History of calculus

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This is a sub-article to Calculus and History of mathematics.

History of Calculus is part of the history of mathematics focused on limits, functions, derivatives, integrals, and infinite series. The subject, known historically as infinitesimal calculus, constitutes a major part of modern mathematics education. It has two major branches, **differential calculus** and **integral calculus**, which are related by the fundamental theorem of calculus. Calculus is the study of change, in the same way that geometry is the study of shape and algebra is the study of operations and their application to solving equations. A course in calculus is a gateway to other, more advanced courses in mathematics devoted to the study of functions and limits, broadly called mathematical analysis. Calculus has widespread applications in science, economics, and engineering and can solve many problems for which algebra alone is insufficient.

History of science

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Historiography
Pseudoscience

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In the Middle Ages
In the Renaissance
Scientific Revolution

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Development of calculus

Integral calculus

Calculating volumes and areas, the basic function of integral calculus, can be traced back to the Moscow papyrus (c. 1820 BC), in which an Egyptian mathematician successfully calculated the volume of a pyramidal frustum.[1][2]

Greek geometers are credited with a significant use of infinitesimals. Democritus is the first person recorded to consider seriously the division of objects into an infinite number of cross-sections, but his inability to rationalize discrete cross-sections with a cone's smooth slope prevented him from accepting the idea. At approximately the same time, Zeno of Elea discredited infinitesimals further by his articulation of the paradoxes which they create.

Antiphon and later Eudoxus are generally credited with implementing the method of exhaustion, which made it possible to compute the area and volume of regions and solids by breaking them up into an infinite number of recognizable shapes. Archimedes developed this method further, while also inventing heuristic methods which resemble modern day concepts somewhat. (See *Archimedes' Quadrature of the Parabola*, *The Method*, *Archimedes on Spheres & Cylinders*.[3]) It was not until the time of Newton that these methods were made obsolete. It should not be thought that infinitesimals were put on rigorous footing during this time, however. Only when it was supplemented by a proper geometric proof would Greek mathematicians accept a proposition as true.

In the third century Liu Hui wrote his *Nine Chapters* and also *Haidao suanjing* (*Sea Island Mathematical Manual*), which dealt with using the Pythagorean theorem (already stated in the *Nine Chapters*), known in China as the *Gougu* theorem, to measure the size of things. He discovered the usage of Cavalieri's principle to find an accurate formula for the volume of a cylinder, showing a grasp of elementary concepts associated with the differential and integral calculus. In the 11th century, the Chinese polymath, Shen Kuo, developed 'packing' equations that dealt with integration.

Indian mathematicians produced a number of works with some ideas of calculus. The formula for the sum of the cubes was first written by Aryabhata *circa* 500 AD, in order to find the volume of a cube, which was an important step in the development of integral calculus.[4]

The next major step in integral calculus came in the 11th century, when Ibn al-Haytham (known as *Alhacen* in Europe), an Iraqi mathematician working in Egypt, devised what is now known as "Alhazen's problem", which leads to an equation of the fourth degree, in his *Book of Optics*. While solving this problem, he was the first mathematician to derive the formula for the sum of the fourth powers, using a method that is readily generalizable for determining the general formula for the sum of any integral powers. He performed an integration in order to find the volume of a paraboloid, and was able to generalize his result for the integrals of polynomials up to the fourth degree. He thus came close to finding a general formula for the integrals of polynomials, but he was not concerned with any polynomials higher than the fourth degree.[4]

In the 17th century, Pierre de Fermat, among other things, is credited with an ingenious trick for evaluating the integral of any power function directly, thus providing a valuable clue to

Newton and Leibniz in their development of the fundamental theorem of calculus.[5] Fermat also obtained a technique for finding the centers of gravity of various plane and solid figures, which influenced further work in quadrature.

At around the same time, there was also a great deal of work being done by Japanese mathematicians, particularly Kowa Seki.[6] He made a number of contributions, namely in methods of determining areas of figures using integrals, extending the method of exhaustion. While these methods of finding areas were made largely obsolete by the development of the fundamental theorems by Newton and Leibniz, they still show that a sophisticated knowledge of mathematics existed in 17th century Japan.

Differential calculus

The Greek mathematician Archimedes was the first to find the tangent to a curve, other than a circle, in a method akin to differential calculus. While studying the spiral, he separated a point's motion into two components, one radial motion component and one circular motion component, and then continued to add the two component motions together thereby finding the tangent to the curve.[7]

The Indian mathematician-astronomer Aryabhata in 499 used a notion of infinitesimals and expressed an astronomical problem in the form of a basic differential equation.[8] Manjula, in the 10th century, elaborated on this differential equation in a commentary. This equation eventually led Bhāskara II in the 12th century to develop the concept of a derivative representing infinitesimal change, and he described an early form of "Rolle's theorem".[8][9][10]

In the late 12th century, the Persian mathematician, Sharaf al-Dīn al-Tūsī, introduced the idea of a function. In his analysis of the equation $x^3 + d = bx^2$ for example, he begins by changing the equation's form to $x^2(b - x) = d$. He then states that the question of whether the equation has a solution depends on whether or not the “function” on the left side reaches the value d. To determine this, he finds a maximum value for the function. Sharaf al-Din then states that if this value is less than d, there are no positive solutions; if it is equal to d, then there is one solution; and if it is greater than d, then there are two solutions. However, his work was never followed up on in either Europe or the Islamic world.[11]

Sharaf al-Dīn was also the first to discover the derivative of cubic polynomials.[12] His *Treatise on Equations* developed concepts related to differential calculus, such as the derivative function and the maxima and minima of curves, in order to solve cubic equations which may not have positive solutions. For example, in order to solve the equation $x^3 + a = bx$, al-Tusi finds the maximum point of the curve $y = bx - x^3$. He uses the derivative of the function to find that the maximum point occurs at $x = \sqrt{\frac{b}{3}}$, and then finds the maximum value for y at $2(\frac{b}{3})^{\frac{3}{2}}$ by substituting $x = \sqrt{\frac{b}{3}}$ back into $y = bx - x^3$. He finds that the equation $bx - x^3 = a$ has a solution if $a \leq 2(\frac{b}{3})^{\frac{3}{2}}$, and al-Tusi thus deduces that the equation has a positive root if $D = \frac{b^3}{27} - \frac{a^2}{4} \geq 0$, where D is the discriminant of the equation.[13]

In the 15th century, an early version of the mean value theorem was first described by Parameshvara (1370–1460) from the Kerala school of astronomy and mathematics in his commentaries on Govindasvāmi and Bhaskara II.[14]

In the 17th century, European mathematicians Isaac Barrow, Pierre de Fermat, Blaise Pascal, John Wallis and others discussed the idea of a derivative. In particular, in *Methodus ad disquirendam maximam et minima* and in *De tangentibus linearum curvarum*, Fermat developed a method for determining maxima, minima, and tangents to various curves that was equivalent to differentiation.[15] Isaac Newton would later write that his own early ideas about calculus came directly from "Fermat's way of drawing tangents."[16]

The first proof of Rolle's theorem was given by Michel Rolle in 1691 after the founding of modern calculus. The mean value theorem in its modern form was stated by Augustin Louis

Cauchy (1789-1857) also after the founding of modern calculus.

Mathematical analysis

Main article: Mathematical analysis

Greek mathematicians such as Eudoxus and Archimedes made informal use of the concepts of limits and convergence when they used the method of exhaustion to compute the area and volume of regions and solids.[17] In India, the 12th century mathematician Bhaskara II gave examples of the derivative and differential coefficient, along with a statement of what is now known as Rolle's theorem.

Mathematical analysis has its roots in work done by Madhava of Sangamagrama in the 14th century, along with later mathematician-astronomers of the Kerala school of astronomy and mathematics, who described special cases of Taylor series, including the Madhava-Gregory series of the arctangent, the Madhava-Newton power series of sine and cosine, and the infinite series of π.[18] *Yuktibhasa*, which some consider to be the first text on calculus, summarizes these results.[19][20][21]

It has recently been conjectured that the discoveries of the Kerala school of astronomy and mathematics were transmitted to Europe, though this is disputed.[22] (See Possibility of transmission of Kerala School results to Europe.)

In the 15th century, a German cardinal named Nicholas of Cusa argued that rules made for finite quantities lose their validity when applied to infinite ones, thus putting to rest Zeno's paradoxes.

Modern calculus

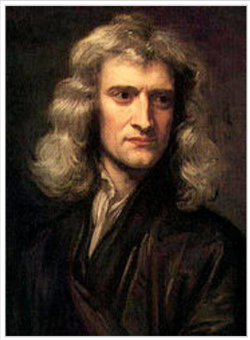
James Gregory was able to prove a restricted version of the second fundamental theorem of calculus in the mid-17th century.

Newton and Leibniz are usually credited with the invention of modern infinitesimal calculus in the late 17th century. Their most important contributions were the development of the fundamental theorem of calculus. Also, Leibniz did a great deal of work with developing consistent and useful notation and concepts. Newton was the first to organize the field into one consistent subject, and also provided some of the first and most important applications, especially of integral calculus.

Important contributions were also made by Barrow, Descartes, de Fermat, Huygens, Wallis and many others.

Newton and Leibniz

Before Newton and Leibniz, the word “calculus” was a general term used to refer to any body of mathematics, but in the following years, "calculus" became a popular term for a field of mathematics based upon their insights.[23] The purpose of this section is to examine Newton and Leibniz’s investigations into the developing field of infinitesimal calculus. Specific importance will be put on the justification and descriptive terms which they used in an attempt to understand calculus as they themselves conceived it.



Isaac Newton

Gottfried Leibniz

By the middle of the seventeenth century, European mathematics had changed its primary repository of knowledge. In comparison to the last century which maintained Hellenistic mathematics as the starting point for research, Newton, Leibniz and their contemporaries increasingly looked towards the works of more modern thinkers.[24] Europe had become home to a burgeoning mathematical community and with the advent of enhanced institutional and organizational bases a new level of organization and academic integration was being achieved. Importantly, however, the community lacked formalism; instead it consisted of a disordered mass of various methods, techniques, notations, theories, and paradoxes.

Newton came to calculus as part of his investigations in physics and geometry. He viewed calculus as the scientific description of the generation of motion and magnitudes. In comparison, Leibniz focused on the tangent problem and came to believe that calculus was a metaphysical explanation of change. These differences in approach should neither be overemphasized nor under appreciated. Importantly, the core of their insight was the formalization of the inverse properties between the integral and the differential. This insight had been anticipated by their predecessors, but they were the first to conceive calculus as a system in which new rhetoric and descriptive terms were created.[25] Their unique discoveries lay not only in their imagination, but also in their ability to synthesize the insights around them into a universal algorithmic process, thereby forming a new mathematical system.

Newton

Newton completed no definitive publication formalizing his Fluxional Calculus; rather, many of his mathematical discoveries were transmitted through correspondence, smaller papers or as embedded aspects in his other definitive compilations, such as the *Principia* and *Opticks*. Newton would begin his mathematical training as the chosen heir of Isaac Barrow in Oxford. His incredible aptitude was recognized early and he quickly learned the current theories. By 1664 Newton had made his first important contribution by advancing the binomial theorem, which he had extended to include fractional and negative exponents. Newton succeeded in expanding the applicability of the binomial theorem by applying the algebra of finite quantities in an analysis of infinite series. He showed a willingness to view infinite series not only as approximate devices, but also as alternative forms of expressing a term.[26]

Many of Newton's critical insights occurred during the plague years of 1665-1666 which he later described as, "the prime of my age for invention and minded mathematics and [natural] philosophy more than at any time since." It was during his plague-induced isolation that the first written conception of Fluxionary Calculus was recorded in the unpublished *De*

Analysi per Aequationes Numero Terminorum Infinitas. In this paper, Newton determined the area under a curve by first calculating a momentary rate of change and then extrapolating the total area. He began by reasoning about an indefinitely small triangle whose area is a function of x and y. He then reasoned that the infinitesimal increase in the abscissa will create a new formula where $x = x + o$ (importantly, o is the letter, not the digit 0). He then recalculated the area with the aid of the binomial theorem, removed all quantities containing the letter o and re-formed an algebraic expression for the area. Significantly, Newton would then “blot out” the quantities containing o because terms “multiplied by it will be nothing in respect to the rest”.

At this point Newton had begun to realize the central property of inversion. He had created an expression for the area under a curve by considering a momentary increase at a point. In effect, the fundamental theorem of calculus was built into his calculations. While his new formulation offered incredible potential, Newton was well aware of its logical limitations at the time. He admits that “errors are not to be disregarded in mathematics, no matter how small” and that what he had achieved was “shortly explained rather than accurately demonstrated.”

In an effort to give calculus a more rigorous explication and framework, Newton compiled in 1671 the *Methodus Fluxionum et Serierum Infinitarum*. In this book, Newton’s strict empiricism shaped and defined his Fluxional Calculus. He exploited instantaneous motion and infinitesimals informally. He used math as a methodological tool to explain the physical world. The base of Newton’s revised Calculus became continuity; as such he redefined his calculations in terms of continual flowing motion. For Newton, variable magnitudes are not aggregates of infinitesimal elements, but are generated by the indisputable fact of motion.

Newton attempted to avoid the use of the infinitesimal by forming calculations based on ratios of changes. In the Methodus Fluxionum he defined the rate of generated change as a fluxion, which he represented by a dotted letter, and the quantity generated he defined as a fluent. For example, if x and y are fluents, then $\dot{x}$ and $\dot{y}$ are their respective fluxions. This revised calculus of ratios continued to be developed and was maturely stated in the 1676 text *De Quadratura Curvarum* where Newton came to define the present day derivative as the ultimate ratio of change, which he defined as the ratio between evanescent increments (the ratio of fluxions) purely at the moment in question. Essentially, the ultimate ratio is the ratio as the increments vanish into nothingness. Importantly, Newton explained the existence of the ultimate ratio by appealing to motion;

“For by the ultimate velocity is meant that, with which the body is moved, neither before it arrives at its last place, when the motion ceases nor after but at the very instant when it arrives... the ultimate ratio of evanescent quantities is to be understood, the ratio of quantities not before they vanish, not after, but with which they vanish”[27]

Newton developed his Fluxional Calculus in an attempt to evade the informal use of infinitesimals in his calculations.

Leibniz

While Newton began development of his fluxional calculus in 1665-1666 his findings did not become widely circulated until later. In the intervening years Leibniz also strove to create his calculus. In comparison to Newton who came to math at an early age, Leibniz began his rigorous math studies with a mature intellect. He was a polymath, and his intellectual interests and achievements involved metaphysics, law, economics, politics, logic, and mathematics. In order to understand Leibniz’s reasoning in calculus his background should be kept in mind. Particularly, his metaphysics which considered the world as an infinite aggregate of indivisible monads and his plans of creating a precise formal logic whereby, “a general method in which all truths of the reason would be reduced to a kind of calculation.” In 1672 Leibniz met the mathematician Huygens who convinced Leibniz to dedicate significant time to the study of mathematics. By 1673 he had progressed to reading Pascal’s *Traité des Sinus du Quarte Cercle* and it was during his largely autodidactic research that Leibniz said a light turned on. Leibniz, like Newton, saw the tangent as a ratio but declared it as simply the ratio between ordinates and abscissas. He continued to argue that the integral was in fact the sum of the ordinates for infinitesimal intervals in the abscissa, in effect, a sum of an infinite number of rectangles. From these definitions the inverse relationship became clear and Leibniz quickly realized the potential to form a whole new system of mathematics. Where Newton shied away from the use of infinitesimals, Leibniz made it the cornerstone of his notation and calculus.

In the manuscripts of 25 October – 11 November 1675, Leibniz records his discoveries and experiments with various forms of notation. He is acutely aware of the notational terms used and his earlier plans to form a precise logical symbolism become evident. Eventually, Leibniz denotes the infinitesimal increments of abscissas and ordinates *dx* and *dy*, and the summation of infinitely many infinitesimally thin rectangles as a long s (ʃ), which became the present integral symbol $\int$.

Importantly, while Leibniz's notation is used by modern mathematics, his logical base was different than our current one. Leibniz embraced infinitesimals and wrote extensively so as, “not to make of the infinitely small a mystery, as had Pascal.” Towards this end he defined them “not as a simple and absolute zero, but as a relative zero... that is, as an evanescent quantity which yet retains the character of that which is disappearing.” Alternatively, he defines them as, “less then any given quantity” For Leibniz, the world was an aggregate of infinitesimal points and the lack of scientific proof for their existence did not trouble him. Infinitesimals to Leibniz were ideal quantities of a different type from appreciable numbers. The truth of continuity was proven by existence itself. For Leibniz the principle of continuity and thus the validity of his Calculus was assured. Three hundred years after Leibniz's work, Abraham Robinson showed that using infinitesimal quantities in calculus could be given a solid foundation.

The rise of Calculus stands out as a unique moment in mathematics. It is the math of motion and change and its invention required the creation of a new mathematical system. Importantly, Newton and Leibniz did not create the same Calculus and they did not conceive of modern Calculus. While they were both involved in the process of creating a mathematical system to deal with variable quantities their elementary base was different. For Newton, change was a variable quantity over time and for Leibniz it was the difference ranging over a sequence of infinitely close values. Notably, the descriptive terms each system created to describe change was different.

Historically, there was much debate over whether it was Newton or Leibniz who first "invented" calculus. This argument, the Leibniz and Newton calculus controversy, involving Leibniz, who was German, and the Englishman Newton, led to a rift in the European mathematical community lasting over a century. Leibniz was the first to publish his investigations; however, it is well established that Newton had started his work several years prior to Leibniz and had already developed a theory of tangents by the time Leibniz became interested in the question. Much of the controversy centers on the question whether Leibniz had seen certain early manuscripts of Newton before publishing his own memoirs on the subject. Newton began his work on calculus no later than 1666, and Leibniz did not begin his work until 1673. Leibniz visited England in 1673 and again in 1676, and was shown some of Newton's unpublished writings. He also corresponded with several English scientists (as well as with Newton himself), and may have gained access to Newton's manuscripts through them. It is not known how much this may have influenced Leibniz. The initial accusations were made by students and supporters of the two great scientists at the turn of the century, but after 1711 both of them became personally involved, accusing each other of plagiarism.

The priority dispute had an effect of separating English-speaking mathematicians from those in the continental Europe for many years and, consequently, slowing down the development of mathematical analysis. Only in the 1820s, due to the efforts of the Analytical Society, Leibnizian analytical calculus became accepted in England. Today, both Newton and

Leibniz are given credit for independently developing the basics of calculus. It is Leibniz, however, who is credited with giving the new discipline the name it is known by today: "calculus". Newton's name for it was "the science of fluents and fluxions".

The work of both Newton and Leibniz is reflected in the notation used today. Newton introduced the notation $\dot{f}$ for the derivative of a function f.[28] Leibniz introduced the symbol $\int$ for the integral and wrote the derivative of a function y of the variable x as $\frac{dy}{dx}$, both of which are still in use.

Integrals

Niels Henrik Abel seems to have been the first to consider in a general way the question as to what differential expressions can be integrated in a finite form by the aid of ordinary functions, an investigation extended by Liouville. Cauchy early undertook the general theory of determining definite integrals, and the subject has been prominent during the 19th century. Frullani's theorem (1821), Bierens de Haan's work on the theory (1862) and his elaborate tables (1867), Dirichlet's lectures (1858) embodied in Meyer's treatise (1871), and numerous memoirs of Legendre, Poisson, Plana, Raabe, Sohncke, Schlömilch, Elliott, Leudesdorf, and Kronecker are among the noteworthy contributions.

Eulerian integrals were first studied by Euler and afterwards investigated by Legendre, by whom they were classed as Eulerian integrals of the first and second species, as follows:

$$\int_0^1 x^{n-1}(1-x)^{n-1}dx$$
$$\int_0^\infty e^{-x}x^{n-1}dx$$

although these were not the exact forms of Euler's study.

If n is an integer, it follows that:

$$\int_0^\infty e^{-x}x^{n-1}dx = (n-1)!,$$

but the integral converges for all positive real n and defines an analytic continuation of the factorial function to all of the complex plane except for poles at zero and the negative integers. To it Legendre assigned the symbol Γ, and it is now called the gamma function. Besides being analytic over the positive reals, Γ also enjoys the uniquely defining property that $\log\Gamma$ is convex, which aesthetically justifies this analytic continuation of the factorial function over any other analytic continuation. To the subject Dirichlet has contributed an important theorem (Liouville, 1839), which has been elaborated by Liouville, Catalan, Leslie Ellis, and others. On the evaluation of $\Gamma(x)$ and $\log\Gamma(x)$ Raabe (1843-44), Bauer (1859), and Gudermann (1845) have written. Legendre's great table appeared in 1816.

Symbolic methods

Symbolic methods may be traced back to Taylor, and the much debated analogy between successive differentiation and ordinary exponentials had been observed by numerous writers before the nineteenth century. Arbogast (1800) was the first, however, to separate the symbol of operation from that of quantity in a differential equation. François (1812) and Servois (1814) seem to have been the first to give correct rules on the subject. Hargreave (1848) applied these methods in his memoir on differential equations, and Boole freely employed them. Grassmann and Hermann Hankel made great use of the theory, the former in studying equations, the latter in his theory of complex numbers.

Calculus of variations

The calculus of variations may be said to begin with a problem of Johann Bernoulli's (1696). It immediately occupied the attention of Jakob Bernoulli and the Marquis de l'Hôpital, but Euler first elaborated the subject. His contributions began in 1733, and his *Elementa Calculi Variationum* gave to the science its name. Lagrange contributed extensively to the theory, and Legendre (1786) laid down a method, not entirely satisfactory, for the discrimination of maxima and minima. To this discrimination Brunacci (1810), Gauss (1829), Poisson (1831), Ostrogradsky (1834), and Jacobi (1837) have been among the contributors. An important general work is that of Sarrus (1842) which was condensed and improved by Cauchy (1844). Other valuable treatises and memoirs have been written by Strauch (1849), Jellett (1850), Hesse (1857), Clebsch (1858), and Carll (1885), but perhaps the most important work of the century is that of Weierstrass. His celebrated course on the theory is epoch-making, and it may be asserted that he was the first to place it on a firm and unquestionable foundation.

Applications

The application of the infinitesimal calculus to problems in physics and astronomy was contemporary with the origin of the science. All through the eighteenth century these applications were multiplied, until at its close Laplace and Lagrange had brought the whole range of the study of forces into the realm of analysis. To Lagrange (1773) we owe the introduction of the theory of the potential into dynamics, although the name "potential function" and the fundamental memoir of the subject are due to Green (1827, printed in 1828). The name "potential" is due to Gauss (1840), and the distinction between potential and potential function to Clausius. With its development are connected the names of Dirichlet, Riemann, von Neumann, Heine, Kronecker, Lipschitz, Christoffel, Kirchhoff, Beltrami, and many of the leading physicists of the century.

It is impossible in this place to enter into the great variety of other applications of analysis to physical problems. Among them are the investigations of Euler on vibrating chords; Sophie Germain on elastic membranes; Poisson, Lamé, Saint-Venant, and Clebsch on the elasticity of three-dimensional bodies; Fourier on heat diffusion; Fresnel on light; Maxwell, Helmholtz, and Hertz on electricity; Hansen, Hill, and Gyldén on astronomy; Maxwell on spherical harmonics; Lord Rayleigh on acoustics; and the contributions of Dirichlet, Weber, Kirchhoff, F. Neumann, Lord Kelvin, Clausius, Bjerknes, MacCullagh, and Fuhrmann to physics in general. The labors of Helmholtz should be especially mentioned, since he contributed to the theories of dynamics, electricity, etc., and brought his great analytical powers to bear on the fundamental axioms of mechanics as well as on those of pure mathematics.

Furthermore, infinitesimal calculus was introduced into the social sciences, starting with Neoclassical economics. Today, it is a valuable tool in mainstream economics.

See also

- Analytic geometry
- Calculus
- Non-standard calculus

Notes

1. **^** There is no exact evidence on how it was done; some, including Morris Kline (*Mathematical thought from ancient to modern times* Vol. I) suggest trial and error.
2. **^** Helmer Aslaksen. Why Calculus? (http://www.math.nus.edu.sg/aslaksen/teaching/calculus.html) National University of Singapore.
3. **^** [1] (http://mathpages.com/home/kmath343.htm) MathPages - Archimedes on Spheres & Cylinders
4. ^ ***a b*** Victor J. Katz (1995), "Ideas of Calculus in Islam and India", *Mathematics Magazine* **68** (3): 163-174 [165-9 & 173-4]
5. **^** Paradís, Jaume; Pla, Josep; Viader, Pelagrí, *Fermat's Treatise On Quadrature: A New Reading* (http://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID848544_code386779.pdf?abstractid=848544&mirid=5) , http://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID848544_code386779.pdf?abstractid=848544&mirid=5, retrieved 2008-02-24
6. **^** http://www2.gol.com/users/coynerhm/0598rothman.html
7. **^** Boyer, Carl B. (1991). "Archimedes of Syracuse". *A History of Mathematics* (Second ed.). John Wiley & Sons, Inc.. pp. 127. ISBN 0471543977. "Greek mathematics sometimes has been described as essentially static, with little regard for the notion of variability; but Archimedes, in his study of the spiral, seems to have found the tangent to a curve through kinematic considerations akin to differential calculus. Thinking of a point on the spiral $r = a\theta$ as subjected to a double motion - a uniform radial motion away from

the origin of coordinates and a circular motion about the origin - he seems to have found (through the parallelogram of velocities) the direction of motion (hence of the tangent to the curve) by noting the resultant of the two component motions. This appears to be the first instance in which a tangent was found to a curve other than a circle.
Archimedes' study of the spiral, a curve that he ascribed to his friend Conon of Alexandria, was part of the Greek search for the solution of the three famous problems."

8. ^ ***a b*** George G. Joseph (2000). *The Crest of the Peacock*, p. 298-300. Princeton University Press. ISBN 0691006598.
9. **^** Ian G. Pearce. Bhaskaracharya II. (http://turnbull.mcs.st-and.ac.uk/~history/Projects/Pearce/Chapters/Ch8_5.html)
10. **^** Broadbent, T. A. A. (October 1968), "Reviewed work(s): *The History of Ancient Indian Mathematics* by C. N. Srinivasiengar", *The Mathematical Gazette* **52** (381): 307–8
11. **^** Victor J. Katz, Bill Barton (October 2007), "Stages in the History of Algebra with Implications for Teaching", *Educational Studies in Mathematics* (Springer Netherlands) **66** (2): 185–201 [192], doi:10.1007/s10649-006-9023-7 (http://dx.doi.org/10.1007%2Fs10649-006-9023-7)
12. **^** J. L. Berggren (1990). "Innovation and Tradition in Sharaf al-Din al-Tusi's Muadalat", *Journal of the American Oriental Society* **110** (2), p. 304-309.
13. **^** O'Connor, John J.; Robertson, Edmund F., "Sharaf al-Din al-Muzaffar al-Tusi (http://www-history.mcs.st-andrews.ac.uk/Biographies/Al-Tusi_Sharaf.html) ", *MacTutor History of Mathematics archive*, http://www-history.mcs.st-andrews.ac.uk/Biographies/Al-Tusi_Sharaf.html.
14. **^** J. J. O'Connor and E. F. Robertson (2000). Paramesvara (http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Paramesvara.html) , *MacTutor History of Mathematics archive*.
15. **^** Pellegrino, Dana. "Pierre de Fermat (http://www.math.rutgers.edu/~cherlin/History/Papers2000/pellegrino.html) ". http://www.math.rutgers.edu/~cherlin/History/Papers2000/pellegrino.html. Retrieved 2008-02-24.
16. **^** Simmons, George F. (2007). *Calculus Gems: Brief Lives and Memorable Mathematics*. Mathematical Association of America. p. 98. ISBN 0883855615.
17. **^** (Smith, 1958)
18. **^** "Madhava (http://www-gap.dcs.st-and.ac.uk/~history/Biographies/Madhava.html) ". *Biography of Madhava*. School of Mathematics and Statistics University of St Andrews, Scotland. http://www-gap.dcs.st-and.ac.uk/~history/Biographies/Madhava.html. Retrieved 2006-09-

13.
19. ^ "An overview of Indian mathematics (http://www-history.mcs.st-andrews.ac.uk/HistTopics/Indian_mathematics.html) ". *Indian Maths*. School of Mathematics and Statistics University of St Andrews, Scotland. http://www-history.mcs.st-andrews.ac.uk/HistTopics/Indian_mathematics.html. Retrieved 2006-07-07.
20. ^ "Science and technology in free India (http://www.kerala.gov.in/keralcallsep04/p22-24.pdf) " (PDF). *Government of Kerala — Kerala Call, September 2004*. Prof.C.G.Ramachandran Nair. http://www.kerala.gov.in/keralcallsep04/p22-24.pdf. Retrieved 2006-07-09.
21. ^ Charles Whish (1835). *Transactions of the Royal Asiatic Society of Great Britain and Ireland*.
22. ^ Almeida, D. F., John, J. K. and Zadorozhnyy, A. 2001. Keralese Mathematics: Its Possible Transmission to Europe and the Consequential Educational Implications. *Journal of Natural Geometry* 20, 77-104.
23. ^ Reyes, Mitchell. "The Rhetoric in Mathematics: Newton, Leibniz, the Calculus, and the Rhetorical Force of the Infinitesimal" Quarterly Journal of speech V.90(2004) Pg 160
24. ^ Such as Kepler, Descartes, Fermat, Pascal and Wallis. Source used; Calinger, Ronald. A Contextual History of Mathematics. (Toronto: Prentice- Hall Inc, 1999) Pg 556
25. ^ Foremost among these was Barrow who had created formulas for specific cases and Fermat who created a similar definition for the derivative. For more information; Boyer 184
26. ^ Calinger 610: Calinger, Ronald. *A Contextual History of Mathematics*. Toronto: Prentice-Hall Inc., 1999.
27. ^ *Principia*, Florian Cajori 8
28. ^ The use of prime to denote the derivative, $f'(x)$, is due to Lagrange.

Further reading

- Roero, C.S., 2005, "Gottfried Wilhelm Leibniz, first three papers on the calculus" in Ivor Grattan-Guinness, ed., *Landmark Writings in Western Mathematics 1640–1940*. Elsevier: 46-58.
- Boyer, Carl. The History of Calculus. New York: Dover Publications, 1949
- Calinger, Ronald. A Contextual History of Mathematics. Toronto: Prentice- Hall Inc, 1999.
- Reyes, Mitchell. "The Rhetoric in Mathematics: Newton,

Leibniz, the Calculus, and the Rhetorical Force of the Infinitesimal" Quarterly Journal of speech V.90(2004): 159-184.

External links

- A history of the calculus in The MacTutor History of Mathematics archive (http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The_rise_of_calculus.html) , 1996.
- Earliest Known Uses of Some of the Words of Mathematics: Calculus & Analysis (http://www.economics.soton.ac.uk/staff/aldrich/Calculus%20and%20Analysis%20Earliest%20Uses.h