr}\tilde{u}_{,r} + A_{\theta r}\frac{\tilde{u}}{r} - N_r^T$$

$$N_\theta = A_{r\theta}\tilde{u}_{,r} + A_{\theta\theta}\frac{\tilde{u}}{r} - N_\theta^T.\tag{34}$$

In (34)

$$(A_{rr}, A_{\theta r}) = \int_0^t \frac{E_r}{1-\nu_{r\theta}\nu_{\theta r}}(1, \nu_{\theta r})\,\mathrm{d}\bar{z}$$

$$(A_{\theta\theta}, A_{r\theta}) = \int_0^t \frac{E_\theta}{1-\nu_{r\theta}\nu_{\theta r}}(1, \nu_{r\theta})\,\mathrm{d}\bar{z}$$

$$N_r^T = \int_0^t \frac{E_r}{1-\nu_{r\theta}\nu_{\theta r}} \left\{ \int_0^T [\alpha_r(T) + \nu_{\theta r}\alpha_\theta(T)] \, dT \right\} dz$$

$$N_\theta^T = \int_0^t \frac{E_\theta}{1-\nu_{r\theta}\nu_{\theta r}} \left\{ \int_0^T [\alpha_\theta(T) + \nu_{r\theta}\alpha_r(T)] \, dT \right\} dz. \tag{35}$$

Obviously, the stiffnesses and thermal terms in (35) depend on the particular relationships $E_r(T)$, $E_\theta(T)$, $\nu_{r\theta}(T)$, $\nu_{\theta r}(T)$, $\alpha_r(T)$ and $\alpha_\theta(T)$. If these relationships are known, the substitution of $T = T(z, r)$ and integrations according to (35) yield $A_{rr}(r), \ldots, N_\theta^T(r)$.

The equation of equilibrium is

$$(rN_r)_{,r} - N_\theta = 0 \tag{36}$$

which upon substitution of (34) yields a second-order differential equation for $\tilde{u}$:

$$rA_{rr}\tilde{u}_{,rr} + [(rA_{rr})_{,r} + A_{\theta r} - A_{r\theta}]\tilde{u}_{,r} + (rA_{\theta r,r} - A_{\theta\theta})\frac{\tilde{u}}{r} - (rN_r^T)_{,r} + N_\theta^T = 0. \tag{37}$$

In a general case this equation has variable coefficients and cannot be integrated in a closed form. The boundary conditions at the external and internal boundaries are:

$$r = \bar{r}_i: \quad N_r = 0; \qquad r = \bar{r}_c: \quad N_r = q. \tag{38}$$

The solution can be found by a collocation method using power series to represent radial displacements:

$$\tilde{u} = \sum_{n=1}^{m} a_n r^n. \tag{39}$$

Substitution of (34) and (39) into (38) yields

$$\sum_n [nA_{rr}(\bar{r}_i) + A_{\theta r}(\bar{r}_i)]a_n\bar{r}_i^{n-1} - N_r^T(\bar{r}_i) = 0$$

$$\sum_n [nA_{rr}(\bar{r}_c) + A_{\theta r}(\bar{r}_c)]a_n\bar{r}_c^{n-1} - N_r^T(\bar{r}_c) = q. \tag{40}$$

An additional $(m-2)$ algebraic equations are obtained from the equilibrium equation if one requires it to be satisfied at $(m-2)$ points throughout the depth of the plate:

$$\sum_{n=2}^{m} \{A_{rr}n(n-1) + [(rA_{rr})_{,r} + A_{\theta r} - A_{r\theta}]n + rA_{\theta r,r} - A_{\theta\theta}\}_{r=r_k} a_n r_k^{n-1}$$
$$+ [(rA_{rr})_{,r} + A_{\theta r} - A_{r\theta} + rA_{\theta r,r} - A_{\theta\theta}]_{r=r_k} a_1 - (rN_{r,r}^T - N_\theta^T)_{r=r_k} = 0 \tag{41}$$

where

$$r_k = \bar{r}_i + \frac{\bar{r}_c - \bar{r}_i}{m-1} k \tag{42}$$

and k varies from one to $(m-2)$.

The substitution of $q = 1$ yields the values of a_n corresponding to a unit load. Now $\tilde{u}(\bar{r}_c)$ can be calculated; this value is equal to one half of the compliance of the stiffener since each annular plate supports two adjacent spans. Therefore, the value of the compliance in (22) should be taken as being equal to $g = 2\tilde{u}(\bar{r}_c; q = 1)$.

(5) *Particular case: cylindrically-orthotropic annular plates with temperature-independent properties*

In this case the A_{ij} given by (35) are constant. If the stiffener is very thin so that $T \simeq T_i$ the thermal terms become

$$N_s^T = (A_{ss}\alpha_s + A_{js}\alpha_j)T_i \quad (s, j = r, \theta). \tag{43}$$

If $T = T(r, z)$, then the thermal terms can be evaluated depending on a particular distribution of temperature. For example, if this distribution is given by (30),

$$N_s^T = 8(A_{ss}\alpha_s + A_{js}\alpha_j)T_e t \sum_{n=1,3,5,\ldots}^{\infty} \frac{f_n(r)}{(n\pi)^2 f_n(\bar{r}_e)}. \tag{44}$$

The equilibrium eqn (37) is simplified:

$$rA_{rr}\bar{u}_{,rr} + (A_{\theta r} - A_{r\theta})\bar{u}_{,r} - A_{\theta\theta}\frac{\bar{u}}{r} = f(r) \tag{45}$$

where $f(r)$ is a function which depends on a particular temperature distribution. The homogeneous equation corresponding to (45) can be reduced to Euler's equation. Then the solution of (45) can be written as

$$\bar{u} = C_1 r^{\beta_1} + C_2 r^{\beta_2} + \phi(r) \tag{46}$$

where C_1 and C_2 are constants of integration,

$$\beta_{1,2} = \frac{1}{2}\frac{A_{rr} + A_{r\theta} - A_{\theta r}}{A_{rr}} \pm \sqrt{\frac{1}{4}\left(\frac{A_{rr} + A_{r\theta} - A_{\theta r}}{A_{rr}}\right)^2 + \frac{A_{\theta\theta}}{A_{rr}}} \tag{47}$$

and $\phi(r)$ is a particular integral of the non-homogeneous equation which can be easily evaluated for the thermal terms given by (43) or (44). Constants of integration are obtained from the boundary conditions (38).

NUMERICAL EXAMPLES

The purpose of the following examples is to illustrate that even moderate temperature gradients in the radial direction (of the order of 100°F) can result in large deflections and stresses of composite shells. The material of the shell considered in the examples was boron/A17178-T6 with the following properties: $E_1 = 213.75$ GPa, $E_2 = 131.01$ GPa, $\nu_{12} = 0.255$, $\alpha_1 = 5.16 \times 10^{-6}$ (F°)$^{-1}$, $\alpha_2 = 10^{-5}$ (F°)$^{-1}$. These properties were assumed to remain constant since fluctuations of temperature in the thickness direction were limited. The direction of the fibers coincided with the shell axis. The geometry of the shell was $h = 0.004$ m, $R = 2$ m and $L = 0.5$ m (except for Fig. 5 were L varied). The ring stiffeners were assumed to be rigid so that $w(\pm L/2) = 0$. In the examples presented here $N_x^0 = 0$, i.e. the shell is neither restricted nor loaded in the axial direction.

The thermal terms given by (15) can be easily calculated in this case; in the following formulae they are complemented by M_y^T which is necessary to calculate the stresses:

$$\begin{aligned} \{N_x^T, M_x^T\} &= (Q_{11}\alpha_1 + Q_{12}\alpha_2)\{F_1, F_2\} \\ \{N_y^T, M_y^T\} &= (Q_{12}\alpha_1 + Q_{22}\alpha_2)\{F_1, F_2\} \end{aligned} \tag{48}$$

where

$$F_1 = A_1 r_i [(\bar{R}+\bar{h}) \ln (\bar{R}+\bar{h}) - 2\bar{h}] + B_1 h$$
$$F_2 = A_1 r_i^2 [\tfrac{1}{2}(\bar{R}^2 - \bar{h}^2) \ln (\bar{R}+\bar{h}) - \bar{R}\bar{h}]$$
$$\bar{R} = R/r_i \quad \bar{h} = h/2r_i$$
A_1 and B_1 are given by (10). (49)

As follows from the calculations, the effect of shear deformability is negligible for a shell with the chosen geometry. Therefore, the analysis was based on eqns (24), (25) where $B_{ij} = 0$ since the properties are independent of z.

The effect of the external temperature on the radial deflections at the midspan is shown in Fig. 3 (deflections are measured in meters in Figs 3–5). It appears that a shell subject to an elevated temperature on the external surface will bend in the outward direction. A uniformly-distributed external pressure ($p > 0$) reduces deflections. On the other hand, as would be expected, internal pressure ($p < 0$) results in larger deformations. An elevated

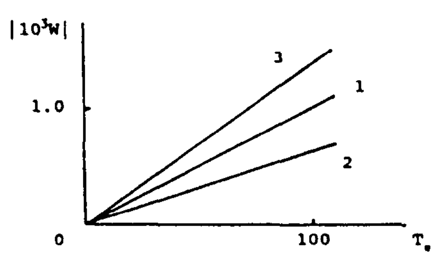

Fig. 3. Effect of external temperature and pressure on radial deformation; $T_i = 0$, curve 1: $p = 0$, curve 2: $p = 50$ kPa, curve 3: $p = -50$ kPa.

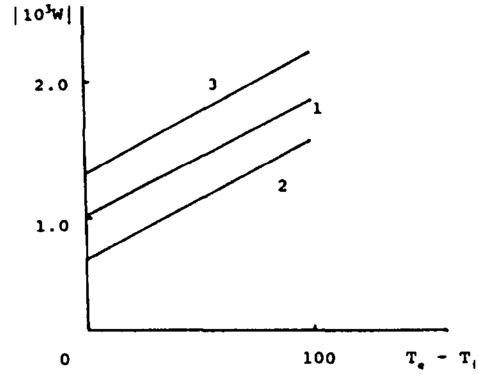

Fig. 4. Effect of temperature gradient and pressure on radial deformation; $T_i = 50$°F, curve 1: $p = 0$, curve 2: $p = 50$ kPa, curve 3: $p = -50$ kPa.

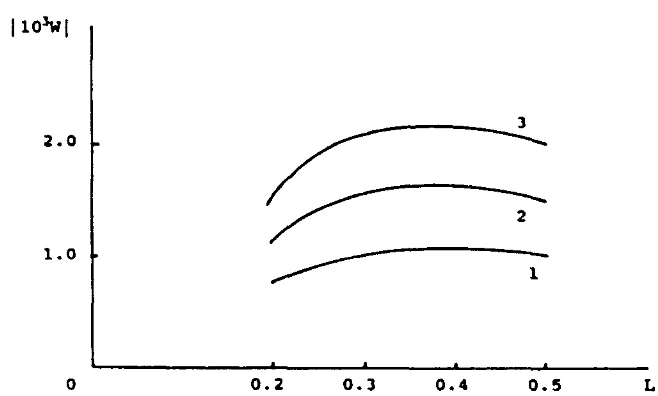

Fig. 5. Effect of spacing of ring stiffeners on radial deformation; $T_i = 0$, $p = 0$, curve 1: $T_e = 100$°, curve 2: $T_e = 150$°, curve 3: $T_e = 200$°.

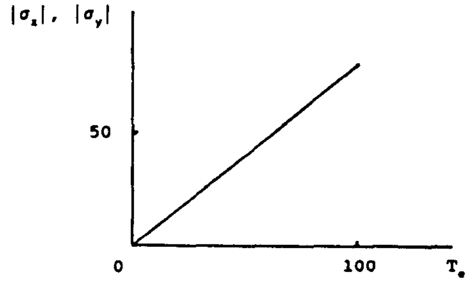

Fig. 6. Maximum stresses at midspan in megapascals: $T_i = 0$, $p = 0$.

temperature results in outward radial deflections, even if it is uniform, as follows from Fig. 4. Additional outward deformations appear as a result of an increasing outside temperature or internal pressure.

The influence of the spacing of the stiffeners on the deflections of the midspan is shown in Fig. 5. Note that there is a value of the span L corresponding to the maximum deflection. This conclusion is not surprising since a similar phenomenon was observed for beams on elastic foundation and for cylindrical shells subject to concentrated loading; see Figs 9.8 and 9.18 in the book by Calladine (1983).

Finally, maximum stresses at the midspan and at the cross-sections over the ring stiffeners are shown in Figs 6 and 7. As follows from these figures, larger stresses exist at the cross-sections where the shell is supported by the ring stiffeners than at the midspan. The lines $|\sigma_x(T_e)|$ and $|\sigma_y(T_e)|$ in Fig. 6 almost coincide. Note that in this example σ_x is a bending stress while σ_y includes both bending and membrane portions. Membrane stresses σ_y are tensile at the midspan and compressive at $x = \pm L/2$. The stresses are quite significant even at a rather small (100 F) difference between the temperatures outside and inside the shell. The maximum stress in the circumferential direction at $x = \pm L/2$, i.e. 96.5 MPa, is equal to 58.5% of the yield stresses in the direction perpendicular to the fibers (165 MPa). Therefore, even a moderate difference between the temperatures on the external and internal surfaces can result in very high stresses in the shell.

CONCLUSIONS

Steady-state thermoelastic bending problems of shear-deformable specially-orthotropic cylindrical shells, reinforced by cylindrically-orthotropic ring stiffeners, are considered. The solutions obtained in the paper make it possible to include the effects of temperature on material properties in the analysis. The analytical solutions for the shells are obtained in a closed form. The solutions for the ring stiffeners which are necessary both to evaluate their compliances as well as to check the strength can be obtained in a closed form (properties independent of temperature) or by the collocation method (general case).

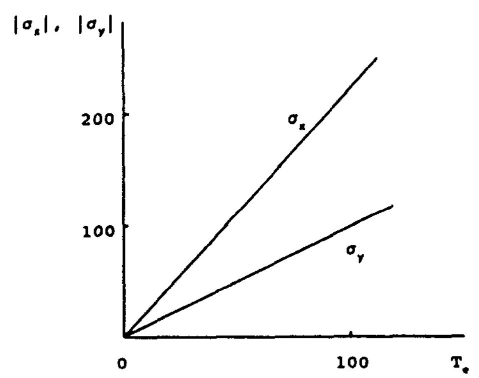

Fig. 7. maximum stresses in stiffener cross-section (in megapascals): $T_i = 0$, $p = 0$.

It is shown that high stresses can exist in shells due to relatively small differences between the outside and inside temperatures.

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