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Research Article

The One Step Optimal Homotopy Analysis Method to Circular Porous Slider

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An incompressible Newtonian fluid is forced through the porous of a circular slider which is moving laterally on a horizontal plan. In this paper, we introduce and apply the one step Optimal Homotopy Analysis Method (one step OHAM) to the problem of the circular porous slider where a fluid is injected through the porous bottom. The effects of mass injection and lateral velocity on the heat generated by viscous dissipation are investigated by solving the governing boundary layer equations using one step optimal homotopy technique. The approximate solution for the coupled nonlinear ordinary differential equations resulting from the momentum equation is obtained and discussed for different values of the Reynolds number of the velocity field. The solution obtained is also displayed graphically for various values of the Reynolds number and it is shown that the one step OHAM is capable of finding the approximate solution of circular porous slider.

1. Introduction

An interesting subject in mathematical physics is the study and analysis of flow between plates [1–6]. An analytical overview of study of porous bearing has been carried out by Morgan and Cameron in [3]. Gorla [7] discussed the fluid dynamical and heat transfer of the circular porous slider bearing. The study of the effects of the Reynolds number on circular porous slider has been investigated in [8] by using the Variational Iteration Method (VIM) which is one of the semi analytical methods. The fluid dynamics in a slider bearing have been discussed in [9] by using the series expansion and asymptotic expansion. Wang [9], in fact, discussed the numerical solution for the porous slider for the large Reynolds number. As it is well known the numerical methods such as finite difference and finite element are time consuming and may be difficult due to stability constraints. Toward this end, in this paper, we introduce and apply an effective method (so-called one step Homotopy Analysis Method) that provides accurate solution and has advantage over the finite difference and finite element methods.

Semi analytical schemes such as Variational Iteration Method (VIM), Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), and Homotopy Analysis Method (HAM) have been widely employed to solve various linear and nonlinear ordinary and partial differential equations. One of the advantages of the semi approximate analytical methods is that these methods generate an infinite series solution and, unlike finite difference methods, semi approximate analytical methods do not have the problem of rounding error. Therefore, in contrast to implicit finite difference methods that require the solution of systems of equation, the semi analytical schemes require only the solution of recursive process.

The HAM was developed by Liao [10] who utilized the idea of homotopy in topology. The interested reader can refer to the much cited book [11] for a systematic and clear exposition on the HAM. It has been reported that HAM, as an analytical method, has an advantage over perturbation methods in that it is not dependent on small or large parameters [12]. According to [12], perturbation methods are based on the existence of small or large parameters and they cannot be applied to all nonlinear equations. Nonperturbative methods, such as δ-expansion and ADM, are independent of small parameters. According to [13], both perturbation techniques and nonperturbative methods cannot provide a simple procedure to adjust or control the convergence region and rate of given approximate series. HAM allows for fine tuning of convergence region and rate of convergence by allowing an auxiliary parameter $\hbar$ to vary [14, 15]. The proper choice of the initial condition, the auxiliary linear operator, and auxiliary parameter $\hbar$ will guarantee the convergence of the HAM solution series [16]. According to [13], compared to the HPM, the HPM solution series will be convergent by considering two factors: the auxiliary linear operator and initial guess [13].

In a series of papers, Marinca et al. [17–20] have introduced and developed a new method, called Optimal Homotopy Asymptotic Method (OHAM). The HPM and HAM are known to be special cases of OHAM. An advantage of OHAM over the HAM is that there is no necessity to identify the $\hbar$-curve. The control and adjustment of the convergence region are also provided in a convenient way. Furthermore, there is a built-in convergence criteria similar to HAM but with greater flexibility [21]. Marinca et al. [17–20] have used this method successfully on problems in mechanics and have also shown its effectiveness and accuracy.

The OHAM have also some disadvantages. A disadvantage of this method is the necessity to solve a set of nonlinear algebraic equations at each order of approximations. Another disadvantage is that OHAM includes many unknown convergence-control parameters which makes it time consuming for calculating. To overcome of this difficulty, Niu and Wang [22] presented a new modification of OHAM called one step optimal homotopy analysis method, to improve the computational efficiency of the HAM. In their approach, only one nonlinear algebraic equation with one unknown variable is solved at each order approximations. An optimal homotopy analysis approach that contains at most three convergence-control parameters at any order of approximations has been introduced by Liao [23].

Our goal of this paper is to apply the one step OHAM introduced by Niu and Wang in [22] for the circular porous slider. The general framework for solving this kind of problem is introduced. Several cases have been given to demonstrate the efficiency of the framework. So far as we are aware, this is the first time that the coupled nonlinear ordinary differential equation resulting from the momentum equations has been solved approximately using the one step OHAM.

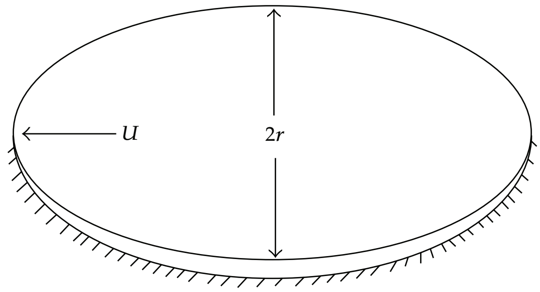

Figure 1: Schematic diagram of the problem.

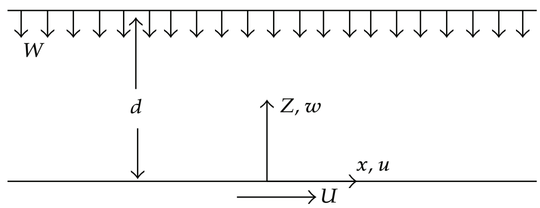

Figure 2: System of coordinate axes.

Our paper is organized as follows. In Section 2, we present description of conservation mass and momentum density Navier-Stokes equations and also transformation. In Section 3, we have introduced the one step OHAM to nonlinear system of equations. In Section 4, solutions are given to illustrate capability of one step OHAM. Finally, in Section 5, we give the conclusion of this study.

2. Formulation

In this paper, the flow field due to a circular porous slider (Figure 1) is calculated by using one step OHAM. A fluid of constant density is forced through the porous bottom of the slider and thus separates the slider from the ground. An incompressible fluid is forced through the porous wall of the slider with a velocity W. Figure 2 shows the slider which is fixed at the plane $z = d$, with a viscous fluid injected through it. The base is the plane at $z = 0$, which is moving in the x-direction with velocity U. For detail, please see [7, 24].

As the gap d is small, it can be assumed that both planes are extended to infinity [24]. Considering the u, v, and w to be the velocity components in the direction x, y, and z, respectively, the conservation mass and conservation momentum density Navier-Stokes equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2.1}$$

$$x\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{2.2}$$

$$x\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \tag{2.3}$$

$$x\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{2.4}$$

where ρ is density of fluid, ν is kinematic viscosity, and P is pressure. The boundary conditions are as follows:

$$\begin{gathered} z=0, \qquad u=U, \qquad v=w=0, \\ z=d, \qquad u=v=0, \qquad w=-W, \end{gathered} \tag{2.5}$$

where U is velocity of the slider in lateral and longitudinal direction and W is velocity of fluid injected through the porous bottom of the slider. For transforming (2.2)–(2.4), the following equations are defined [7, 8, 24]:

$$\eta=\frac{z}{d}, \tag{2.6}$$

$$u=Uf(\eta)+W\frac{x}{d}h'(\eta), \tag{2.7}$$

$$v=\frac{W}{d}h'(\eta), \tag{2.8}$$

$$w=-2Wh(\eta). \tag{2.9}$$

By substituting (2.7)–(2.9) into Navier-Stoks equations (2.2)–(2.4), it can be obtained that [7, 8, 24]

$$h''''(\eta)+2Rh(\eta)h'''(\eta)=0, \tag{2.10}$$

$$f''(\eta)+2Rh(\eta)f'(\eta)-Rh'(\eta)f(\eta)=0, \tag{2.11}$$

$$-\frac{P}{\rho}=\frac{WK}{2d^2}\left(x^2+y^2\right)+\frac{1}{2}w^2-\nu\frac{\partial w}{\partial z}+A, \tag{2.12}$$

where $R=Wd/\nu$ is the cross-flow Reynolds number, A and K are constants which will have to be determined. The boundary conditions for the transformation are as follows:

$$\begin{gathered} h(0)=h'(0)=h'(1)=0, \qquad h(1)=\frac{1}{2}, \\ f(0)=1, \qquad f(1)=0. \end{gathered} \tag{2.13}$$

The series solution for small values of R was obtained by Wang in [9]. Gorla [7] solved nonlinear equation (2.10) and (2.11) by using the fourth-order Runge-Kutta method together with the shooting method.

3. One Step Optimal Homotopy Analysis Method

To illustrate the basic idea of the one step Optimal Homotopy Analysis Method, we consider the system of nonlinear differential equation

$$A_i(u_1, u_2, \ldots, u_n) + f_i(\eta) = 0, \quad \eta \in \Omega,\ i = 1, 2, \ldots, n, \tag{3.1}$$

$$B_{i,j}\left(u_j, \frac{du_j}{d\eta}\right) = 0, \quad \eta \in \Gamma,\ i, j = 1, 2, \ldots, n, \tag{3.2}$$

where A_i are differential operators and B_i are boundary operators, $u_i(\eta)$ are unknown functions, η denotes independent variable, Γ is the boundary of the domain Ω, and $f_i(\eta)$ are known analytic functions. In general, A_i, $i = 1, 2, \ldots, n$ can be decomposed as

$$A_i = L_i + N_i, \tag{3.3}$$

where L_i are linear operators and N_i are nonlinear operators. The optimal $\phi_i(x; q) : \Omega \times [0, 1] \to \mathbb{R}$ which satisfies

$$\begin{aligned} H_i(\phi_1(\eta; q), \ldots, \phi_n(\eta; q), q) = (1 - q)\{&L_i(\phi_1(\eta; q), \ldots, \phi_n(\eta; q)) + f_i(\eta)\} \\ &- H_i(q)\{A(\phi_1(\eta; q), \ldots, \phi_n(\eta; q)) + f_i(\eta)\}. \end{aligned} \tag{3.4}$$

Here $\eta \in \Omega$ and $q \in [0, 1]$ is an embedding parameter, $H_i(q)$ are nonzero auxiliary functions for $q \neq 0$, and $H_i(0) = 0$. For $q = 0$ and $q = 1$

$$\begin{aligned} H_i(\phi_1(\eta; 0), \ldots, \phi_n(\eta; 0), 0) &= L_i(\phi_1(\eta; 0), \ldots, \phi_n(\eta; 0)) + f_i(\eta), \\ H_i(\phi_1(\eta; 1), \ldots, \phi_n(\eta; 1), 1) &= H_i(1)\{A(\phi_1(\eta; 1), \ldots, \phi_n(\eta; 1)) + f_i(\eta)\} = 0. \end{aligned} \tag{3.5}$$

When $q = 0$ and $q = 1$, $\phi_i(\eta; 0) = u_{i,0}(\eta)$ and $\phi_i(\eta; 1) = u_i(\eta)$, respectively. The zeroth-order problem $u_{i,0}(\eta)$ is obtained from (3.2) and (3.4) with $q = 0$ giving

$$L_i(\phi_i(\eta; 0)) + f_i(\eta) = 0, \qquad B_i\left(u_{i,0}, \frac{du_{i,0}}{dx}\right) = 0. \tag{3.6}$$

The auxiliary functions $H_i(q)$ are chosen in the form

$$H_i(q) = \sum_{j=1}^{\infty} C_{i,j} q^j, \tag{3.7}$$

where $C_{i,j}$ are constants. The (3.7), in fact, is a simple case of auxiliary function $H(\tau, q)$ in [25, 26]. Marinca and Herişanu [25, 26] proposed the auxiliary function $H(\tau, q)$ that has the following form:

$$H_i(q) = qC_{i,1} + q^2 C_{i,2} + \cdots + p^m C_{i,m}(\tau), \tag{3.8}$$

where $C_{i,1}, C_{i,2}, \ldots, C_{i,m-1}$ can be constants and the last value $C_{i,m}(\tau)$ can be a function depending on the variable τ. To get an approximate solution, $\phi_i(\eta; q, C_{i,j})$, $i, j = 1, 2, \ldots, n$ are expanded in a Taylor's series about q as

$$\phi_i(\eta; q, C_{i,j}) = u_{i,0}(\eta) + \sum_{j=1}^{\infty} u_{i,j}(\eta; C_{i,j}) q^j, \quad j = 1, 2, 3, \ldots. \tag{3.9}$$

Substituting (3.9) into (3.4) and equating the coefficient of like powers of q, the first- and second-order problems are given as [21]

$$\begin{gathered} L_i(u_{i,1}(\eta)) = C_{i,1} N_{i,0}(u_{i,0}(\eta)), \qquad B_i\left(u_{i,1}, \frac{du_{i,1}}{d\eta}\right) = 0, \\ L_i(u_{i,2}(\eta)) - L(u_{i,1}(\eta)) = C_{i,2} N_{i,0}(u_{i,0}(\eta)) + C_{i,1}\left[L(u_{i,1}(\eta)) + N_{i,1}(u_{i,0}(\eta), u_{i,1}(\eta))\right], \\ B_i\left(u_{i,2}, \frac{du_{i,2}}{d\eta}\right) = 0, \end{gathered} \tag{3.10}$$

and the general governing equations for $u_{i,j}(\eta)$ are given as [21]

$$\begin{aligned} L_i(u_{i,j}(\eta)) &= L_i(u_{i,j-1}(\eta)) + C_{i,j} N_{i,0}(u_{i,0}(\eta)) \\ &\quad + \sum_{k=1}^{j-1} C_{i,k}\left[L(u_{i,j-k}(\eta)) + N_{i,j-k}(u_{i,0}(\eta), \ldots, u_{i,j-k}(\eta))\right], \quad j = 2, 3, \ldots, \\ &B_i\left(u_{i,j}, \frac{du_{i,j}}{d\eta}\right) = 0, \end{aligned} \tag{3.11}$$

where $N_{i,j-k}(u_{i,0}(\eta), u_{i,1}(\eta), \ldots, u_{i,j-k}(\eta))$ is the coefficient of q^{j-i} in the expansion of $N(\phi(x; q))$ about the embedding parameter q and

$$N_i(\phi_i(\eta; q, C_{i,j})) = N_{i,0}(u_{i,0}(\eta)) + \sum_{j\geq 1} N_{i,j}(u_{i,0}, u_{i,1}, \ldots, u_{i,j}) q^j. \tag{3.12}$$

It can be seen in the number of papers that the convergence of the series (3.9) depends upon the auxiliary constants $C_{i,1}$, $C_{i,2}$, $C_{i,3}$, ... [17–21]. If the series is convergent at $q = 1$, then

$$\tilde{u}_i(\eta; q) = u_{i,0}(x) + \sum_{j=1}^{m} u_{i,j}(\eta; C_{i,j}), \tag{3.13}$$

which m denotes the mth order of approximation. Substituting (3.13) into (3.1) gives the following expression for the residual:

$$R_i(\eta; C_{i,j}) = L_i(\tilde{u}_i(\eta; C_{i,j})) + f_i(\eta) + N(\tilde{u}_i(\eta; C_{i,j})). \tag{3.14}$$

If $R_i(\eta; C_{i,j}) = 0$, then $\tilde{u}_i(\eta; C_{i,j})$ are the exact solutions of nonlinear system differential equations. For the determination of auxiliary constants $C_{i,j}$, the least squares can be used. Consider

$$\Delta_{i,j}(C_{i,j}) = \int_{\Omega} R_i^2(\eta; C_{i,j})\,dx. \tag{3.15}$$

It is to be noted that, at the first order of approximation, the square residual error $\Delta_{i,1}$ only depends on $C_{i,1}$. To obtain the optimal value of $\Delta_{i,1}$, we need to solve the following system of nonlinear algebraic equation:

$$\frac{d\Delta_{i,1}}{dC_{i,1}} = 0, \quad i = 1, 2, \ldots, n. \tag{3.16}$$

For the second-order approximation, the square residual error $\Delta_{i,2}$ are functions with respect to $C_{i,1}$ and $C_{i,2}$. The values of $C_{i,1}$ have been obtained. To obtain the optimal value of $\Delta_{i,2}$, we need to solve the following system of nonlinear algebraic equations:

$$\frac{d\Delta_{i,2}}{dC_{i,2}} = 0, \quad i = 1, 2, \ldots, n. \tag{3.17}$$

By repeating the above process, the square residual error $\Delta_{i,n}$ will contain only the unknown convergence-control parameter $C_{i,n}$. To obtain the optimal value of square residual error $\Delta_{i,n}$, we should solve the following system of nonlinear algebraic equations:

$$\frac{d\Delta_{i,n}}{dC_{i,n}} = 0, \quad i = 1, 2, \ldots, n. \tag{3.18}$$

Contrary to Marinca's approach which requires the solution of a set of m nonlinear algebraic equation for m unknown convergence-control parameters C_1, C_2,..., C_m, in one step OHAM the square residual error is minimized at each equation of system and each order so as to obtain the optimal convergence-control parameter only one by one. In fact, it is needed to solve only one nonlinear algebraic equation to obtain the $C_{i,n}$ at each order of approximation. An advantage of one step OHAM is that it is easy to implement and obtain high order of approximation with less CPU time [22]. The disadvantage of the OHAM is the need to solve a set of coupled nonlinear algebraic equations for the unknown convergence-control parameters $C_1, C_2, C_3, \ldots, C_m$ which will be obtained from relation (3.9). For low order of m, the nonlinear algebraic system can be easily solved but for large m it is more difficult. Therefore, the necessary CPU time increases exponentially [22].

Table 1: The values of $C_{1,n}$, $C_{2,n}$, $\Delta_{1,n}$, and $\Delta_{2,n}$ given by one step OHAM with $R = 0.01$.

Order	$h(\eta)$		$f(\eta)$	
	$C_{1,n}$	$\Delta_{1,n}$	$C_{2,n}$	$\Delta_{2,n}$
1	-0.998447	2.62513×10^{-9}	-0.998305	1.37745×10^{-10}
2	1.25250×10^{-6}	1.08397×10^{-14}	1.01375×10^{-6}	4.53986×10^{-16}
3	2.31374×10^{-9}	7.26901×10^{-20}	1.56460×10^{-9}	2.43297×10^{-21}
4	5.43527×10^{-12}	5.93503×10^{-25}	2.90453×10^{-12}	1.58185×10^{-26}
5	1.27548×10^{-14}	5.36086×10^{-30}	2.88430×10^{-15}	1.24666×10^{-31}

Table 2: The values of $C_{1,n}$, $C_{2,n}$, $\Delta_{1,n}$, and $\Delta_{2,n}$ given by one step OHAM with $R = 0.1$.

Order	$h(\eta)$		$f(\eta)$	
	$C_{1,n}$	$\Delta_{1,n}$	$C_{2,n}$	$\Delta_{2,n}$
1	-0.984506	2.55432×10^{-5}	-0.983137	1.33655×10^{-6}
2	1.23397×10^{-4}	1.04900×10^{-8}	1.96362×10^{-5}	4.38245×10^{-10}
3	2.25989×10^{-6}	6.95465×10^{-12}	1.52751×10^{-6}	2.32214×10^{-13}
4	5.26737×10^{-8}	5.60423×10^{-15}	2.82907×10^{-8}	1.49055×10^{-16}
5	1.37987×10^{-9}	5.03925×10^{-18}	5.59958×10^{-10}	1.05699×10^{-19}

Table 3: The values of $C_{1,n}$, $C_{2,n}$, $\Delta_{1,n}$, and $\Delta_{2,n}$ given by one step OHAM with $R = 1$.

Order	$h(\eta)$		$f(\eta)$	
	$C_{1,n}$	$\Delta_{1,n}$	$C_{2,n}$	$\Delta_{2,n}$
1	-0.852494	0.195328	-0.843116	9.97874×10^{-3}
2	0.0100734	7.10009×10^{-3}	7.9411×10^{-3}	2.88876×10^{-4}
3	1.63641×10^{-3}	3.95865×10^{-4}	1.09686×10^{-3}	1.28659×10^{-5}
4	3.41259×10^{-4}	2.63846×10^{-6}	1.87783×10^{-4}	6.84637×10^{-7}
5	8.02990×10^{-5}	1.94829×10^{-7}	3.53364×10^{-5}	4.01397×10^{-8}

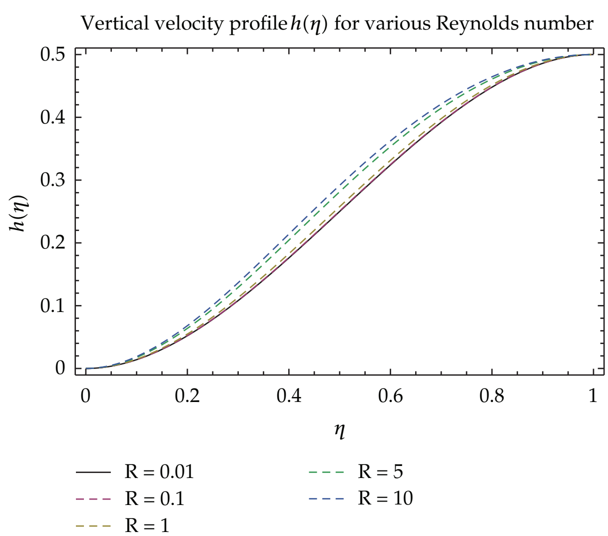

Figure 3: Effect of the various Reynolds number on vertical velocity profile $h(\eta)$.

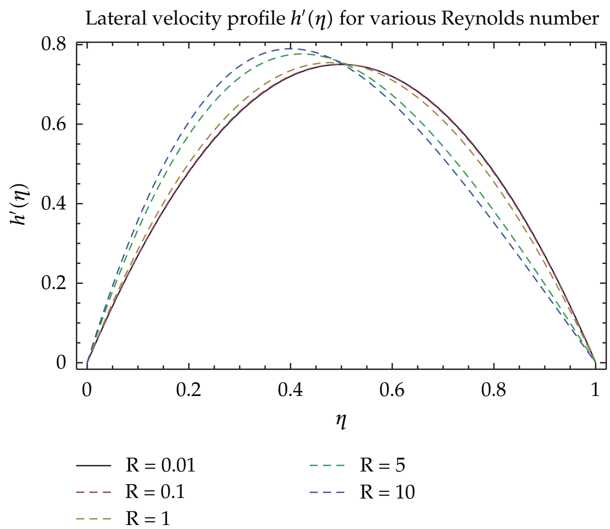

Figure 4: Effect of the various Reynolds number on lateral velocity profile $h'(\eta)$.

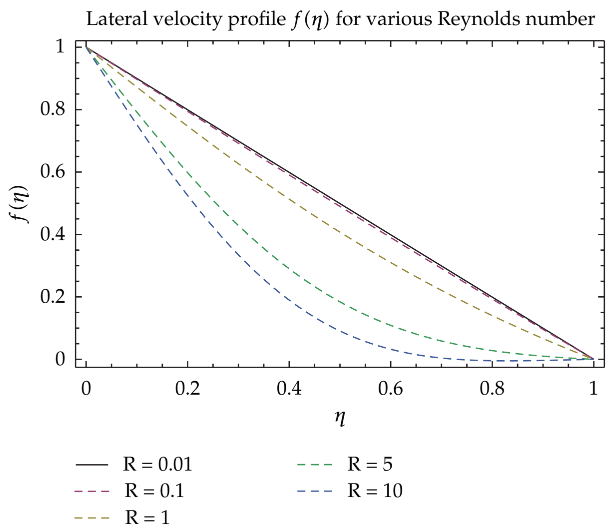

Figure 5: Effect of the various Reynolds number on lateral velocity profile $f(\eta)$.

Table 4: The values of residual functions given by 6 terms of one step OHAM.

t/R	Eh		Ef	
	$R = 0.01$	$R = 0.1$	$R = 0.01$	$R = 0.1$
0.1	6.22549×10^{-16}	6.01324×10^{-10}	3.64726×10^{-16}	3.37381×10^{-10}
0.3	3.55792×10^{-15}	3.44806×10^{-9}	5.68122×10^{-16}	5.25215×10^{-10}
0.5	3.12250×10^{-15}	3.06883×10^{-9}	6.76542×10^{-17}	5.02513×10^{-11}
0.7	8.53484×10^{-16}	7.32369×10^{-10}	3.55618×10^{-16}	3.39019×10^{-10}
0.9	1.84575×10^{-15}	1.77685×10^{-9}	1.89085×10^{-16}	1.51579×10^{-10}

4. Application of One Step OHAM

According to OHAM formulation described in Section 3, we start with

$$\begin{aligned}
L_1(\phi_1(\eta;q),\phi_2(\eta;q)) &= \frac{d^4\phi_1(\eta;q)}{d\eta^4},\\
L_2(\phi_1(\eta;q),\phi_2(\eta;q)) &= \frac{d^2\phi_2(\eta;q)}{d\eta^2},\\
N_1(\phi_1(\eta;q),\phi_2(\eta;q)) &= 2R\phi_1(\eta;q)\frac{d^3\phi_1(\eta;q)}{d\eta^3},\\
N_2(\phi_1(\eta;q),\phi_2(\eta;q)) &= 2R\phi_1(\eta;q)\frac{d\phi_2(\eta;q)}{d\eta} - R\phi_2(\eta;q)\frac{d\phi_1(\eta;q)}{d\eta}.
\end{aligned} \tag{4.1}$$

We can easily choose the initial approximation as

$$h_0(\eta) = -\eta^3 + 1.5\eta^2, \qquad f_0(\eta) = 1 - \eta. \tag{4.2}$$

By applying the OHAM, we can obtain components of OHAM series solution (3.9). By substituting the solution obtained into (2.10) and (2.11), we can obtain the following residual function as

$$\begin{aligned}
Eh &= \tilde{h}''''(\eta) + 2R\tilde{h}(\eta)\tilde{h}'''(\eta),\\
Ef &= \tilde{f}''(\eta) + 2R\tilde{f}'(\eta)\tilde{h}(\eta) - R\tilde{f}(\eta)\tilde{h}'(\eta).
\end{aligned} \tag{4.3}$$

The square residual errors at the m order of approximation are defined by [22]

$$\begin{aligned}
\Delta_{1,m} &= \int_0^1 (Eh)^2 d\eta,\\
\Delta_{2,m} &= \int_0^1 (Ef)^2 d\eta,
\end{aligned} \tag{4.4}$$

where

$$\begin{aligned}
\tilde{h}(\eta) &= h_0(\eta) + \sum_{k=1}^{m} h_k(\eta),\\
\tilde{f}(\eta) &= f_0(\eta) + \sum_{k=1}^{m} f_k(\eta).
\end{aligned} \tag{4.5}$$

Table 5: Wall gradients of vertical functions for various Reynolds number.

R	One step OHAM		Runge-Kutta in [7]	
	$h''(0)$	$h''(1)$	$h''(0)$	$h''(1)$
0.01	3.001857	−2.99685	3.001855	−2.99685
0.1	3.018610	−2.96882	3.018610	−2.96866
1	3.18914	−2.71042	3.189800	−2.71104
5	3.983929	−1.96626	3.984910	−1.96266

For $m = 1$, it is found that

$$\begin{aligned}
\Delta_{1,1} &= 13.3714R^2 + 26.7429R^2C_{1,1} + 4.15547R^3C_{1,1} + 13.3714R^2C_{1,1}^2 \\
&\quad + 4.15547R^4C_{1,1} - 0.01459R^4C_{1,1}^3 - 0.00402R^5C_{1,1}^3 + 2.37546 \times 10^{-5}R^6C_{1,1}^4, \\
\Delta_{2,1} &= 0.6428R^2 - 0.01686R^3C_{1,1} + 0.0002378R^4C_{1,1}^2 + 1.28571R^2C_{2,1} + 0.2349R^2C_{2,1} \\
&\quad - 0.01686R^3C_{1,1}C_{2,1} - 9.07251R^4C_{1,1}C_{2,1} - 2.56612 \times 10^{-6}R^5C_{1,1}^2C_{2,1} + 0.6428R^2C_{2,1}^2 \\
&\quad + 0.2349R^3C_{2,1}^2 + 0.035057R^4C_{2,1}^2 + 7.32839 \times 10^{-4}C_{1,1}C_{2,1}^2 \\
&\quad + 5.06569 \times 10^{-7}R^5C_{1,1}C_{2,1}^2 + 8.27189 \times 10^{-9}R^6C_{1,1}^2C_{2,1}^2, \\
&\ \ \vdots,
\end{aligned} \tag{4.6}$$

and so on. By considering (4.6), it is clear that the $\Delta_{1,1}$ and $\Delta_{2,1}$ contain convergence-control parameter $C_{1,1}, C_{2,1}, C_{1,2}$. Thus the approach introduced in Section 3 gives optimal value of the first convergence-control parameter $C_{1,1}$ and $C_{2,1}$ by solving the system of equation

$$\begin{aligned}
\frac{d\Delta_{1,1}}{dC_{1,1}} &= 0, \\
\frac{d\Delta_{2,1}}{dC_{2,1}} &= 0.
\end{aligned} \tag{4.7}$$

For $m = 2$, the square residual error $\Delta_{2,1}$ and $\Delta_{2,2}$ are only dependent $C_{1,2}$ and $C_{2,2}$ since $C_{1,1}$ and $C_{1,2}$ are known. Thus, the optimal values of $C_{1,2}$ and $C_{2,2}$ are obtained by solving the following system of equations:

$$\begin{aligned}
\frac{d\Delta_{2,1}}{dC_{1,2}} &= 0, \\
\frac{d\Delta_{2,2}}{dC_{2,2}} &= 0,
\end{aligned} \tag{4.8}$$

and so on.

In this approach, the optimal values of convergence-control parameters $C_{1,1}, C_{2,1}, C_{1,2}, \ldots$ are obtained one by one until an accurate enough approximation [22].

Table 6: Wall gradients of vertical functions for various Reynolds number.

R	One step OHAM		Runge-Kutta in [7]	
	$h'''(0)$	$h'''(1)$	$h'''(0)$	$h'''(1)$
0.01	−6.00771	−5.97774	−6.00771	−5.97774
0.1	−6.077462	−5.78058	−6.077400	−5.78052
1	−6.802182	−4.09444	−6.805200	−4.09418
5	−10.80969	−0.733029	−10.805200	−0.72054

Table 7: Wall gradients of lateral functions for various Reynolds number.

R	One step OHAM		Runge-Kutta in [7]	
	$f'(0)$	$f'(1)$	$f'(0)$	$f'(1)$
0.01	−1.00300	−0.995509	−1.00300	−0.995512
0.1	−1.029842	−0.955911	−1.029859	−0.955942
1	−1.285961	−0.632878	−1.285215	−0.632150
5	−2.19631	−0.0838189	−2.198724	−0.085460

In Tables 1, 2, and 3, we have calculated the convergence-control parameters $C_{i,j}$ $(i, j = 1, \ldots, n)$ and square residual error $\Delta_{i,j}$ for various orders of n at $R = 0.01$, $R = 0.1$, and $R = 1$ by using one step OHAM introduced in Section 3. It is to be noted that the square residual error decreases quickly as the order of approximation increases. In Table 4, we have calculated the residual functions Eh and Ef for various $t \in (0, 1)$ and $R = 0.01$ and $R = 0.1$. It can be seen that the errors are very small.

In Tables 5 and 6, we have compared the values of $h''(0)$, $h''(1)$, $h'''(0)$ and $h'''(1)$ between the solution obtained by using one step OHAM and solution obtained by using fourth-order Rung-Kutta method in [7].

In Table 7, we have compared the values of $f'(0)$ and $f'(1)$ between the solution obtained by using one step OHAM and solution obtained by using fourth-order Runge-Kutta method in [7].

In Figure 3, we have displayed the vertical velocity $h(\eta)$ for various Reynolds number. It can be seen that $h(\eta)$ increases within the gap width as the Reynolds number increases. In Figure 4, we have shown the lateral velocity $h'(\eta)$. It is clear from Figure 4 that for increasing the Reynolds number R, the $h'(\eta)$ increases whereas the position of maximum velocity tends to move closer to the moving wall. In Figure 5, it can be seen that in the case of $R = 0.01$, the lateral velocity $f(\eta)$ is linear. These results are consistent with those obtained in [7].

5. Conclusion

In this paper, the one step Optimal Homotopy Analysis Method (one step OHAM) has been successfully introduced and applied for solving the problem of circular porous slider. The influence of the Reynolds number has been discussed through graphs. The graphical behavior of f, h, and h' for different values of the Reynolds number (small and big Reynolds number) is presented graphically for fifth-order approximation solution using one step OHAM. In optimal homotopy asymptotic method (OHAM), the control and adjustment of the convergence of the series solution using the control parameters C_i's are achieved in a simple way. A disadvantage of OHAM is that it is necessary to solve a set of nonlinear algebraic equations with m unknown convergence-control parameters $C_1, \ldots, C_m$ and this is time consuming, specially for large m. In contrast to OHAM, in one step OHAM introduced in this paper, algebraic equations with only one unknown convergence-control parameter at each level should be solved. In fact, the one step OHAM is easy to implement and obtain high order of approximation with less CPU time.

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