ACCEPTANCE SINGLE SAMPLING PLAN USING VAGUE PARAMETERS

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ABSTRACT

The purpose of this paper is to present the acceptance single sampling plan when the fraction of nonconforming items is a vague number. We have shown that the operating characteristic (oc) curves of the plan whose vague values depends on the ambiguity proportion parameter in the lot when that sample size and acceptance numbers is fixed having a higher and lower bounds. Finally we have concluded the discussion by a numerical example, whose values are represented by a vague triangular set.

Keywords: Statistical Quality Control, Acceptance Single Sampling, Vague Number.

I INTRODUCTION

In the classical set theory introduced by Cantor, German mathematician values of elements in a set are only one of 0 and 1. That is, for any element there are only two possibilities in or not in the set. The theory cannot handle the data with ambiguity and uncertainty.

Zadeh proposed fuzzy theory in 1965. The most important feature is that fuzzy set A is a class of objects that satisfy a certain property each object x has a membership degree of A, denoted by $\mu_A(x)$. This membership function has the following characteristics: The single degree contains the evidence for both supporting and opposing x. It cannot only represent one of the two evidences but it cannot represent both at the same time too.

In order to deal with this problem, Gau and Buehrer proposed the concept of vague set in 1993 by replacing the values of an element in a set with a subinterval of [0, 1]. Namely a true membership function t_v(x) and false membership function f_v(x) are used to describe the boundaries of membership degree. These two boundaries form a subinterval (t_v(x), 1-f_v(x)) of [0, 1]. The vague set theory improves description of the object in real world, becoming a promising tool to deal with inexact, uncertain or vague knowledge. Many researchers have applied this theory to many situation such as fuzzy control decision making, knowledge discovery. And the tool has presented more challenging than with the fuzzy set theory in applications.

Statistical quality control is an efficient method of improving a firm and process quality of production. Sampling for acceptance or rejection of a lot is an important field in statistical quality control.

Acceptance single sampling is one of the sampling methods for acceptance or rejection which is long with classical attribute quality characteristic. In sampling plans, the fraction of defined items is considered as a crisp value but in practice, the fraction of defined items value must be known exactly. Many times these values are estimated or it is provided by experiment. The vagueness present in the value of p with personal judgement experiment or estimated may be treated by means of vague set theory. As known vague set theory is a powerful mathematical tool for modelling uncertainty with the evidence of both supporting and opposing, we define the imprecise proportion parameter as a vague number. With this definition, the number of non conforming items in the sample following a poisson distribution will have the vague parameter.

Classical acceptance sampling plans have been studied by many researchers. They are thoroughly selaborated by Schilling (1982). Single sampling by attributes with relaxed requirements were discussed by Ohta and Ichihashi (1988),and Grzegorzewski(1998,2001b). Grzegrozewski (2000b,2002) also considered sampling plan by variables with fuzzy requirements. Sampling plan by attributes for vague data were considered by Hrniewicz (1992).

The paper is organized as follows: section 2 gives the basic concepts of vague set theory and the arithmetic operations of triangular vague set. In section 3, we discuss about acceptance sampling plan with vague parameter. Section 4 deals with OC bands with vague parameter. Section 5 illustrates the above discussions with numerical examples and the conclusion is discussed in section 6.

II PRELIMINARIES AND DEFINITIONS

1.1 Basic Concepts of Vague Set.

Let U be the universe of discourse U {u_1, u_2, ..., u_n}. A vague set $\hat{A}$ [Chen and Shiy-Ming (2003), Lu.A and Nu.W (2004,2005)] in U is characterized by a truth membership function $t_{\hat{A}}$ $t_{\hat{A}}$: U→ $[0,1]$ and a false membership function $f_{\hat{A}}$ $f_{\hat{A}}$: U→ $[0,1]$, where $t_{\hat{A}}(u_i)$ is a lower bound of the grade of membership of u_i derived from the evidence for u_i. $f_{\hat{A}}(u_i)$ is a lower bound on the negation of grade of membership of u_i derived from the evidence against u_i such that $t_{\hat{A}}(u_i) + f_{\hat{A}}(u_i) \leq 1$. The grade of membership of u_i in the vague set $\hat{A}$ is bounded by a subinterval $[t_{\hat{A}}(u_i), 1 - f_{\hat{A}}(u_i)]$. For example, a vague set $\hat{A}$ in the universe of discourse U is shown in the figure 1:

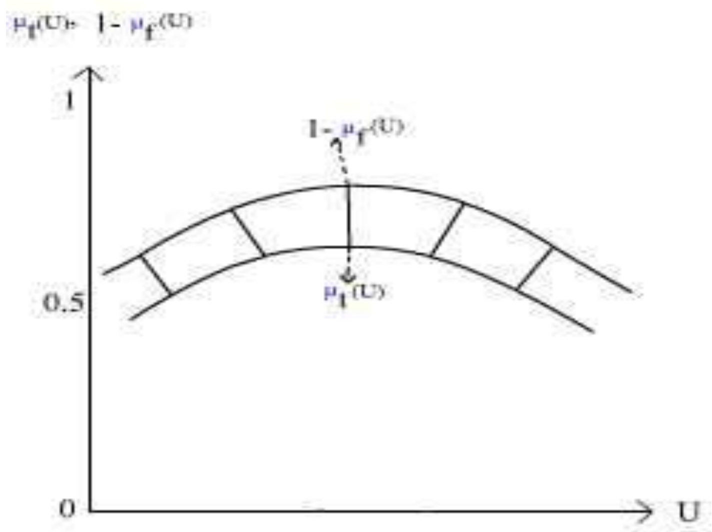

Fig 1: Vague Set

Definition 2.2.

Let $\hat{A}$ be a vague set of the universe of discourse U with truth membership function $t_{\hat{A}}$ and the false membership function $f_{\hat{A}}$ respectively. The vague set $\hat{A}$ is convex [Chen and Shiy-Ming (2003), Lu.A and Nu.W (2004,2005)], if and only if for every u_i in U,

$$t_{\hat{A}}(\lambda u_1 + (1-\lambda)u_2) \geq min\left(t_{\hat{A}}(u_1), t_{\hat{A}}(u_2)\right)$$

$$1 - f_{\hat{A}}(\lambda u_1 + (1-\lambda)u_2) \geq min\left(1 - f_{\hat{A}}(u_1), 1 - f_{\hat{A}}(u_2)\right), \text{ where } \lambda \in [0, 1].$$

Definition 2.3.

A vague set $\hat{A}$ of universe of discourse U is called a normal vague set [Chen and Shiy-Ming (2003), Lu.A and Nu.W (2004,2005)], if $u_i \in$ U, $1 - f_{\hat{A}(U)} = 1$. That is $f_{\hat{A}}(u_i)$=0.

Definition 2.4.

A vague number [Chen and Shiy-Ming (2003), Lu.A and Nu.W (2004,2005)] is a vague subset in the universe of discourse U that is both convex and normal.

In the following, we present the arithmetic operation of triangular vague set.

2.5 Arithmetic Operation of Triangular Vague sets.

Let us consider the triangular vague set $\hat{A}$, where the triangular vague set $\hat{A}$ can be parameterized by a tuple $\langle[(a,b,c);\mu_1],[(a,b,c);\mu_2]\rangle$, where μ_1 is the truth membership for (a,b,c) and μ_2 is the negation of false membership for (a,b,c). For convenience, the tuple can also be abbreviated into $\langle[(a,b,c);\mu_1,\mu_2]\rangle$, where $0\leq\mu1\leq\mu2\leq\mu3\leq\mu4\leq1$ are as follows:

$$\hat{A}\oplus\hat{B} = \langle[(a_1,b_1,c_1);\mu_1,\mu_2]\rangle\oplus\langle[(a_2,b_2,c_2);\mu_3,\mu_4]\rangle$$
$$= \langle[(a_1+a_2, b_1+b_2, c_1+c_2); min(\mu_1,\mu_3); min(\mu_2,\mu_4)]\rangle$$

$$\hat{A}\otimes\hat{B} = \langle[(a_1,b_1,c_1);\mu_1,\mu_2]\rangle\otimes\langle[(a_2,b_2,c_2);\mu_3,\mu_4]\rangle$$
$$= \langle[(a_1\times a_2, b_1\times b_2, c_1\times c_2); min(\mu_1,\mu_3); min(\mu_2,\mu_4)]\rangle$$

Based on the above two operations, we now define the multiplication of two vague matrices. Let $\hat{P}$ and $\hat{Q}$ be the two matrices whose entries are triangular vague sets represented as $\hat{p}_{ij} = \langle[(p_{ij}^1, p_{ij}^2, p_{ij}^3); \mu_{ij}^1; \mu_{ij}^2]\rangle$, where μ_{ij}^1 is the truth membership of $(p_{ij}^1, p_{ij}^2, p_{ij}^3)$, μ_{ij}^2 is the negation of false membership of $(p_{ij}^1, p_{ij}^2, p_{ij}^3)$ and $\hat{q}_{ij} = \langle[(q_{ij}^1, q_{ij}^2, q_{ij}^3); \mu_{ij}^3; \mu_{ij}^4]\rangle$, where μ_{ij}^3 is the truth membership of $(q_{ij}^1, q_{ij}^2, q_{ij}^3)$ and μ_{ij}^2 is the negation of false membership of $(q_{ij}^1, q_{ij}^2, q_{ij}^3)$ respectively given by

$$\hat{P} = (\hat{p}_{ij}) = \begin{pmatrix} \hat{p}_{11} & \hat{p}_{12} & \cdots & \hat{p}_{1n} \\ \hat{p}_{21} & \hat{p}_{22} & \cdots & \hat{p}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{p}_{n1} & \hat{p}_{n2} & \cdots & \hat{p}_{nn} \end{pmatrix} \qquad \hat{Q} = (\hat{q}_{ij}) = \begin{pmatrix} \hat{q}_{11} & \hat{q}_{12} & \cdots & \hat{q}_{1n} \\ \hat{q}_{21} & \hat{q}_{22} & \cdots & \hat{q}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{q}_{n1} & \hat{q}_{n2} & \cdots & \hat{q}_{nn} \end{pmatrix}.$$

Then the multiplication of $\hat{P}$ and $\hat{Q}$ is defined by $\hat{P}\otimes\hat{Q} = \underset{k}{\oplus}(\hat{p}_{ik}\otimes\hat{q}_{kj})$

Definition 2.6.

Let x be a random variable having the poisson mass function P(x) stands for the probability that xϵX, then P(x)=$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, x=0, 1, 2, ... and the parameter λ>0. Now if $\tilde{\lambda} > 0$ for λ is a fuzzy number, then $\tilde{p}$(x) to be fuzzy probability that X=x, we can find α-cut of this fuzzy number as $\tilde{p}(x) = \left\{\frac{e^{-\lambda}\lambda^x}{x!} / \lambda \epsilon \lambda[\alpha]\right.$ for every α∈[0, 1]. Let X be a random variable having the fuzzy probability distribution and $\tilde{p}$ in the definition (1) be small, which means that all p∈$\tilde{p}$[α] are sufficiently small, then $\tilde{p}$[a, b][α] is given by

$$\tilde{p}[a, b][\alpha] = \left\{\sum_{x=a}^{b} \frac{e^{-\lambda}\lambda^x}{x!} / \lambda \in n\tilde{p}[\alpha]\right\}$$

III ACCEPTANCE SAMPLING PLAN WITH VAGUE PARAMETER

Suppose that we want to inspect a lot with a large size of N. First take a randomized sample of size 'n' from the lot, then inspect all items in the sample and the number of defective items (d) will be count down. If the number of observed defective items is lesser than equal to acceptance number then the lot will be accepted, otherwise the lot rejection. If the size of lot be large the random variable d had a binomial distribution with parameter n and p in which p indicates the lot s defective items. However there exists the size of sample be large and p is small then random variable'd' has a Poisson distribution with λ=np. So, the probability for the number of defective items to be exactly equal to d is

$$p(d) = \frac{e^{-np}(np)^d}{d!}$$

and the probability of acceptance of the lot (p_a) is

$$p_a = p(d \le c) = \sum_{d=0}^{c} \frac{e^{-np}(np)^d}{d!}.$$

Suppose we want to inspect a lot with the large size of N, such that the proportion of damaged item is not known precisely, so we represent this parameter with a vague number $\hat{p}$ with the evidence of both favourable and non favourable as follows: $\hat{p} = \{(p_1, p_2, p_3), \mu_t, 1 - \mu_f\}$.

A single sampling plan with a vague parameter if defined by the sample size n and acceptance number c and if the number of observation defective product is less than or equal to c, the lot will be acceptance. If N is a large number, then the number of defective items in this sample (d) has a binomial distribution with a vague value and if $\hat{P}$ is a small, then random variable (d) has a vague value whose parameter $\hat{\lambda}$=n$\hat{P}$ and so the vague probability for number of defective items in a sample size that is exactly equal to d is defined by means of α-cut given by

$\hat{P}$(d-defective)=$\left\{\hat{P}(\alpha), \mu_{t_{\hat{p}(\alpha)}}, 1 - \mu_{f_{\hat{p}(\alpha)}}\right\}$, where $\hat{P}(\alpha) = [P^1[\alpha], P^2[\alpha]]$ and $P^1[\alpha] = min\left\{\frac{e^{-\lambda}\lambda^d}{d!} | \lambda \epsilon n\hat{P}(\alpha)\right\}$

$$P^2[\alpha] = max\left\{\frac{e^{-\lambda}\lambda^d}{d!} | \lambda \epsilon n\hat{P}(\alpha)\right\}$$

and the vague acceptance probability is given by

$$\hat{P} = \left\{\tilde{P}_a, \mu_{t_{\hat{p}_a}}, 1 - \mu_{f_{\hat{p}_a}}\right\}$$

Where $\tilde{P}_a$ is a triangular number $\tilde{P}_a = (P_a^1, P_a^2, P_a^3)$ defined by $\tilde{P}_a = \left\{\sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \mid \lambda \epsilon \tilde{\lambda}[\alpha]\right\}$.

This triangular number $\tilde{P}_a$ can be determined by means of α-cut,

$\tilde{P}_a(\alpha) = [P^1{}_a(\alpha), P^2{}_a(\alpha)]$ and

$$P_a^1(\alpha) = min\left\{\sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \mid \lambda \epsilon \tilde{\lambda}(\alpha)\right\}$$

$$P_a^2(\alpha) = max\left\{\sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \mid \lambda \epsilon \tilde{\lambda}(\alpha)\right\}$$

IV OC-BAND WITH VAGUE PARAMETER

Operating characteristic curve is one of the important criteria in the sampling plan. By this curve one could be determined the probability of acceptance or rejection of a lot having some specific defective items. The OC curve represents the performance of the acceptance sampling plans by plotting the probability of acceptance a lot versus its production quality which is expressed by the proportion of non conforming items in the lot [B.P.M. Duate, P.M. Saraiva (2008)]. OC curve aids in selection of plans that are effective in reducing risk and indicates discriminating power of the plan.

Suppose that the event A is the event of acceptance of a lot. Then the vague probability of acceptance a lot in terms of fraction of defective items having vague values represented as a triangular vague value. The uncertainty degree of a proportion parameter is one of the factors that bandwidth depends on that. Knowing the uncertainty degree of proportion parameter with the evidence of acceptance membership value and non acceptance membership value and the variation of its position as horizontal axis we have different vague number $\hat{P} = \left(\tilde{P}, \mu_{t_{\tilde{P}}}, 1 - \mu_{f_{\tilde{P}}}\right)$ and hence we will have different proportion (P) which the OC bands are plotted in terms of it. To achieve this aim, we consider the structure of $\hat{P}$ as follows:

$\hat{P} = \left(\tilde{P}, \mu_{t_{\tilde{P}}}, 1 - \mu_{f_{\tilde{P}}}\right)$, where $\tilde{P} = (k, a_2 + k, a_3 + k)$ which can be obtained by using α-cuts.

Let $\hat{\lambda} = \left(\tilde{\lambda}, \mu_{t_{\tilde{P}}}, 1 - \mu_{f_{\tilde{P}}}\right)$. Then $\hat{\lambda} = n\hat{P} = \left((nk_1, na_2 + nk, na_3 + nk), \mu_{t_{\tilde{\lambda}}}, 1 - \mu_{f_{\tilde{\lambda}}}\right)$ with which variation k in the domain of the interval between 0 and 1- a_3. The OC band is plotted according to the following calculation:

The triangular number $\tilde{P}$ can be obtained by using α-cut as follows:

$\tilde{P}[\alpha] = [P_1(\alpha), P_2(\alpha)]$

$$= [k + a_2\alpha, a_3 + k - (a_3 - a_2)\alpha]$$

and $\tilde{\lambda}$ are obtained as $\tilde{\lambda}[\alpha] = [\lambda_1(\alpha), \lambda_2(\alpha)]$

$= [nk + na_2\alpha, na_3 + nk - n(a_3 - a_2)\alpha]$ using α-cuts,

$\therefore \hat{P}_a = \left(\tilde{P}_a, \mu_{t_{\tilde{P}_a}}, 1 - \mu_{f_{\tilde{P}_a}}\right)$, where $\tilde{P}_a = \left[P_{a_1}[\alpha], P_{a_2}[\alpha]\right]$

$$= \left[min\left\{\sum_A \frac{e^{-\lambda}\lambda^d}{d!} \mid \lambda \epsilon \tilde{\lambda}[\alpha]\right\}, max\left\{\sum_A \frac{e^{-\lambda}\lambda^d}{d!} \mid \lambda \epsilon \tilde{\lambda}[\alpha]\right\}\right]$$

V EXAMPLE

Example 5.1.

Let us assume that experience of a management company shows that half percent are ill-packed. Major customers choose and inspect 60 items of this product available in a large store to buy them. If the number of non conforming items in this sample equals zero or one, the customers will buy all products in the store. If the non confirming increases, the customer will not buy them. Because of the proportion of defective products has explained linguistically we can consider that as a vague number $\widehat{P} = \left((0,0.005,0.01), 0.9,0.85\right)$.

Therefore the probability of purchasing will be described in the following:

If n=60, c=1, then $\widehat{P} = \left((0,0.005,0.01), 0.9,0.85\right)$

$\hat{\lambda} = \left((0,0.3,0.6), 0.85,0.8\right)$ = ([0.3α,0.6-0.3α],0.85,0.8) and

$\widehat{P_a} = \left(\left[(1.6 - 0.3\alpha)e^{-(0.6-0.3\alpha)}, (1 + 0.3\alpha)e^{-0.3\alpha}\right], 0.85,0.8\right)$.

Using 0-cut and 1-cut, we get $\widehat{P_a} = \left((0.8781, 0.9391,1), 0.9,0.8\right)$ means that having more evidence of acceptance memberships, it is expected that for every 100 lots in such a process 88 to 100 lots will be accepted. In this example, if we take c = 1 for a_2 = 0.005, a_3 = 0.01, then we have at 0 - cut

$\hat{\lambda} = ([nk, nk + 0.01n]; 0.85; 0.8), 0 < k < 0.99$ and

$\widehat{P_a} = \left(\left[(1 + nk + 0.01n)\, e^{-(nk+0.01n)}, (1 + nk)\, e^{-nk}\right]; 0.9; 0.8\right)$

Table 5.1: Vague Probability of Acceptance

k	$\widehat{P}$	$\widehat{P_a}$
0	<(0,0.05,0.1);0.9;0.85>	<(0.8781,0.939,1);0.9;0.8>
0.01	<(0.01,0.015,0.02);0.9;0.85>	<(0.6626,0.7735,0.8781);0.9;0.85>
0.02	<(0.02,0.025,0.03);0.9;0.85>	<(0.4628,0.5627,0.66260;0.9;0.8>
0.03	<(0.03,0.035,0.04);0.9;0.85>	<(0.3084,0.3856,0.4628);0.9;0.8>
0.04	<(0.04,0.045,0.05);0.9;0.85>	<(0.1991,0.2575,0.4628);0.9;0.8>
0.05	<(0.05,0.055,0.06);0.9;0.85>	<(0.1257,0.16240.1991);0.9;0.8>

Fig 2 shows the OC band of the example. This figure represents that then the OC band having high evidence of grade of membership and negation of grade of membership against the evidence, when the process quality decrease from a very good state to a moderate state.

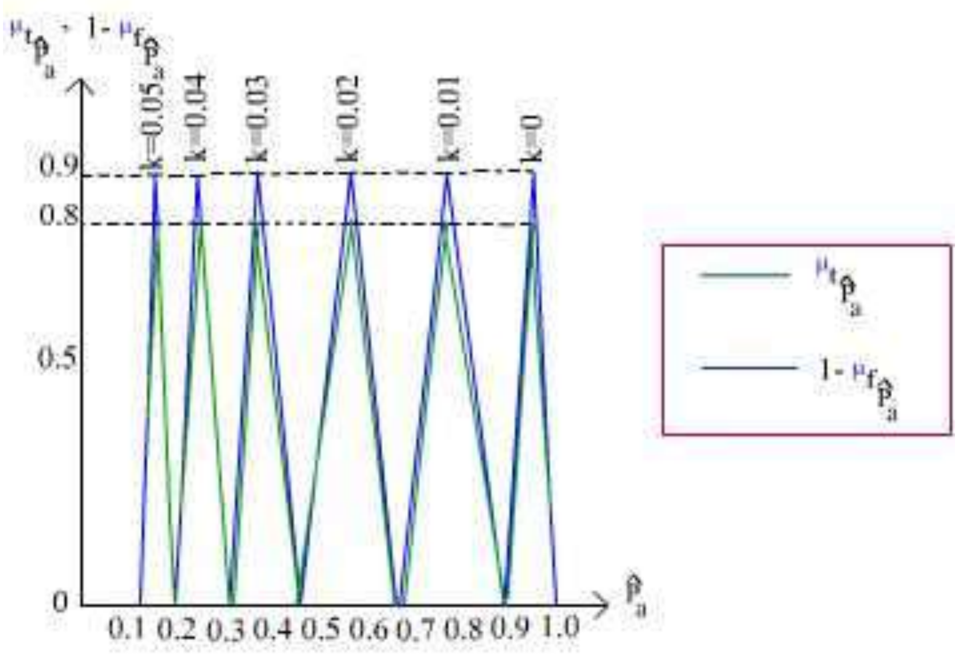

Fig 2: OC Band for a single Sampling Plan with Vague Parameter

Example 5.2: Suppose that c = 0, n = 20 in example 5. 1, then we have $a_2 = 0.005$, $a_3 = 0.01$

$\hat{P} = \langle (k, k+0.005, k+0.01); 0.8; 0.9 \rangle$ and $\hat{\lambda} = \langle (20k, 20k+0.1, 20k+0.2); 0.85; 0.95 \rangle$, for $0 \le k \le 0.99$.

Therefore OC curve in terms of vague probabilities is as follows: $\hat{P}_{ab} = \langle (e^{-(0.2+20k)}, (0.5)(e^{-(0.2+20k)} + e^{-(20k)}), e^{-20k}); 0.85; 0.9 \rangle$

Table 5.2: Vague Probability of Acceptance

k	$\hat{P}_{ab}$
0	<(0.8787,0.9094,1);0.85;0.9>
0.01	<(0.6703,0.7445,0.8787);0.85;0.9>
0.02	<(0.5488,0.6095,0.6703;0.85;0.9>
0.03	<(0.4493,0.4991,0.5488);0.85;0.9>

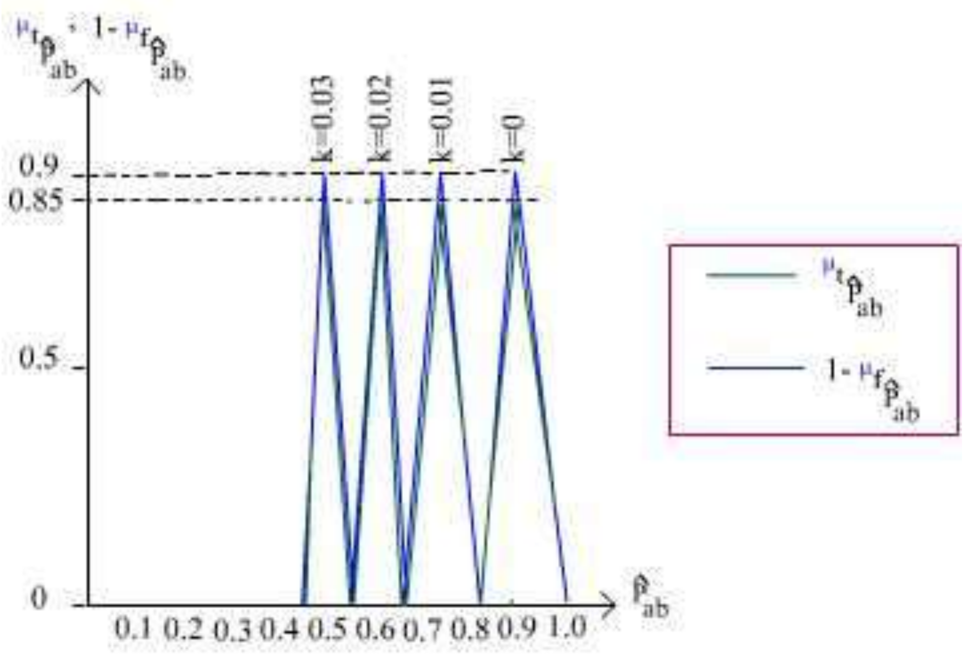

Fig 3: OC Band for a single Sampling Plan with Vague Parameter

The above figure shows OC band for n = 20 indicating that 0.85 is the lower bound for the membership of evidence for the OC bands and 0.9 is the upper bound for the negation of membership against the evidence for the vague probability of acceptance for proportion of defective items whose flow follows reduction of values and it will be more the increase of n.

VI CONCLUSION

In this paper, we have proposed a method for designing acceptance single sampling plan with vague quality characteristic using Poisson distribution represented as triangular vague probabilities. As it was shown that OC curves of the plan is like a band having a higher and lower bounds, we had shown that we made an attempt to capture the vagueness of the plan of OC bands.

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