Transient State Analysis of an Active Magnetic Bearing (AMB) System with Six Degree Of Freedoms Using MATLAB

Dr.–Ing. Ogbonnaya I. Okoro

Department of Electrical Engineering, University of Nigeria, Nsukka, Enugu State.
E-mail: oiokoro@yahoo.co.ukT

ABSTRACT

The control of an AMB system becomes imperative if the eccentricity of the rotor about its operating point is to be sustained without unduly dragging the entire system into a state of instability. For a single-input-single-output (SISO) design, only one degree of freedom is actively controlled. However, practical applications will require multiple-input-multiple-output (MIMO) control which consequently will require the study of the dynamic behavior of the system with the system variables expressed in state variable form.

This paper presents the dynamic analysis of an AMB system with the view of studying the behavior of the state variables in transient state. Insight into the response curves will indeed form the basis for studying the system control with six degree of freedoms (6-DOFs).

(Key words: control, rotor, active magnetic bearing, transient state, degree of freedoms).

INTRODUCTION

The availability of components for power electronics and information processing has made research in control design and in modeling the dynamics of the rotor an interesting feature. In 1975, Schweitzer [1] proposed theoretical and experimental solutions for active damping of self-excited vibrations of centrifuges.

Essential contributions for the introduction of magnetic bearings to industrial applications have been reported by Higuchi [2, 3]. The most frequently used bearing type, the electromagnetic bearing, has some specific features which render it particularly useful for some applications, such as [4]:

- Low bearing losses at high operating speeds.
- Lower maintenance costs and higher life time.
- Rotor can be allowed to rotate at high speeds.
- Dynamic of the contact-free hovering depends mainly on the implemented control law.

A rigid body has six degree of freedoms of motion. If the body is elastic, the number of degree of freedoms becomes infinite. Most research works have assumed one or two degree of freedoms for an active magnetic bearing [5, 6]. However, this method of analysis does not adequately represent the dynamic behavior of the active magnetic bearing system, neither does it take into consideration the various factors that affect the system dynamic performance in the other degree of freedoms. Also, in practice, the method has proved to be inadequate for control applications involving rotor stability about its operating points.

In this paper, therefore, the dynamic behavior of an active magnetic bearing system with six degree of freedoms, as shown in Figure 1, is proposed. The mathematical model representing the dynamic performance of the system is developed and the system variables expressed in state variable form. In this way, the entire analysis is fashioned in such a way that it is amenable to computer simulation. The simulation results are then presented and discussed.

MATHEMATICAL MODEL DEVELOPMENT

The rotor in Figure 1 is supported radially in two bearings—Bearing A and Bearing B. In order to develop the mathematical model of the system,

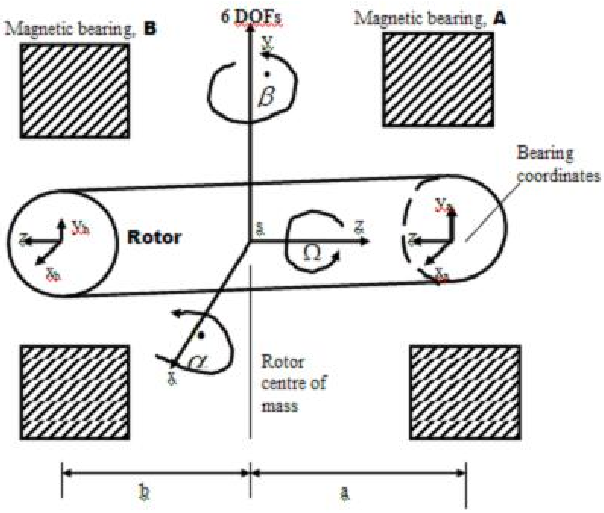

Figure 1: The Proposed Magnetic Bearing System.

the following assumptions are made [7]:

- When the rotor is at rest, its center of mass (s) coincides with the origin of the position of the inertially fixed coordinate system.
- Deviations from the reference position are small compared to the rotor dimensions.
- The rotor is rigid and symmetric.

The small movements of the rotor are described by displacements x_a, x_b, y_a, y_b of its center of mass (s) with respect to its inclinations α, β, and Ω about the x-, y-, and z-axis. The constant angular velocity of the rotor about its longitudinal axis (z) is Ω. The equations of motion for the magnetic bearing system become:

$$I_x \ddot{\alpha} + I_z \Omega \dot{\beta} = bF_{yb} - aF_{ya} \quad \text{(1)}$$

$$I_y \ddot{\beta} - I_z \Omega \dot{\alpha} = aF_{xa} - bF_{xb} \quad \text{(2)}$$

$$m\ddot{x} = F_{xa} - F_{xb} \quad \text{(3)}$$

$$m\ddot{y} = F_{ya} + F_{yb} \quad \text{(4)}$$

Assumption two connotes that the angle (α) should be small ($\sin\alpha \approx \alpha$) resulting in the following relationships:

$$\alpha = \frac{1}{a+b}(y_b - y_a) \quad \text{(5)}$$

$$\beta = \frac{1}{a+b}(x_a - x_b) \quad \text{(6)}$$

$$y = \frac{1}{2}(y_a + y_b) \quad \text{(7)}$$

$$x = \frac{1}{2}(x_a + x_b) \quad \text{(8)}$$

Combining equations (5-8) with equations (1-4) yields:

$$\ddot{x}_a = -A_o \frac{I_z}{I_y}\Omega \dot{y}_a + A_o \frac{I_z}{I_y}\Omega \dot{y}_b + \left(\frac{1}{m} + \frac{a^2}{I_y}\right)F_{xa} + \left(\frac{1}{m} - \frac{ab}{I_y}\right)F_{xb} \quad \text{(9)}$$

$$\ddot{x}_b = B_o \frac{I_z}{I_y}\Omega \dot{y}_a - B_o \frac{I_z}{I_y}\Omega \dot{y}_b + \left(\frac{1}{m} - \frac{ab}{I_y}\right)F_{xa} + \left(\frac{1}{m} + \frac{b^2}{I_y}\right)F_{xb} \quad \text{(10)}$$

$$\ddot{y}_a = A_o \frac{I_z}{I_y}\Omega \dot{y}_a - A_o \frac{I_z}{I_y}\Omega \dot{x}_b + \left(\frac{1}{m} + \frac{a^2}{I_x}\right)F_{ya} + \left(\frac{1}{m} - \frac{ab}{I_x}\right)F_{yb} \quad \text{(11)}$$

$$\ddot{y}_b = -B_o \frac{I_z}{I_x}\Omega \dot{x}_a + B_o \frac{I_z}{I_x}\Omega \dot{x}_b + \left(\frac{1}{m} - \frac{ab}{I_x}\right)F_{ya} + \left(\frac{1}{m} + \frac{b^2}{I_x}\right)F_{yb} \quad \text{(12)}$$

Where,

$$A_o = \frac{a}{a+b} \quad \text{(13a)}$$

$$B_o = \frac{b}{a+b} \quad \text{(13b)}$$

The moment of inertia of the system becomes [8]:

$$I_x = \frac{m}{4}r^2 + \frac{m}{12}L_s^2 \quad \text{(14)}$$

$$I_y = \frac{m}{4} r^2 + \frac{m}{12} L_s^2 \quad \textbf{(15)}$$

$$I_z = \frac{m}{2} r^2 \quad \textbf{(16)}$$

The electrical model of the system gives:

$$\frac{di(x,y)}{dt} = \frac{U}{L} - \frac{R}{L} i(x,y) - \frac{Ki}{L} \dot{x}(x,y) \quad \textbf{(17a)}$$

and the magnetic force,

$$F(x,i) = K_s x + K_i i(x,y) \quad \textbf{(17b)}$$

Where,

U = voltage
R = Resistance
L = inductance
K_i = Force-current factor
K_s = Force-displacement factor

STATE VARIABLES REPRESENTATION

The merit of the state-variable method is that it results easily to the form amenable to digital and/or analog computer methods of solution [9].

The proposed magnetic bearing system can be represented in state-variable form as:

$$\dot{x} = Ax + Bu + \beta uVv + \beta gVg + \beta zVz \quad \textbf{(18)}$$

Where,

$$x = \begin{bmatrix} x_a & x_b & y_a & y_b & \dot{x}_a & \dot{x}_b & \dot{y}_a & \dot{y}_b \end{bmatrix}^t \quad \textbf{(19)}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_a & \dot{x}_b & \dot{y}_a & \dot{y}_b & \ddot{x}_a & \ddot{x}_b & \ddot{y}_a & \ddot{y}_b \end{bmatrix}^t \quad \textbf{(20)}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -A_o \frac{I_z}{I_y}\Omega & A_o \frac{I_z}{I_y}\Omega \\ 0 & 0 & 0 & 0 & 0 & 0 & B_o \frac{I_z}{I_y}\Omega & -B_o \frac{I_z}{I_y}\Omega \\ 0 & 0 & 0 & 0 & A_o \frac{I_z}{I_x}\Omega & -A_o \frac{I_z}{I_x}\Omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -B_o \frac{I_z}{I_x}\Omega & B_o \frac{I_z}{I_x}\Omega & 0 & 0 \end{bmatrix} \quad \textbf{(21)}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} + \frac{a^2}{I_y}\right) & \left(\frac{1}{m} - \frac{ab}{I_y}\right) & 0 & 0 \\ \left(\frac{1}{m} - \frac{ab}{I_y}\right) & \left(\frac{1}{m} + \frac{b^2}{I_y}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{1}{m} + \frac{a^2}{I_x}\right) & \left(\frac{1}{m} - \frac{ab}{I_x}\right) \\ 0 & 0 & \left(\frac{1}{m} - \frac{ab}{I_x}\right) & \left(\frac{1}{m} + \frac{b^2}{I_x}\right) \end{bmatrix} \quad \textbf{(22)}$$

$$\mathbf{u} = \begin{bmatrix} F_{xa} & F_{xb} & F_{ya} & F_{yb} \end{bmatrix}^t \quad \textbf{(23)}$$

$$\boldsymbol{\beta}\mathbf{u} = \Omega^2 \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \left(-a\frac{I_{yz}}{I_y} - ey\right) & \left(a\frac{I_{xz}}{I_y} + ex\right) \\ \left(b\frac{I_{yz} - ey}{Iy}\right) & \left(-b\frac{I_{xz}}{I_y} + ex\right) \\ \left(a\frac{I_{xz}}{I_x} + ex\right) & \left(a\frac{I_{yz}}{I_x} + ey\right) \\ \left(-b\frac{I_{xz}}{I_x} + ex\right) & \left(-b\frac{I_{yz}}{I_x} + ey\right) \end{bmatrix} \quad \textbf{(24)}$$

$$\mathbf{vu} = \begin{bmatrix} \cos(\Omega t) \\ \sin(\Omega t) \end{bmatrix} \quad \text{.......} \quad \textbf{(25)}$$

$$\mathbf{vz} = \begin{bmatrix} F_{zxa} & F_{zxb} & F_{zya} & F_{zyb} \end{bmatrix}^t \quad \text{.......} \quad \textbf{(26)}$$

$$\boldsymbol{\beta}\mathbf{g} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dfrac{-1}{m} & \dfrac{-1}{m} \end{bmatrix}^t \quad \textbf{(27)}$$

$$vg = mg \quad \text{.......} \quad \textbf{(28)}$$

$$\boldsymbol{\beta}\mathbf{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \left(\dfrac{1}{m} + \dfrac{a^2}{I_y}\right) & \left(\dfrac{1}{m} - \dfrac{ab}{I_y}\right) & 0 & 0 \\ \left(\dfrac{1}{m} - \dfrac{ab}{I_y}\right) & \left(\dfrac{1}{m} + \dfrac{b^2}{I_y}\right) & 0 & 0 \\ 0 & 0 & \left(\dfrac{1}{m} + \dfrac{a^2}{I_x}\right) & \left(\dfrac{1}{m} - \dfrac{ab}{I_x}\right) \\ 0 & 0 & \left(\dfrac{1}{m} - \dfrac{ab}{I_x}\right) & \left(\dfrac{1}{m} + \dfrac{b^2}{I_x}\right) \end{bmatrix} \quad \textbf{(29)}$$

SIMULATION AND RESULTS

Table 1 shows the active magnetic bearing parameters used for the simulation.

In order to study the transient behavior of the active magnetic bearing system, the state-variable form of equation (18), the electrical model, and magnetic force of equation (17) were solved numerically using the fourth-order Runge-Kutta method [10]. By incorporating the developed algorithm into the MATLAB m-file [11], the system time response curves for the displacement, velocity, acceleration, current and force at various coordinate systems are developed. The displacement–time graphs are shown in Figures (2-5) while Figures (6-9) show the velocity-time graphs for the proposed system. The time function of the system currents and forces are shown in Figures (10-13) and Figures (14-17) respectively.

Table 1: Magnetic Bearing Parameters.

Force-displacement factor, K_s	1109.0N/m
Force-current ratio, K_i	125N/A
Mass of rotor, m	63.48Kg
Length of rotor; Ls	859mm
Rotor radius, r	99mm
Rotor speed, Ω	50s^{-1}
Distance between the rotor centre of mass and the middle of the magnetic bearing A, a	350mm
Distance between the rotor centre of mass and the middle of the magnetic bearing B, b	350mm
Displacement of the rotor from the centre of mass(x-axis),ex	0.1mm
Displacement of the rotor from the centre of mass(y-axis),ey	0.1mm
Products of inertia, I_{xz}	0.1Kgm^2
Products of inertia, I_{xyz}	0.1Kgm^2
Moment of inertia, I_x	4.05Kgm2
Moment of inertia, I_y	4.05Kgm2
Moment of inertia, I_x	0.31Kgm2
Coupling factor, $\dfrac{I_z}{I_y}$	0.076
Inductance, L	10.3mH
Resistance, R	0.5Ω
Voltage, U	1.0V

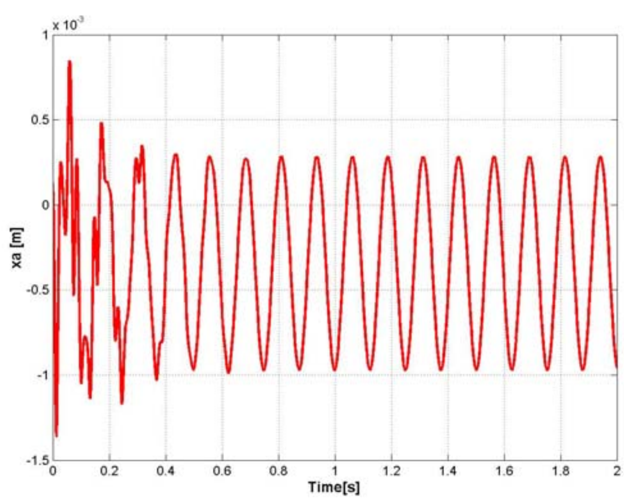

Figure 2: Graph of Displacement, x_a against Time.

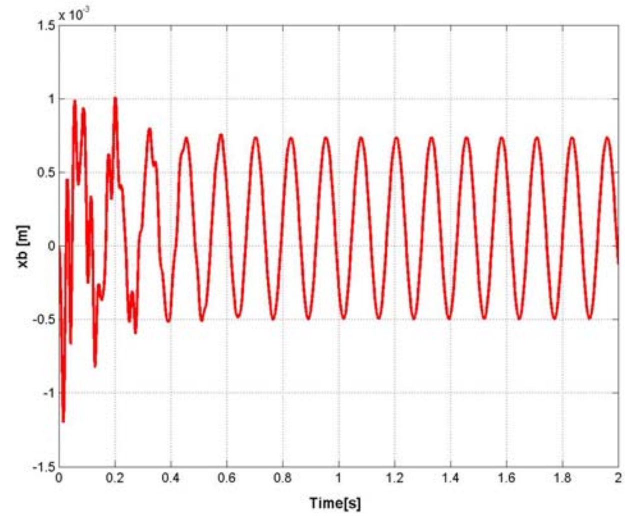

Figure 3: Graph of Displacement, x_b against Time.

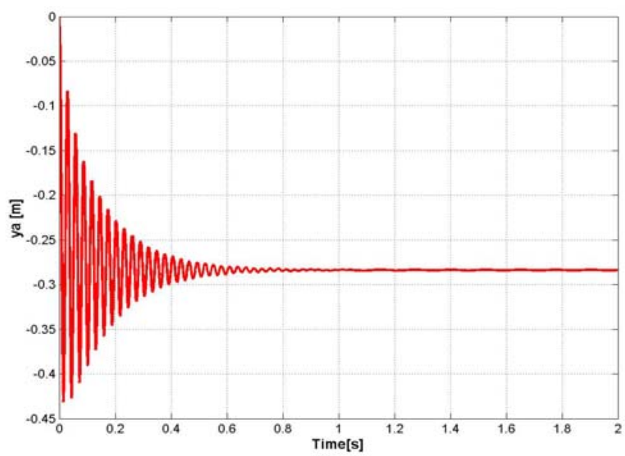

Figure 4: Graph of Displacement, y_a against Time.

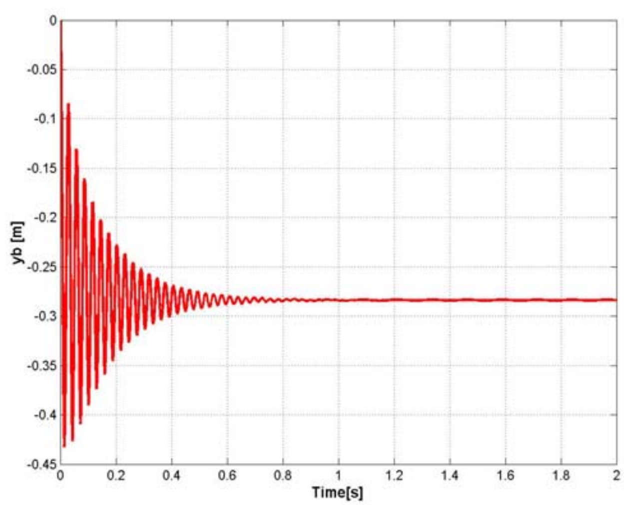

Figure 5: Graph of Displacement, y_b against Time.

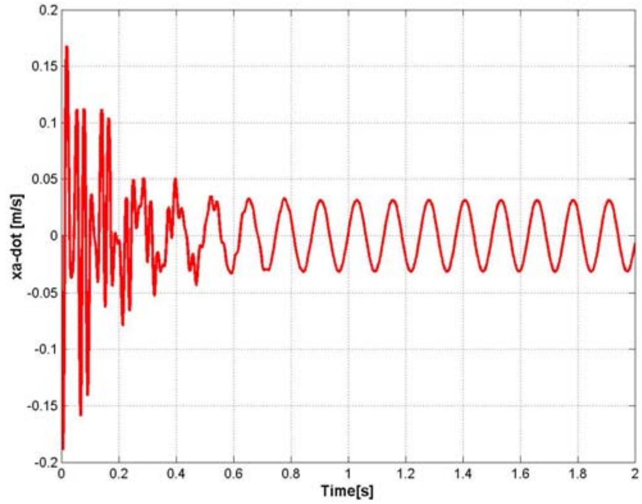

Figure 6: Graph of Velocity, x_a_dot against Time.

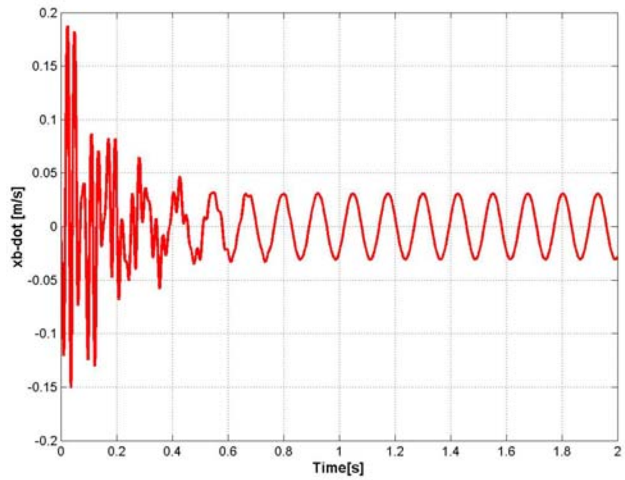

Figure 7: Graph of Velocity, x_b_dot against Time.

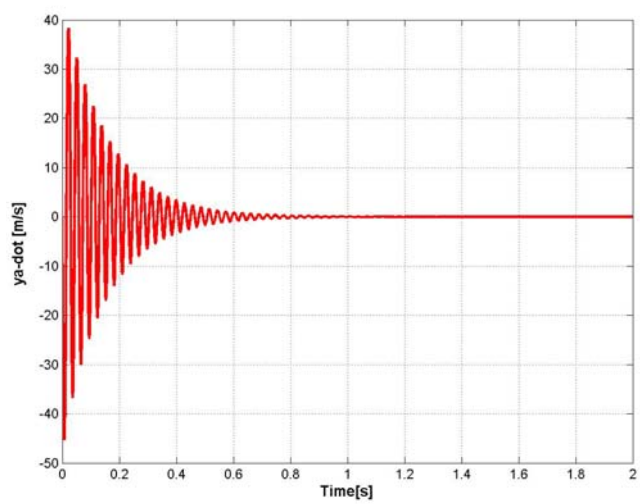

Figure 8: Graph of Velocity, y_a_dot against Time.

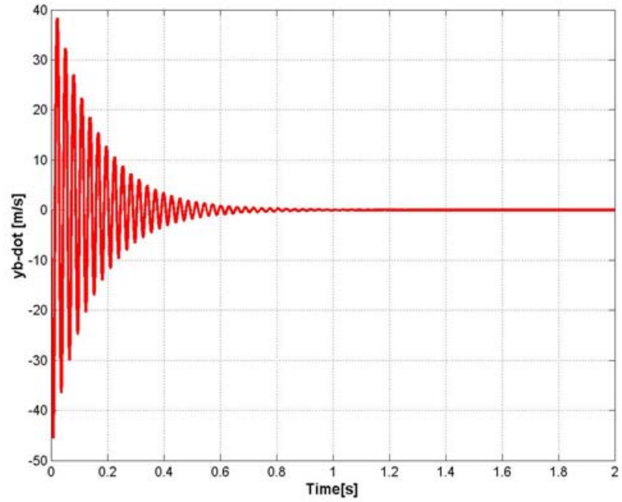

Figure 9: Graph of Velocity, y_b_dot against Time.

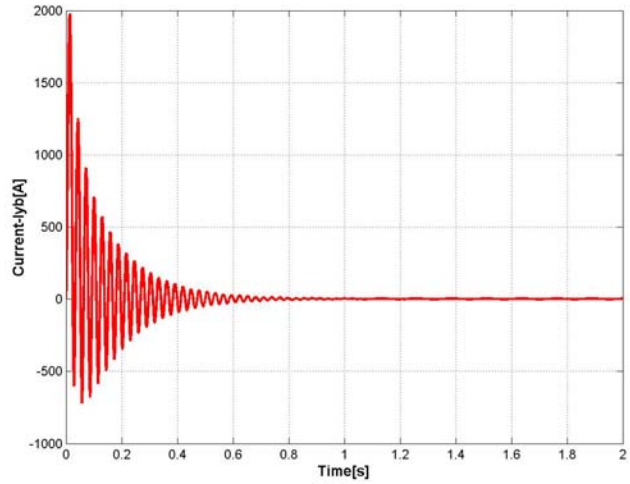

Figure 12: Graph of Current, Iy_b against Time.

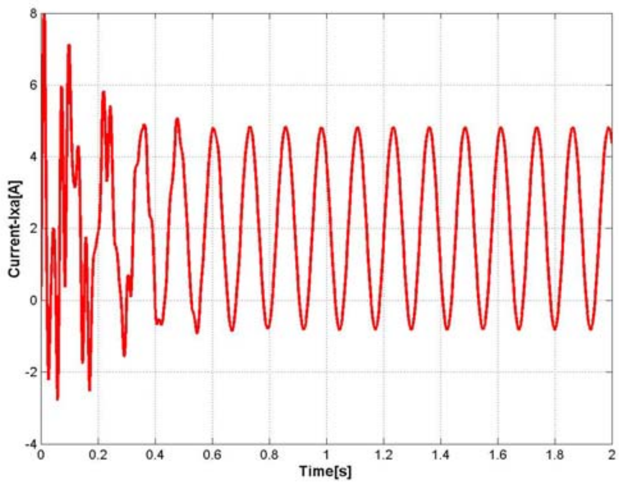

Figure 10: Graph of Current, Ix_a against Time.

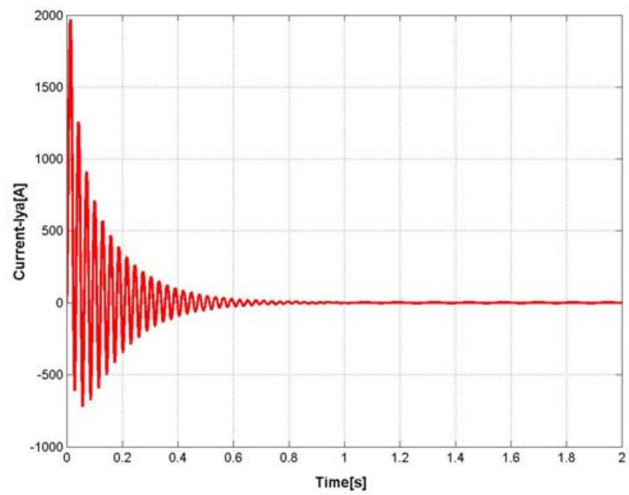

Figure 13: Graph of Current, Iy_a against Time.

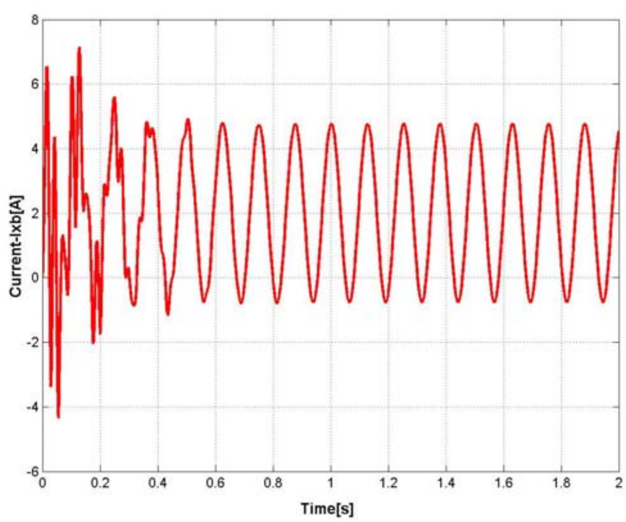

Figure 11: Graph of Current, Ix_b against Time.

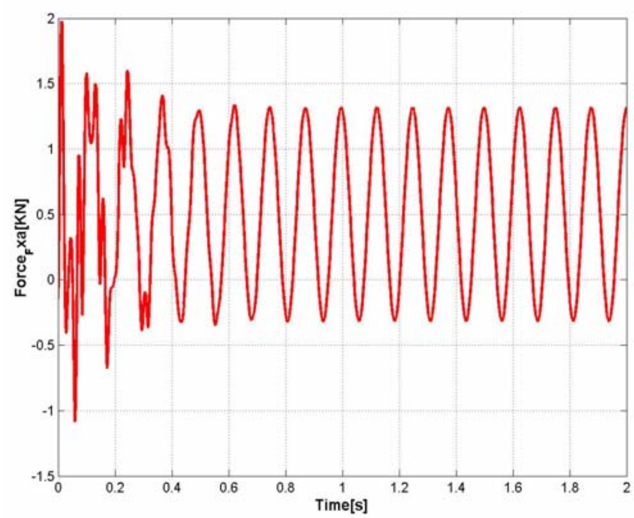

Figure 14: Graph of Force, Fx_a against Time.

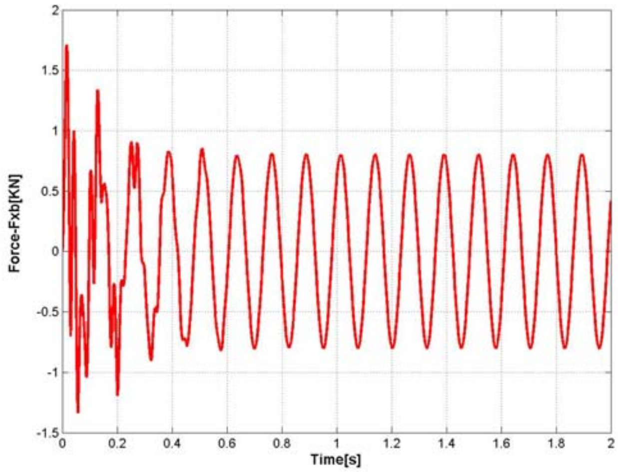

Figure 15: Graph of Force, Fx_b against Time.

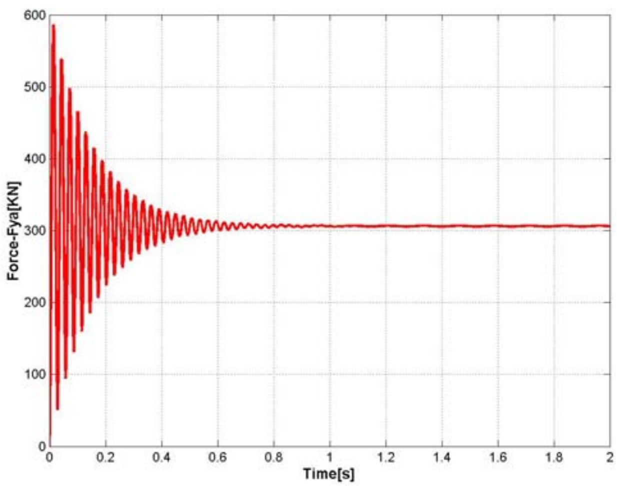

Figure 16: Graph of Force, Fy_a against Time.

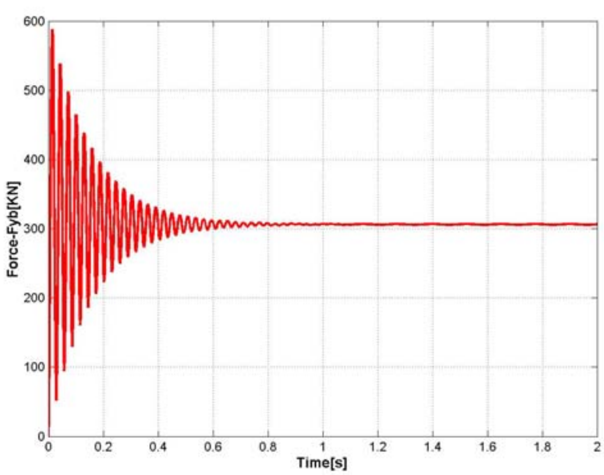

Figure 17: Graph of Force, Fy_b against Time.

Figures 2 and 3 show that the rotor displacement in the x-axis is continuously oscillatory, unlike in Figure 4 and Figure 5 where the oscillation approaches zero after 0.6s.

In the same vein, the velocity-time graphs of Figures 6 and 7 reveal the oscillatory nature of the velocity in the x-axis whereas Figures 8 and 9 show a momentary oscillation for about 0.6s with pronounced initial magnitude of the velocity.

Figure 10 and Figure 11 show that the current in the x-axis is continuously oscillatory compare to currents in the y-axis which readily approach zero at about 0.6s with high initial current magnitude as shown in Figure 12 and Figure 13.

Again, the start-up force in the y-axis is greater than that of x-axis as Figures 13-14 and Figures 16-17 respectively indicate. Figures 14-15 are continuously oscillatory while Figures 16-17 approach zero after 0.6s.

CONCLUSION

This paper has shown that it is possible to use the commercially available software package, MATLAB®, to model and simulate the transient behavior of active magnetic bearing systems. By using the developed program, it becomes simple to compute and study the effect of the state variables on the system during transient state.

The program, although it is specific for AMB system with six degree of freedoms, can easily be modified and adopted for any degree of freedom of interest. The analysis and simulation results presented in this paper will be of immense benefit in the development and realization of a decentralized controller for the proposed active magnetic bearing system. Work in this direction is in progress and will soon be reported by the author.

ACKNOWLEDGEMENT

The author would like to thank Prof. Dr.-Ing. B. Weidemann for his useful discussion on the subject and to DAAD (The German Academic Exchange Service) for their financial assistance during the research work at the University of Kassel, Germany.

REFERENCES

[1] Schweitzer, G. 1975. "Stabilization of Self-Excited Rotor Vibrations by an Active Damper". *IUTAM Symposium on Dynamics of Rotors*. Lyngby, Springer-Verlag: Berlin.

[2] Higuchi, T. 1988. "Applications of Magnetic Bearings in Robotics". *First International Symp. on Magnetic Bearings.* Zürich. Springer-Verlag: Berlin.

[3] Higuchi, T. 1990. "Magnetic Bearings". *Proc. Second International Symposium on Magnetic Bearings.* Tokyo.

[4] Larsonneur, R. 1990. "Design and Control of Active Magnetic Bearing Systems for High Speed Rotation". Dissertation, ETH. Zürich. No. 9140, 1990.

[5] Vischer, D. and Bleuler, H. 1990. "A New Approach to Sensorless and Voltage Controlled AMBs Based on Network Theory Concepts". *2nd International Symposium on Magnetic Bearings*. Tokyo.

[6] Vischer, D. 1988. "Sensorlose und Spannungsgesteuerte Magnetlager". Disertation, ETH No. 8665. Zürich.

[7] Schweitzer, G., Bleuler, H., and Traxler, A. 1994. *Active Magnetic Bearings: Basics, Properties and Applications of Active Magnetic Bearings*. Vdf Hochschulverlag AG: Zürich.

[8] Walker, J.D. 1972. *Applied Mechanics*. Hodder and Stoughton: Great Britain.

[9] Perdikaris, G.A. 1996. *Computer Controlled Systems: Theory and Applications*. Kluwer Academic Publishers: Netherlands.

[10] Gerald, C. F. 1978. *Applied Numerical Analysis*. Addison-Wesley Publishing Company: London.

[11] MATLAB. 1997. "The MATLAB Compiler User's Guide". In: *Mathworks Handbook*. Mathworks: Natick.

ABOUT THE AUTHOR

Dr.-Ing. Ogbonnaya Inya Okoro was born in Unwana, Afikpo-North Local Government Area of Ebonyi State, Nigeria in 1969. He received the B.Eng. and M.Eng. in Electrical Engineering from University of Nigeria, Nsukka/Nigeria, in 1991 and 1997, respectively. From 1998 until 2002 he received a DAAD scholarship at the "IEE-Elektrische Maschinen" of the Universität Kassel/Germany and obtained his Doctor of Engineering degree at the Universität Kassel/Germany in September 2002. Since November 2002 he has been with the Department of Electrical Engineering, Faculty of Engineering, University of Nigeria, Nsukka/Nigeria. His research interests are in the areas of dynamic simulation and control of induction machines as well as in the thermal and dynamic analysis of AC machines. He is a member of the IEEE, the Nigerian Society of Engineers, and the Solar Energy Society of Nigeria.

(Department of Electrical Engineering, University of Nigeria, Nsukka, Enugu State, Nigeria, E-mail: ogbonnayaokoro@hotmail.com)

SUGGESTED CITATION

Okoro, O.I. 2005. "Transient State Analysis of an Active Magnetic Bearing (AMB) System with Six Degree of Freedoms Using MATLAB". *Pacific Journal of Science and Technology*. 6(1):56-63.

HPacific Journal of Science and TechnologyH