

DYNAMIC INTERACTION BETWEEN RETAINING WALLS AND RETAINED STRUCTURES

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ABSTRACT

The seismic response of retaining walls that support soil layers has been examined by various researchers in the past. However, in engineering practice retaining walls are very frequently used to support, apart from soil layers, structures founded on the retained soil layers. Therefore, during a seismic event it is evident that the dynamic response of each component of this complex system (wall, soil, and structure) may affect substantially the response of the rest, and vice versa. This phenomenon, which could be adequately described as “*dynamic wall–soil–structure interaction*” (*DWSSI*), is a rather complicated issue that combines: (a) the dynamic interaction between the wall and the retained soil layers, and (b) the “standard” one-dimensional dynamic interaction of a structure with its underlying soil layers. In the present study, using numerical simulations, the influence of the wall flexibility on the free-field ground shaking behind the wall is investigated. Subsequently, a simple structure founded on the retained soil is included in the numerical models. A parametric study is being performed in order to examine at what extend the presence of the wall may affect the inertial accelerations imposed on the structure (with respect to its position and its fundamental eigen-period). In addition, it is investigated how the location and/or the characteristics of the structure may affect the dynamic earth pressures induced on the retaining wall. Numerical results provide a clear indication of the direct dynamic interaction between a retaining wall and its retained structures.

Keywords: retaining walls, dynamic response, soil-structure interaction

INTRODUCTION

Retaining systems are extensively used worldwide for serving various purposes in structures and infrastructures. Deep excavations, bridge abutments, or harbor quay-walls are some of the cases where a rigid gravity or a flexible cantilever retaining wall is constructed. Despite their structural simplicity, the seismic response of walls (that retain even a single soil layer) is a rather complicated problem. What makes that response so complicated is the dynamic interaction between the wall and the retained soil, especially when material and/or geometry nonlinearities are present (Kramer, 1996; Iai, 1998; Wu & Finn, 1999). Consequently, the performance of retaining walls during earthquakes is a subject being still examined by many researchers, experimentally, analytically, or numerically (Veletsos & Younan, 1997; PIANC, 2001; Psarropoulos et al., 2005). Depending on the expected material behavior of the retained soil and the possible mode of the wall displacement, there exist two main categories of analytical methods used in the design of retaining walls against earthquakes: (a) the pseudo-static limiting-equilibrium Mononobe–Okabe type solutions which assume yielding walls resulting to plastic behavior of the retained soil (Okabe, 1926; Mononobe & Matsuo, 1929; Seed & Whitman, 1970), and

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(b) the elasticity-based solutions that regard the retained soil as a visco-elastic continuum (Scott, 1973; Wood, 1975; Veletsos & Younan, 1997).

However, in many real cases retaining walls are used to support, apart from soil layers, structures founded on the retained soil. It is evident that during a seismic event the dynamic response of each component of this complex system (wall, soil layer, structure) may affect substantially the response of the others. In other words, the presence of a retaining wall will affect not only the ground surface shaking of the retained soil, but the dynamic response of any type of retained structure as well. In addition, the existence of a structure behind the wall is expected to alter the dynamic earth pressures developed on the wall. Therefore, the phenomenon of *dynamic wall–soil–structure interaction (DWSSI)* is a rather complicated issue that includes: (a) the dynamic interaction between a wall and a retained soil layers, and (b) the “standard” one-dimensional dynamic soil–structure interaction of a structure with its underlying soil (Tsompanakis et al., 2006). The aforementioned dynamic interaction issues are not considered with the proper realism in the current seismic norms used in modern engineering practice, like the Eurocode 8 (EC8, 2004) or the Greek Seismic Code (EAK, 2000). Regarding the design of retaining structures, the dynamic interaction between a retaining wall and the retained soil is ignored; while on the other hand, the issue of *dynamic soil–structure interaction* taken into consideration in a simplistic way is considered a-priori to be beneficial for a structure, which seems not to be always the case (Mylonakis & Gazetas, 2000).

The objective of the present study is to examine more thoroughly the phenomenon of *dynamic wall–soil–structure interaction*. For this purpose, two-dimensional numerical simulations are performed, utilizing the finite-element method, in order to investigate some of the most important aspects of this complex phenomenon. Firstly, the influence of the wall flexibility on the ground surface shaking behind the wall is investigated (Figure 1(a)), while emphasis is given on the amplification of the base acceleration, a fact generally ignored by the seismic norms. Subsequently, a simple structure founded on the retained soil is included in the numerical models (Figure 1(b)). A parametric study has been performed in order to examine how the location of the structure may affect the earth pressures induced on the retaining wall. In addition, the parametric study investigates at what extent the presence of the wall may affect the inertial forces imposed on the structure with respect to its position. In all cases, the wall is characterized by its height H , its relative flexibility d_w , and its relative compliance of the foundation d_θ , while the soil material is considered as visco-elastic with shear-wave velocity V_s , density ρ , and critical damping ratio ζ .

In general, dynamic response of any system depends on the seismic excitation characteristics (both in the time and in the frequency domain). In a recent preliminary investigation of *DWSSI* (Tsompanakis et al., 2006), both real earthquake records and pulses were used. In the present numerical study, in order to understand more clearly various aspects of the complex phenomena incorporated in the *DWSSI*, the excitations were limited to harmonic and simple pulses. Results provide a clear indication of the direct dynamic interaction between a retaining wall and its retained structures. That fact justifies the necessity for a more elaborate consideration of this interrelated phenomenon on the seismic design, not only of the retaining walls, but of the nearby structures as well.

NUMERICAL MODELLING

In order to examine more efficiently the *DWSSI* phenomenon, numerical analyses were based on the study of Veletsos & Younan (1997) who developed an analytical approach for evaluating the magnitude and distribution of the dynamic displacements, pressures, and forces induced by horizontal ground shaking on walls that are both flexible and elastically constrained against rotation at their base. Their analytical methodology permitted the assessment of the effects and the relative importance of the factors involved. In their model the soil was considered to act as a uniform, infinitely extended visco-elastic stratum of height H . The properties of the soil were regarded constant, defined by the density ρ , the shear modulus G , and Poisson's ratio ν . The material damping was presumed to be of the constant hysteretic type and was defined by the critical damping ratio ζ . The layer was retained by a

vertical, flexible wall, elastically constrained against rotation at its base; it was free at its upper surface and it was fixed on a rigid base (thus no radiation damping was expected). The properties of the wall were described by its thickness t_w , mass per unit of surface area μ_w , modulus of elasticity E_w , Poisson's ratio ν , and critical damping ratio ζ_w . The base of both the wall and the soil stratum were considered to be excited by a space-invariant horizontal motion, assuming an equivalent force-excited system.

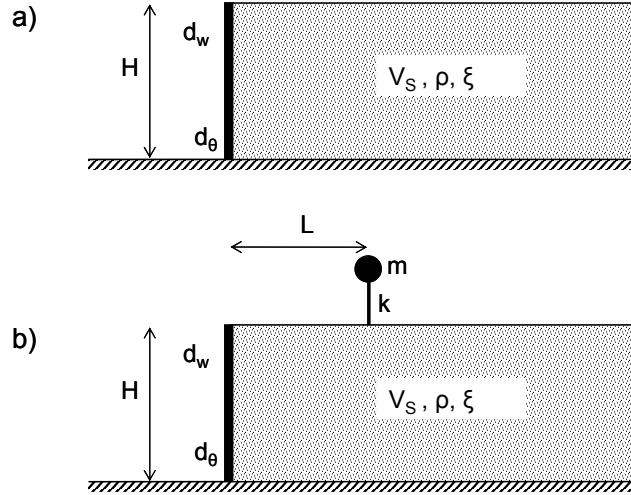


Figure 1. The retaining systems examined in this study: (a) a wall retaining a single soil layer, (b) a wall retaining a soil layer on which a simple structure is founded at distance L

In the present study, in order to examine the effects of DWSSI on both retaining wall and retained structures, two-dimensional (2-D) numerical simulations of the two retaining systems depicted in Figure 1 were conducted. The simulations were performed utilizing the ABAQUS (Version 6.4, 2003) finite-element code, which is capable of performing dynamic linear analyses using Rayleigh type of material damping (resulting to a critical damping ratio of ζ for the frequencies of interest). Although soil nonlinearities are expected to have a significant impact on the DWSSI, it was not examined in this preliminary investigation of this complex phenomenon. An introductory study on this important issue was performed in a companion paper (Tsompanakis et al., 2007). The structure was modelled as a lumped mass m on top of a weightless column discretized with beam elements of flexural stiffness k . The wall was discretized also using beam elements of unit longitudinal dimension and thickness equal to $t_w = 0.20\text{m}$. The main parameters that affect the response of the system are:

(a) the relative (with respect to the retained soil) flexibility of the wall, defined by:

$$d_w = \frac{GH^3}{D_w} \quad (1)$$

and (b) the relative (with respect to the retained soil) flexibility of the rotational base constraint, defined by:

$$d_\theta = \frac{GH^2}{R_\theta} \quad (2)$$

D_w in Equation (1) denotes the flexural rigidity per unit of length of the wall:

$$D_w = \frac{E_w t_w^3}{12(1 - \nu_w^2)} \quad (3)$$

while R_θ in Equation (2) is the stiffness of the rotational base constraint.

Three cases were examined in this study: (a) a rigid fixed-base wall ($d_w = 0, d_\theta = 0$), (b) a flexible fixed-base wall ($d_w = 5, d_\theta = 0$), and (c) a flexible wall with rotational compliance ($d_w = 5, d_\theta = 5$). Given the value of d_w , the modulus of elasticity of the wall E_w is evaluated using Equations (1) and (2), while the Poisson's ratio ν_w is taken as 0.2. The wall mass per unit of surface area is presumed to be $2.5t/m^2$. The simplifying assumptions that no de-bonding or relative slip is allowed to occur at the wall-soil and the structure-soil interfaces were used.

In general, the soil material properties (G, γ) and the wall height alone do not affect the dynamic pressures on the wall, as the wall flexibility is examined in relation to soil stiffness and the earth pressures are normalized with γ and H (Veletsos & Younan, 1997; Psarropoulos et al., 2005). Taking that point into account, all the analyses were performed considering an 8m-high wall. The retained soil layer is characterized by a relatively low shear-wave velocity V_s equal to 100m/s and a unit weight γ of $18kN/m^3$. The retained soil was discretized using four-node quadrilateral plain-strain elements. Horizontal and vertical viscous dashpots were used at the right-hand side of the model in order to simulate the radiation of energy from P and S waves, respectively. We have to mention that although the efficiency of the viscous dashpots is in general quite acceptable, it depends strongly on the angle of incidence of the impinging wave. Therefore, the dashpots were placed far away from the wall in order to simulate the semi-infinite stratum more accurately.

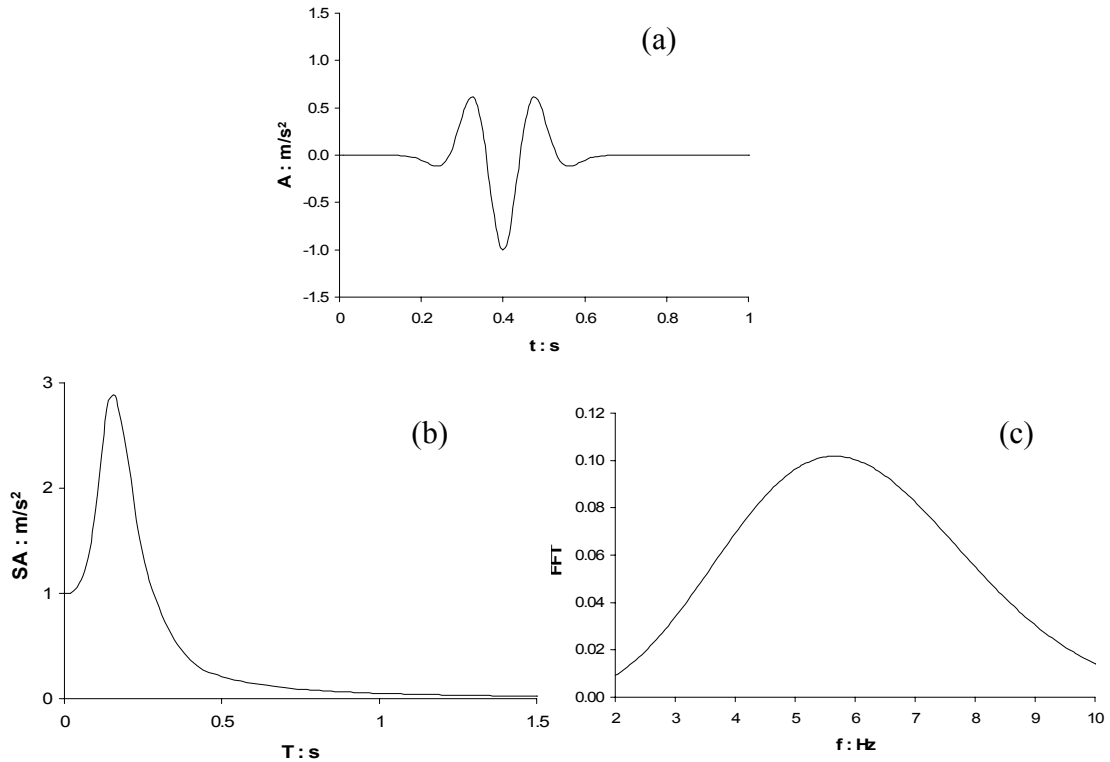


Figure 2. Acceleration time-history (a), response spectrum (b), and Fourier spectrum (c) of the Ricker pulse excitation (with central frequency $f_o = 4Hz$) that has been used in the analyses

As mentioned before, apart from harmonic excitations, simple pulses have also been used. A simple Ricker pulse with central frequency $f_o = 4Hz$ has been selected as pulse excitation (Ricker, 1960). Despite the simplicity of its waveform, this wavelet covers a broad range of frequencies up to nearly $3f_o$ ($\approx 12Hz$). The acceleration time-history (scaled to $0.10g$) and the corresponding response and Fourier spectra of the pulse are shown in Figure 2.

EFFECTS OF THE WALL ON THE RETAINED-SOIL RESPONSE

The dynamic response of a single soil layer under 1-D conditions has been studied by many researchers and analytical solutions for harmonic excitation can be found in the literature (Roesset, 1977; Kramer, 1996). In the case of the harmonic excitation the response is controlled by the ratio T/T_{SOIL} , where T is the dominant period of the excitation, and T_{SOIL} the fundamental period of the soil layer. For the case of one-dimensional (1-D) conditions T_{SOIL} is given by Kramer (1996):

$$T_{SOIL} = \frac{4H}{V_s} \quad (4)$$

in which H is the height of the soil layer, and V_s its shear-wave velocity. In our case the fundamental period of the soil layer $T_{SOIL} = 0.32s$ (or equivalently, the fundamental frequency of the soil layer $f_{SOIL} \approx 3.1Hz$). The duration of the sinusoidal pulse was such that steady state conditions were reached. In that case the maximum soil amplification factor (AF_{SOIL}) is given by:

$$AF_{SOIL} = \frac{2}{\pi\xi} \frac{1}{2n+1} \quad (5)$$

where ξ is the critical damping ratio and n is the mode number. For the first mode ($n = 0$) and $\xi = 5\%$, $AF_{SOIL} \approx 12.5$

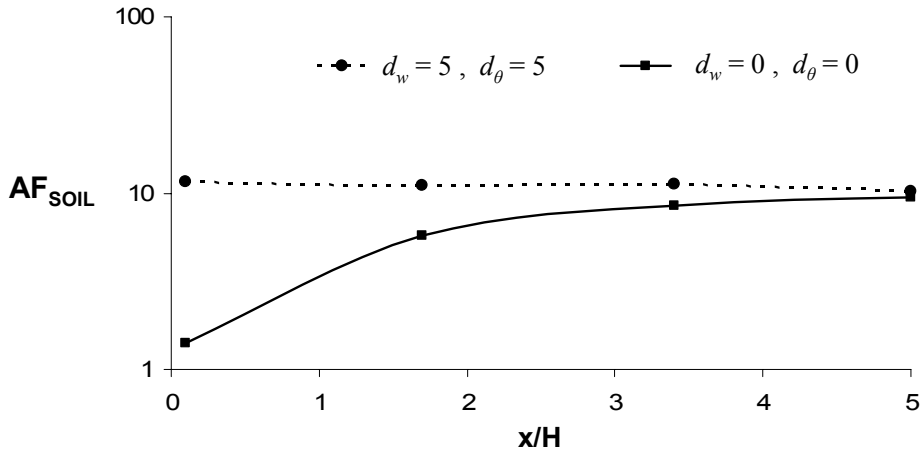


Figure 3. Distribution of the soil amplification factor (AF_{SOIL}) along the surface of the backfill in the case of the harmonic excitation at resonance ($T = T_{SOIL}$) for the two extreme cases of wall flexibility ($d_w = d_\theta = 0$, and $d_w = d_\theta = 5$). Note that vertical axis is in logarithmic scale

The presence of a retaining wall essentially imposes a vertical boundary condition, leading thus to a two-dimensional (2-D) dynamic response. In this study the response of the soil layer under 1-D conditions is compared with the corresponding 2-D due to the existence of the wall (see Figure 1a). The distribution of the amplification factor (AF_{SOIL}) on the surface of the backfill in the case of the harmonic excitation at resonance ($T = T_{SOIL}$) is plotted in Figure 3. It is evident that, for the rigid fixed-base wall case ($d_w = d_\theta = 0$), the motion in the vicinity of the wall is practically induced by the wall itself, and therefore no amplification is observed ($AF_{SOIL} \approx 1$). The amplification factor converges to its maximum value ($AF_{SOIL} \approx 12.5$) at a distance longer than $4H$ from the wall, since at that distance 1-D conditions are present (*free-field motion*). On the other hand, the flexible wall system ($d_w = d_\theta = 5$) permits shear deformation, and consequently, higher levels of acceleration are developed behind the wall. Thus, the response of the retained soil layer resembles the 1-D conditions. As shown in Figure 4, the trends of AF_{SOIL} distribution along the surface in the case of Ricker pulse excitation are similar, despite the substantial decrease in amplification levels.

Figure 5 depicts the Transfer Functions (TF) calculated for the response of point B which is just behind the wall. Each TF is defined as:

$$TF = \frac{FFT_B}{FFT_A} \quad (6)$$

where FFT_B is the Fourier spectrum of the acceleration time history calculated at point B, and FFT_A is the Fourier spectrum of the acceleration time history of the Ricker pulse excitation that is applied at the base of the model (point A). The Fourier spectrum of the acceleration time history of the Ricker pulse excitation has been given in Figure 2(b) and it is evident that it covers smoothly the frequency range between 2 and 10Hz. Therefore it is clear that TF actually comprises the soil amplification AF_{SOIL} at a specific point in the frequency domain. This observation is justified in Figure 5, where AF_{SOIL} for a series of harmonic excitations of the model are depicted. As it was expected, for frequencies close to the fundamental frequency of the soil layer f_{SOIL} , resonance phenomena take place, and TF converges to its maximum possible value. This value is close to 12 in the case of the flexible wall ($d_w = d_\theta = 5$), whereas it is close to 1 for the case of rigid fixed-base wall ($d_w = d_\theta = 0$). Note that these values of AF_{SOIL} can also be observed in Figure 3, at the point just behind the wall.

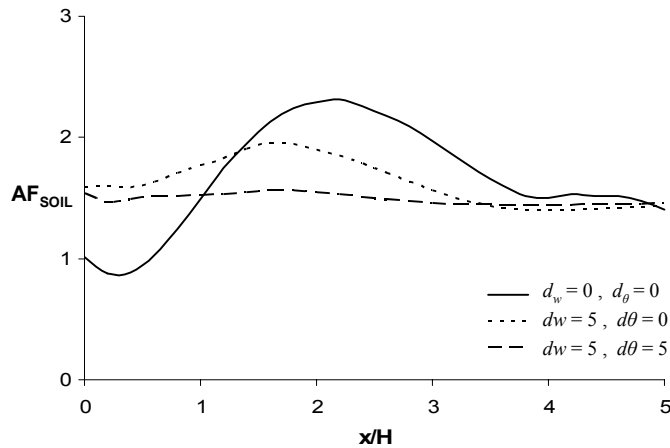


Figure 4. Distribution of the soil amplification factor (AF_{SOIL}) along the surface of the backfill in the case of the high-frequency Ricker pulse excitation

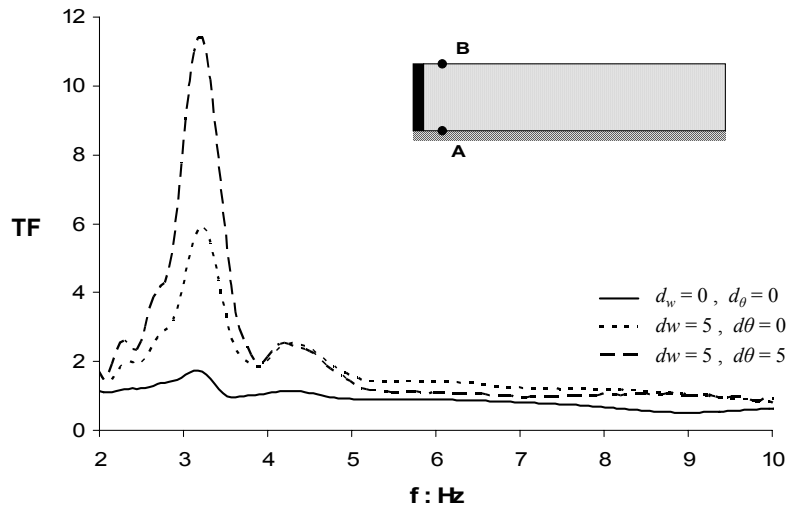


Figure 5. Transfer Functions (TF) calculated for the response of point B, which is just behind the wall, for a series of harmonic excitations. All three cases of wall flexibility are presented

EFFECTS ON THE RESPONSE OF THE STRUCTURE

As it was previously mentioned, prescriptive seismic norms are not capable of taking realistically into consideration the main “components” of the dynamic wall-soil-structure interaction: (a) the dynamic

interaction between a retaining wall and the retained soil layer, and (b) the “standard” 1-D dynamic soil-structure interaction, e.g. the foundation of a structure on a soil layer and the related kinematic or inertial interaction with it. The first is coped with in a very simplistic way, while the latter is usually considered to be either neutral or even beneficial. However, this is not always the case, since dynamic soil-structure interaction may also be detrimental, depending on the circumstances (Mylonakis & Gazetas, 2000). Therefore, in a case of a complex wall-soil-structure system, elaborate numerical modelling of the whole problem is unavoidable, as it is not realistic to study the wall-soil system and the soil-structure system independently.

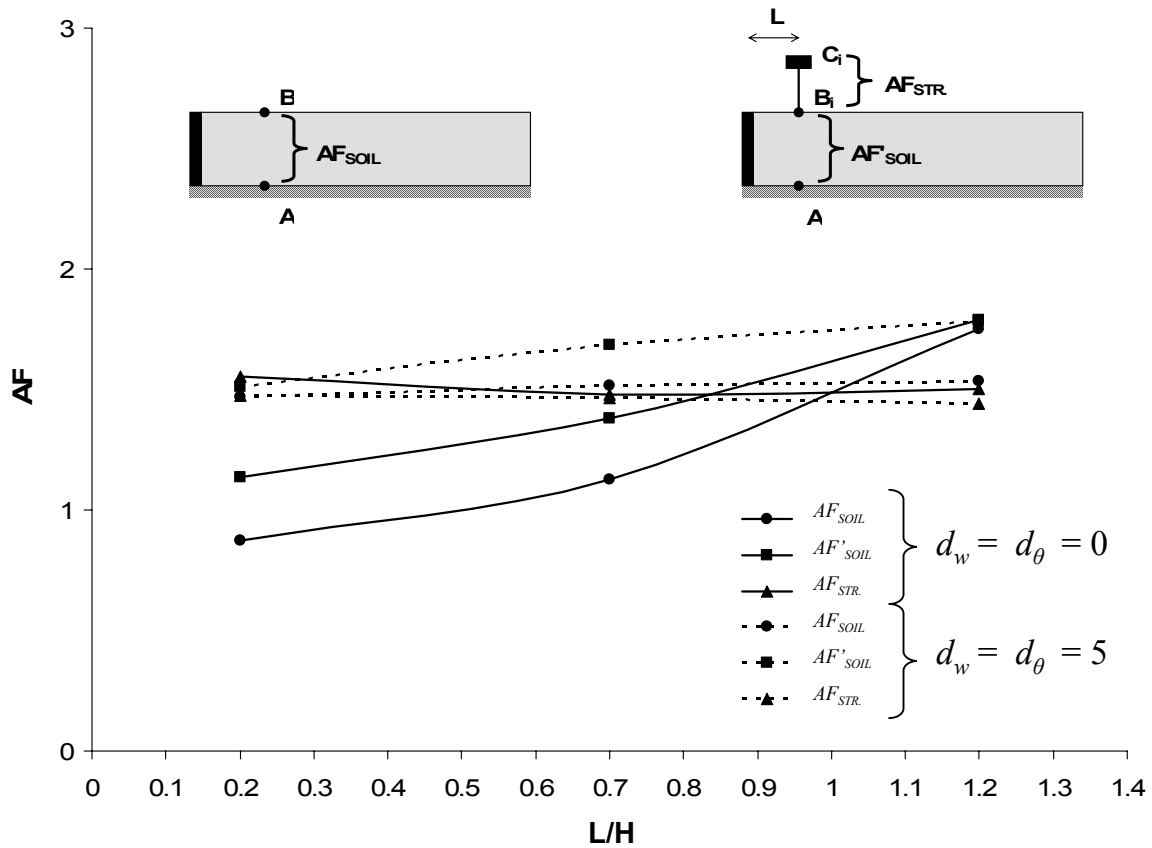


Figure 6. Ricker pulse excitation: Comparison between the soil amplification factors calculated at three points B_i at the surface behind the wall ($L/H = 0.2, 0.7, 1.2$) without the structure (AF_{SOIL}) or with the structure (AF'_{SOIL}). The amplification factor of the structure (AF_{STR}) is also included

In this study the impact of a potential simple structure has been also examined. The simplified model of the structure is shown in Figure 1(b) and it consists of a concentrated mass m located on the top of a single column that provides the stiffness k of the structure. Since the structure can be realized as a fixed-base single-degree-of-freedom (SDOF) system, its eigen-period T_{STR} could be easily calculated by:

$$T_{STR} = 2\pi\sqrt{\frac{m}{k}} \quad (7)$$

As the relation between T_{SOIL} and T_{STR} was expected to play a significant role on the overall response, two cases were examined. In the first, T_{SOIL} coincides with T_{STR} (implying a kind of resonance), while in the second case T_{SOIL} is three times higher than T_{STR} (implying a relatively stiff structure). In Figure 6 the soil amplification factors AF_{SOIL} (without the structure) and AF'_{SOIL} (with the structure) are compared for three surface points B_i behind the wall: $L/H = 0.2, 0.7, 1.2$, or $L = 1.6, 5.6, 9.6$ m, respectively. It is obvious that in the case of the rigid fixed-base wall ($d_w = d_\theta = 0$) the difference

between AF_{SOIL} and AF'_{SOIL} decreases, while moving away from the wall. On the contrary, in the case of the flexible wall ($d_w = d_\theta = 5$) that difference between AF_{SOIL} and AF'_{SOIL} seems to increase.

As it was expected, in the case of the rigid wall the response of the structure depends substantially on its distance from the wall, and differs considerably from the corresponding response of the structure under 1-D conditions (i.e., when there is no wall). On the other hand, for the case of the flexible wall the location of the structure does not alter the response and the behaviour of the structure is affected only by the 1-D conditions. In the same figure the amplification of the structure alone, AF_{STR} , is also included for comparison. This factor represents the amplification of acceleration between the top of the structure and its base. It is evident that the wall flexibility and the distance of the structure from the wall L have no substantial effect on its response.

Finally, in order to examine the effect of the absolute value of the mass of the structure on the amplification factor of the structure AF_{STR} , a structure with five times more mass was incorporated in the models. However, since T_{STR} should be kept constant in order to make the comparison feasible, the ratio between m and k was kept unchanged, thus the second structure was also five times more stiff. According to Figure 7, this substantial increase of mass seems to have certain influence on the response of the structure that follows the same trend for both cases of wall's flexibility.

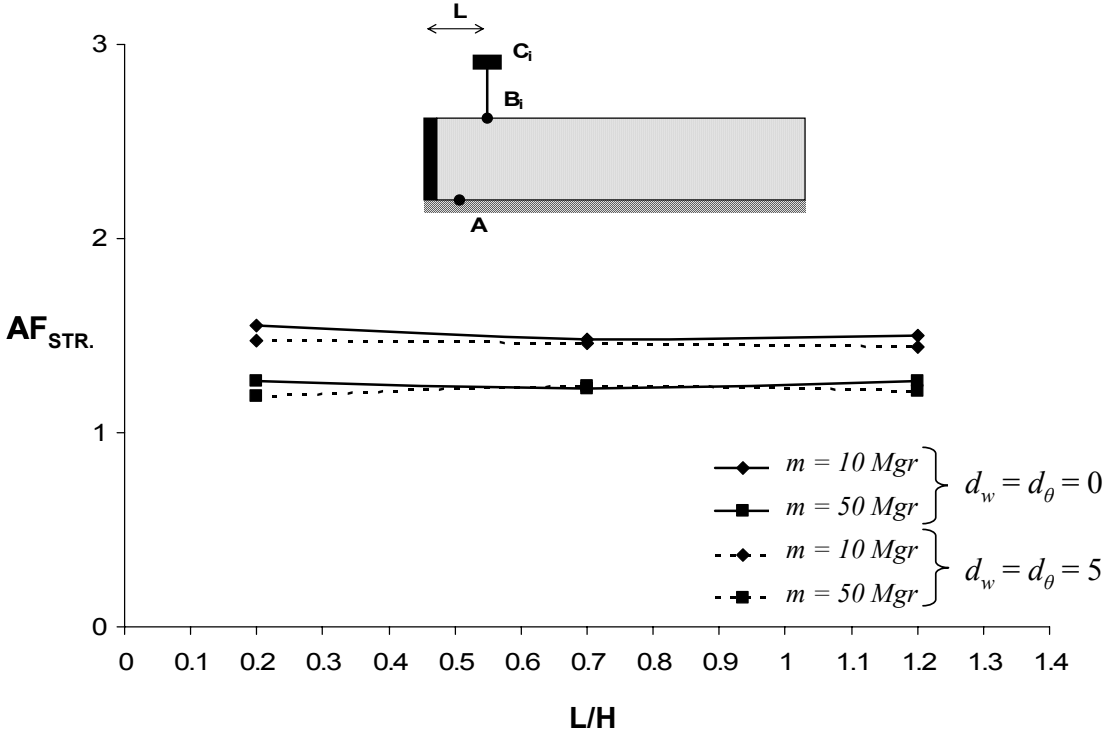


Figure 7. Ricker pulse excitation: Effect of the absolute value of the mass of the structure on the amplification factor of the structure (AF_{STR}). Note that T_{STR} is kept constant

EFFECT OF THE STRUCTURE ON THE WALL DISTRESS

Numerical results have proven that the existence of a structure may increase or reduce the dynamic earth pressures developed on the wall. Figure 8 shows the height-wise distribution of the normalized induced dynamic earth pressures for the two extreme systems examined, in the case that the structure is close to the wall. This phenomenon may be attributed to the impact of the structure on the eigen-frequencies of the whole system, and can be explained by the results plotted in Figure 9. That Figure

shows the Pressure Amplification Factor (PAF) as a function of frequency which can be defined using the following expression:

$$PAF = \frac{FFT[P(t)]}{FFT_A} \quad (6)$$

where $FFT[P(t)]$ is the Fourier spectrum of the normalized induced dynamic earth force time history $P(t)$, and FFT_A is the Fourier spectrum of the acceleration time history of the Ricker pulse excitation that is applied at the base of the model (point A). It is evident that in the case of low-frequency excitations the values of PAF converge to the values proposed by Veletsos & Younan (1997) and Psarropoulos et al. (2005). However, the presence of the structure behind the wall has an impact not only on the amplitude of the developed dynamic earth but on its frequency content. This impact however seems to depend strongly on the characteristics of the structure.

CONCLUSIONS

The scope of the present study was to investigate the dynamic interaction between retaining walls, retained soil and retained structures. In all cases examined it was proven that the characteristics of the wall as well as the seismic excitation affect substantially the dynamic behaviour of the whole system. The rigid wall imposes a boundary that clearly alters the 1-D conditions of the backfill, while the flexible wall does not transform the model into 2-D. Furthermore, it has been shown that the amplification of the acceleration levels on the retained soil and structure depends also on the seismic motion. In addition, it has been presented that the existence of a retaining wall may alter considerably the dynamic response of a structure founded on the retained soil. Moreover, the distress of the wall may be affected significantly by the presence of retained structures.

These results of this preliminary investigation provide a clear indication of the direct dynamic interaction between the wall, the retained soil, and the retained structures. That fact justifies the necessity for a more elaborate consideration, both in seismic codes and engineering practice, of this interrelated phenomenon during the seismic design, not only of the retaining walls but of the nearby structures as well.

ACKNOWLEDGEMENTS

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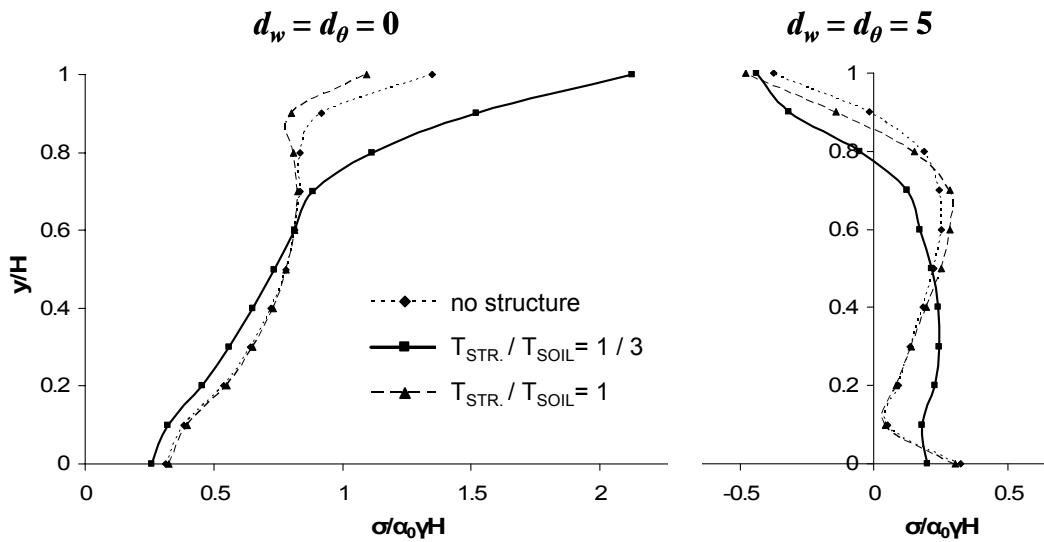


Figure 8. Ricker pulse excitation: Height-wise distribution of the normalized induced dynamic earth pressures for the two extreme cases of wall flexibility examined

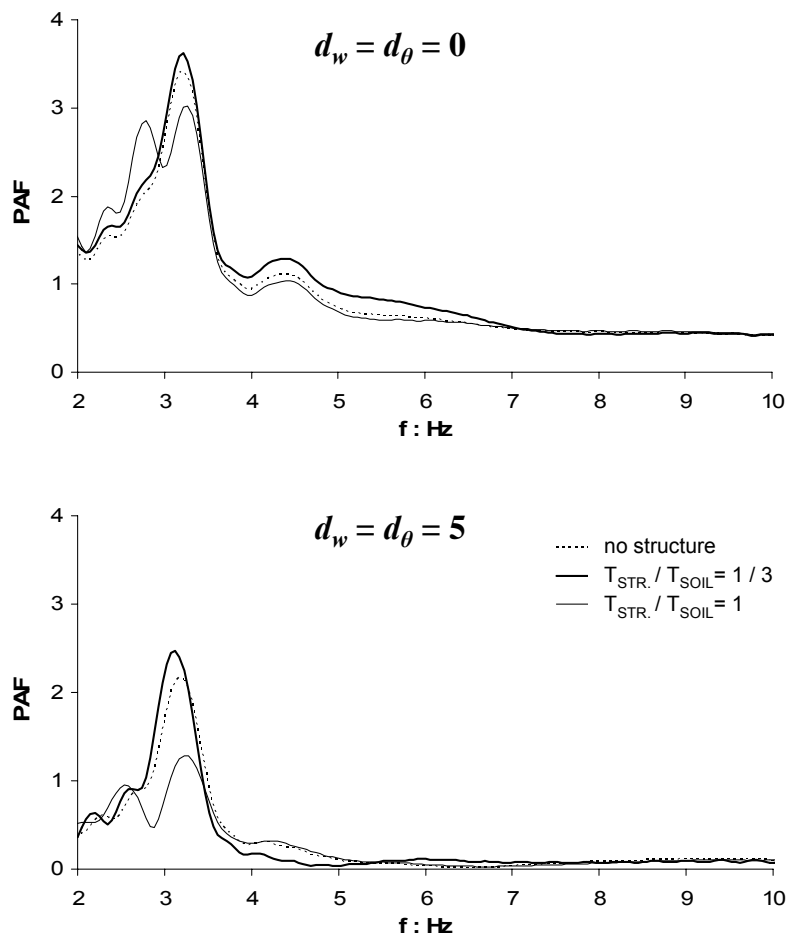


Figure 9. Ricker pulse excitation: the Pressure Amplification Factors (PAF), calculated for the two extreme cases of wall flexibility examined

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