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PDE and Martingale Methods in Option Pricing

Bocconi University Press

Springer

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B&SS – Bocconi & Springer Series
ISSN print edition: 2039-1471 ISSN electronic edition: 2039-148X

ISBN 978-88-470-1780-1 e-ISBN 978-88-470-1781-8

DOI 10.1007/978-88-470-1781-8

Library of Congress Control Number: 2010929483

Springer Milan Dordrecht Heidelberg London New York

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9 8 7 6 5 4 3 2 1

Cover-Design: K Design, Heidelberg

Typesetting with LaTeX: PTP-Berlin, Protago TeX-Production GmbH, Germany (www.ptp-berlin.eu)
Printing and Binding: Grafiche Porpora, Segrate (Mi)

Printed in Italy

Springer-Verlag Italia srl – Via Decembrio 28 – 20137 Milano
Springer is a part of Springer Science+Business Media (www.springer.com)

Preface

This book gives an introduction to the mathematical, probabilistic and numerical methods used in the modern theory of option pricing. It is intended as a textbook for graduate and advanced undergraduate students, but I hope it will be useful also for researchers and professionals in the financial industry.

Stochastic calculus and its applications to the arbitrage pricing of financial derivatives form the main theme. In presenting these, by now classic, topics, the emphasis is put on the more quantitative rather than economic aspects. Being aware that the literature in this field is huge, I mention the following incomplete list of monographs whose contents overlap with those of this text: in alphabetic order, Avellaneda and Laurence [14], Benth [43], Björk [47], Dana and Jeanblanc [84], Dewynne, Howison and Wilmott [340], Dothan [100], Duffie [102], Elliott and Kopp [120], Epps [121], Follmer and Schied [134], Glasserman [158], Huang and Litzenberger [171], Ingersoll [178], Karatzas [200; 202], Lamberton and Lapeyre [226], Lipton [239], Merton [252], Musiela and Rutkowski [261], Neftci [264], Shreve [310; 311], Steele [315], Zhu, Wu and Chern [349].

What distinguishes this book from others is the attempt to present the matter by giving equal weight to the probabilistic point of view, based on the martingale theory, and the analytical one, based on partial differential equations. The present book does not claim to describe the latest developments in mathematical finance: that target would indeed be very ambitious, given the speed of progress of research in the field. Instead, I have chosen to develop some of the essential ideas of the classical pricing theory to devote space to the fundamental mathematical and numerical tools when they arise. Thus I hope to provide a sound background of basic knowledge which may facilitate the independent study of newer problems and more advanced models.

The theory of stochastic calculus, for continuous and discontinuous processes, constitutes the bulk of the book: Chapters 3 on stochastic processes, 4 on Brownian integration and 9 on stochastic differential equations may form the material for an introductory course on stochastic calculus. In these chapters, I have constantly sought to combine the theoretical concepts to the insight on the financial meaning, in order to make the presentation less abstract and more motivated: in fact many theoretical concepts naturally lend themselves to an intuitive and meaningful economic interpretation.

The origin of this book can be traced to courses on option pricing which I taught at the master program in Quantitative Finance of the University of Bologna, which I have directed with Sergio Polidoro since its beginning, in 2004. I wrote the first version as lecture notes for my courses. During these years, I substantially improved and extended the text with the inclusion of sections on numerical methods and the addition of completely new chapters on stochastic calculus for jump processes and Fourier methods. Nevertheless, during these years the original structure of the book remained essentially unchanged.

I am grateful to many people for the suggestions and helpful comments with which supported and encouraged the writing of the book: in particular I would like to thank several colleagues and PhD students for many valuable suggestions on the manuscript, including David Applebaum, Francesco Caravenna, Alessandra Cretarola, Marco Di Francesco, Piero Foscari, Paolo Foschi, Ermanno Lanconelli, Antonio Mura, Cornelis Oosterlee, Sergio Polidoro, Valentina Prezioso, Enrico Priola, Wolfgang Runggaldier, Tiziano Vargiolu, Valeria Volpe. I also express my thanks to Rossella Agliardi, co-author of Chapter 13, and to Matteo Camaggi for helping me in the translation of the book.

It is greatly appreciated if readers could forward any errors, misprints or suggested improvements to: `andrea.pascucci@unibo.it`
Corrections received after publication will be posted on the website:
`http://www.dm.unibo.it/~pascucci/`

Bologna, November 2010 *Andrea Pascucci*

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