Asynchronous Networks and Erlang Formulas

Erik Chromý, Matej Kavacký

Dept. of Telecommunications, Faculty of Electrical Engineering and Information Technology,
Slovak University of Technology, Ilkovicova 3, 812 19 Bratislava, Slovak Republic
{chromy, kavacky}@ktl.elf.stuba.sk

***Abstract*: :** The paper addresses the idea of utilization of Erlang formulas in asynchronous ATM and IP networks. Based on the common properties of synchronous and asynchronous networks we have proposed the utilization of Erlang formulas not only for synchronous networks, but also for asynchronous networks. It is possible to describe traffic in asynchronous networks by calculation of following parameters – loss, link utilization and bandwidth. We present some simulation results from Matlab.

***Keywords*:** Erlang equation, Next Generation Networks, Asynchronous Transfer Mode, IP.

1. Introduction

Today, much attention is focused on description of traffic in asynchronous packet networks. There are various methods for traffic description. Many of them are complex with high computational requirements. The description by Markov chains is one of them. Therefore the Erlang formulas can be very efficient and simple way how we can describe traffic parameters in asynchronous networks.

Erlang formulas use traffic parameters such as loss, probability of delay, bandwidth and link utilization. These parameters are especially important from the Quality of Service (QoS) providing point of view. Hence the Erlang formulas seem to help us in the field of Quality of Service in Next Generation Networks (NGN).

Erlang theory is described in detail in [3], [4], [13]. Recently the Erlang formulas have been used for Contact centers traffic description. There is not much known related work in the field of Erlang theory for asynchronous networks, therefore this article propose the idea of this possibility.

2. ATM Networks

Asynchronous Transfer Mode (ATM) was the emerging network technology in the beginning of 90-ties. Today, it is replacing by expansion of IP networks [7]. Despite this fact, many companies still offer services based on ATM technology [8]. The main reasons for ATM development were:

- Increasing demand for telecommunication and information technologies and services.
- Convergence of data and voice communication.

ATM networks are connection oriented networks. ATM technology combines fast packet transfer with synchronous transfer through virtual circuits and virtual paths. It is cell switching mode which offers low transfer latency with high level of Quality of Service and ensures the support for voice, data and video service with high transfer rates.

2.1. ATM Quality of Service

Each category of service has different QoS requirements [1, 5, 6]. In the Tab. 1 we can see the ATM services from the Quality of Service point of view.

Table 1. Service categories and their QoS requirements.

Service category	Bandwidth guarantee	Delay variation guarantee	Impact of congestion
CBR	Yes	Yes	No
VBR	Yes	Yes	No
ABR	Yes	No	Yes
UBR	No	No	No

- CBR - Constant Bit Rate,
- VBR-rt - Variable Bit Rate-real time,
- ABR - Available Bit Rate,
- UBR - Unspecified Bit Rate.

Some of QoS parameters between network and end telecommunication equipment are following:

- CDV - Cell delay variation,
- maxCTD - maximum cell transfer delay,
- CLR - cell loss ratio.

3. IP Networks

Networks based on Internet Protocol (IP) provide datagram service. IP transfer is non-connection oriented and its main feature is that it is best effort service. It means that the packet will be transferred in the best way, without forced delay and without unnecessary packet losses. The transfer of IP datagrams is done without guarantee of packet delivery [2, 6].

3.1 IP and Quality of Service

Services such as voice and real-time video are very sensitive on particular traffic parameters. The main of these parameters are delay, loss and error rate. We have to note that various types of services have also different bandwidth requirements. Therefore there is need for guarantee of these parameters. This guarantee is called Quality of Service.

QoS requirements are following:

- end-to-end delay,

- jitter,
- packet losses,
- bandwidth,
- link utilization.

4. Erlang formulas

We have used the two Erlang formulas to describe the traffic in asynchronous networks. Particular calculations we have performed in Matlab environment.

4.1. The first Erlang Formula (Erlang B)

The Erlang B formula represents the ratio of lost calls. Therefore it is sometimes called “loss Erlang formula”. It is defined as follows:

$$B = \frac{\frac{A^N}{N!}}{\sum_{k=0}^{N} \frac{A^k}{k!}} = \frac{A^N}{N!} \cdot \frac{1}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots + \frac{A^N}{N!}} \qquad (1)$$

where:
B - ratio of lost calls [%],
A - total offered traffic [Erl],
N - number of channels (links).
The first Erlang formula can be written also in the following form:

$$B = \frac{1}{1 + \sum_{k=1}^{m} \left(\frac{N}{A}\right) \cdot \left(\frac{N-1}{A}\right) \cdots \left(\frac{N-k+1}{A}\right)} \, . \qquad (2)$$

The use of the equation (2) can significantly decrease the computational requirements and we can also calculate the load of the system with the higher values [3, 4].

The following conditions must be met for the first Erlang formula:

1) The flow of requests (calls) originates randomly with exponential distribution of incoming requests, which means the higher distance between requests, the lower number of these cases.
2) Service time has similar distribution, i.e. exponential decreasing of requests with higher service time.
3) The flow of requests is steady, as if it comes from infinite number of request sources.
4) There is full availability of requests to all served links.
5) Rejected requests do not return to incoming flow, therefore there are not repeated requests.
6) No two requests will originate together [3].

IP traffic brings radical changes into telephone networks. The condition 2 is not fulfilled. And because the traffic from one source is considerable increasing, also the condition 5 can not be met.

4.2. The second Erlang Formula (Erlang C)

The second Erlang formula also assumes infinite number of traffic sources. These sources generate the traffic *A* for *N* lines. The incoming request is inserted into waiting queue if all links *N* are occupied. Waiting queue can store infinite number of requests concurrently. This Erlang equation calculates probability of creation of waiting queue in the case of traffic *A*, and if it is assumed that the blocked calls will remain in the system until they are served [3].

$$C = \frac{\frac{A^N}{N!} \cdot \frac{N}{N-A}}{\sum_{k=0}^{N-1} \frac{A^k}{k!} + \frac{A^N}{N!} \cdot \frac{N}{N-A}} \text{ ,where } N>A \qquad (3)$$

where:
A - total offered traffic [Erl],
N - number of channels (links),
C - probability of waiting for service.

4.3 Representation of the Erlang C formula through Erlang B formula

The second Erlang formula (3) can be simplified through following modifications:

- Dividing numerator and denominator by $\sum_{k=0}^{N} \frac{A^k}{k!}$ and
- consecutive use of Erlang B equation (1).

By these modifications we can state the Erlang C formula in the form [4]:

$$C = \frac{N \cdot B}{N - A \cdot (1 - B)} \, . \qquad (4)$$

4.4. Common characters of asynchronous and synchronous networks

Erlang equations were primary intended for traffic description in synchronous networks. Our idea is to use these formulas also for asynchronous networks, so we have to find common parameters for synchronous and asynchronous networks.

Table 2. Common parameters of synchronous and asynchronous networks.

Synchronous network		Asynchronous network	
B [%]	Lost calls ratio	B [%]	Loss rate
C	Probability of waiting for service	C	Probability of delay
A [Erl]	Total offered traffic	A [%]	Link utilization
N	Number of channels (links)	N [Mbit/s]	Bandwidth

Probability of delay *C* for asynchronous networks represents the latency which occurs during transmission in the case of the heaviest traffic. This delay occurs in IP networks due to waiting queues in buffers in network nodes. Unfortunately, there is no similar parameter for jitter in synchronous networks, hence through Erlang formulas it can not be estimated.

5. Results for Erlang B formula

In this part we present results obtained by calculations through Erlang B formula. Input parameters were given as follows:

- One of parameters was constant.
- Other parameter was increased in given step sequence.

It is known that we can calculate the loss *B* through Erlang B formula if we have given link utilization *A* and bandwidth *N*. But it is also possible to calculate the link utilization *A* by method of bisection, if we know the bandwidth *N* and loss *B* [4].

5.1. Bandwidth and loss in the case of constant link utilization

The task is to obtain the loss *B* in the case of constant link utilization and increasing bandwidth. The results are shown in the Fig. 1.

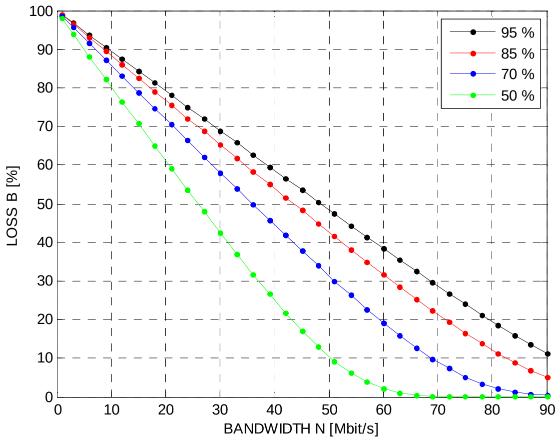

Fig. 1. Dependency between loss and bandwidth in the cases of constant link utilization.

We can see the following tendencies:

- The loss *B* is decreasing if the bandwidth is increasing and link utilization is constant.
- The loss *B* is increasing if the link utilization *A* is increasing.

5.2. The loss and link utilization in the case of constant bandwidth

In this part we have observed the loss B in the case of increasing link utilization together with constant bandwidth through Erlang B formula. The obtained results are shown in the Fig. 2.

We can see following tendencies:

- By increasing link utilization *A* the loss *B* is also increasing if the bandwidth *N* is constant.
- By increasing bandwidth *N* the loss *B* is decreasing if the link utilization *A* is constant.

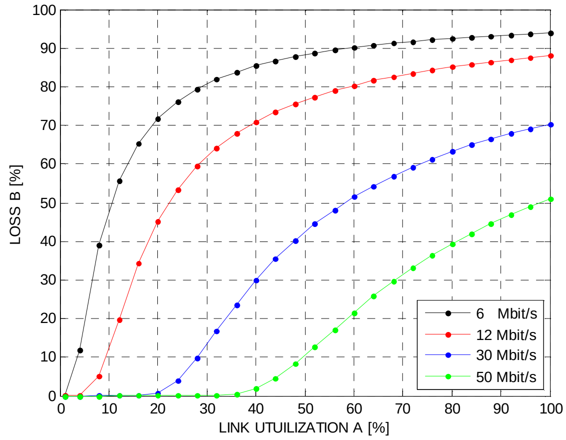

Fig. 2. Dependency between loss and link utilization in the cases of constant bandwidth.

6. Results for Erlang C formula

This part presents the results obtained by use of the Erlang C formula (4). By this equation we can calculate the possibility of delay *C* and loss *B* if we know the two parameters – the link utilization *A* and bandwidth *N*. Also in this case the input parameters were given as follows:

- One of parameters was constant.
- Other parameter was increased in given step sequence.

6.1. Link utilization and probability of delay in the case of constant bandwidth

In this task we have obtained the dependency of probability of delay if the bandwidth is constant and link utilization is increasing. The obtained results are shown in the Fig. 3.

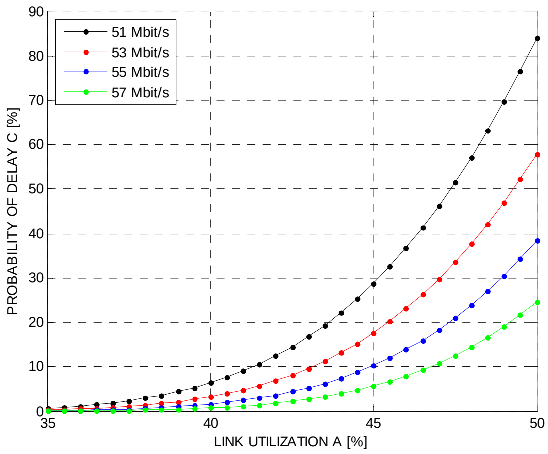

Fig. 3. Probability of delay in the case of constant bandwidth and increasing link utilization.

We can see following tendencies:

- In the case of constant probability of delay *C* and increasing bandwidth *N* the link utilization *A* can be higher.
- In the case of constant link utilization *A* and increasing bandwidth *N* the probability of delay *C* is decreasing.
- In the case of increasing link utilization *A* and constant bandwidth *N* the probability of delay *C* is increasing.

6.2. Link utilization and bandwidth in the case of constant probability of delay

By use of the Erlang B formula (4) and the method of bisection we have obtained the dependency of the link utilization *A* if the probability of delay *C* is constant and bandwidth *N* is increasing. The results are depicted in the Fig. 4.

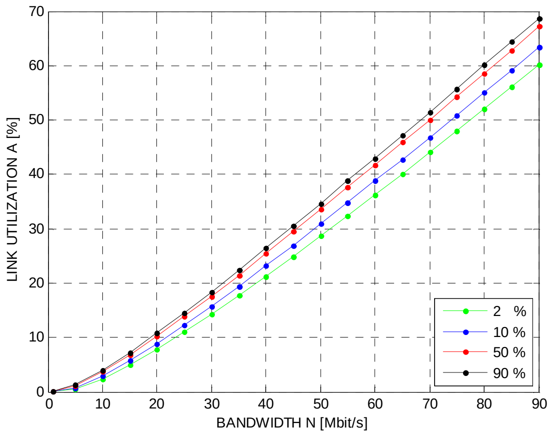

Fig. 4. Link utilization in the case of constant probability of delay and increasing bandwidth.

We can see following tendencies:

- In the case of constant link utilization *A* and increasing bandwidth *N* the probability of delay *C* is decreasing.
- In the case of constant bandwidth *N* and increasing probability of delay *C* the link utilization *A* is increasing.
- In the case of increasing bandwidth *N* and constant probability of delay *C* the link utilization *A* is increasing.

6.3. Probability of delay and loss if the link utilization and bandwidth are changing

By use of the Erlang C formula (4) we have obtained dependencies of probability of delay C and loss B if the bandwidth N and link utilization A were changing. The obtained results are shown in the Fig. 5 and 6.
We can see following tendencies:

- In the case of increasing link utilization *A* together with decreasing bandwidth *N* the loss *B* and probability of delay *C* are increasing.
- In the case of constant bandwidth *N* together with increasing link utilization *A* the loss *B* and probability of delay *C* are increasing.
- In the case of constant link utilization *A* together with increasing bandwidth *N* the loss and probability of delay *C* are decreasing.

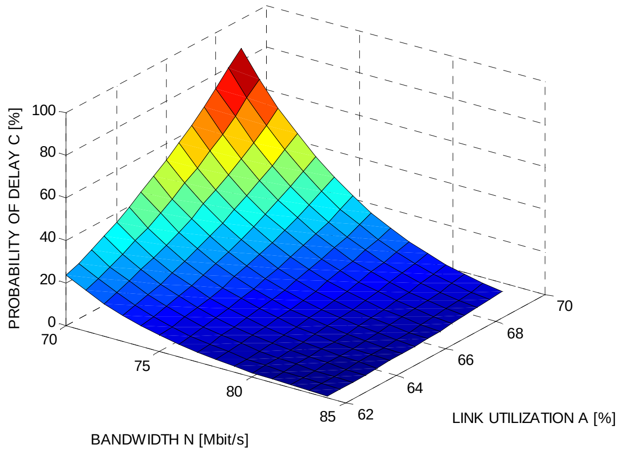

Fig. 5. Dependency between probability of delay, bandwidth and link utilization.

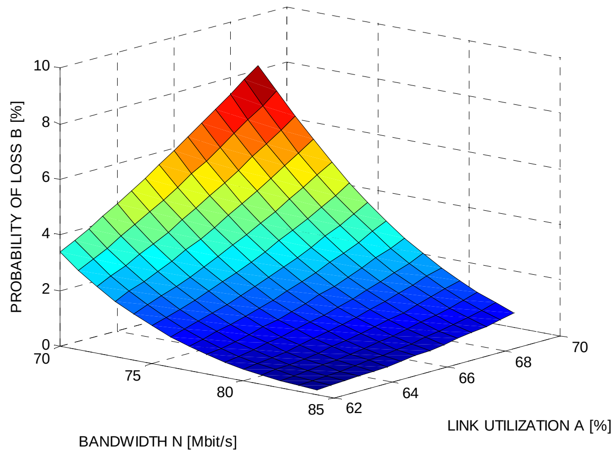

Fig. 6. Dependency between loss, bandwidth and link utilization.

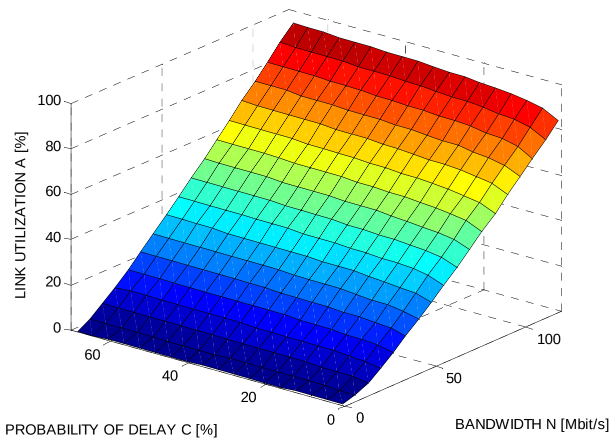

Fig. 7. Dependency between link utilization, probability of delay and bandwidth.

6.4. Link utilization in the case of changes in probability of delay and bandwidth

By use of the Erlang B formula (4) we have obtained the dependency of the link utilization *A* if the probability of delay *C* and bandwidth *N* are changing. From the figure 7 we can see that:

- In the case of increasing bandwidth N together with probability of delay C the link utilization A is also increased.

7. Conclusion

We have proposed the idea of utilization of two Erlang formulas for description of traffic in asynchronous networks. By our calculations through Erlang B and C formulas we have obtained the tendencies that suggest the possibility of use of Erlang equations in ATM or IP networks. Erlang B formula does not contain parameter for probability of delay, therefore this formula is more suitable for ATM network, because the probability of delay in this network is omissible. Through the Erlang C formula we can estimate the probability of delay, which usually occurs in IP networks. Description of traffic gives the opportunities to monitor the Quality of Service parameters. It seems that simplicity and unpretentiousness of Erlang formulas can be their strong advantage against other methods for traffic description in asynchronous networks, but the more future research is necessary in this field.

Acknowledgement

This contribution is the partial result of the Research & Development Operational Program for the project Support of Center of Excellence for Smart Technologies, Systems and Services, ITMS 26240120005, co-funded by the ERDF.

References

[1] MULLER, N. J.: Desktop Encyclopedia of Telecommunications. Edition: 3, *McGraw-Hill Professional*, 2002, ISBN 0071381481.

[2] PUŽMANOVÁ, R.: Modern communication networks from A to Z. *Computer Press Brno*, 2006, ISBN 80-2511278-0.

[3] KLIKA, O.: Theory and experience of operation load in telecommunications. Praha, 1973, OS-31-047-73.

[4] COOPER, R. B.: Introduction to Queueing Theory: 2nd Edition, *Elsevier*, 1980, ISBN 0444010653.

[5] KRAUSE, M., TIPTON, H. F.: Information Security Management Handbook. Edition: 4, *CRC Press*, 1999, ISBN 0849308003.

[6] TAYLOR, T.: QoS in Packet Network. *Springer*, 2004, ISBN 038723389.

[7] HALÁS, M., KYRBASHOV, B.: New trend in IP telephony signalization protocols, In *8th conference KTTO 2008*, VŠB - Technical university of Ostrava, Czech Republic, 24-25.4.2008, pp. 67-69, ISBN 978-80-248-1719-4.

[8] JANATA, V.: Relation between MPLS and Access Network. In *8th Conference KTTO 2008*, VŠB - Technical university of Ostrava, Czech Republic, 24-25.04.2008, pp. 58-60, ISBN 978-80-248-1719-4.

[9] HALÁS, M., KYRBASHOV, B., KOVÁČIK, M.: Influence of Encryption on VoIP Connection Quality, In *RTT 2008. Research in Telecommunication Technology: 9th International Conference*. Vyhne, Slovak Republic, 10.-12.9.2008, pp. 50-53, ISBN 978-80-227-2939-0.

[10] CHROMÝ, E., MIŠUTH, T.: The Erlang Formulas and Traffic Description in Contact Centers. In: RTT 2009. Research in Telecommunication Technology : 11th International Conference. Srby, Czech Republic, 2.-4.9.2009. - Prague : CTU, 2009. - ISBN 978-80-01-04410-0

[11] QIAO, S. *A Robust and Efficient Algorithm for Evaluating Erlang B Formula*. Hamilton, Ontario Canada : McMaster University, 1998. http://www.cas.mcmaster.ca/~qiao/publications/erlang.pdf

[12] Diagnostic Strategies. *Traffic Modeling and Resource Allocation in Call Centers*. : Needham (Mass., USA) : Diagnostic Strategies, 2003. http://www.fer.hr/_download/repository/A4_1Traffic_Modeling.pdf.

[13] Westbay Engineers Ltd. *The Extended Erlang B Traffic Model*. : Westbay Engineers Ltd., 2008. http://www.erlang.com/calculatormanual/index.html?the_extended_erlang_b_traffic_model.htm

[14] FREEMAN, R.: Fundamentals of Telecommunications, Second Edition. John Wiley & Sons, Inc. Hoboken, New Jersey, IEEE Press, 2005, ISBN 0-471-71045-8