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# Convergence Analysis of BNC Turbo Detection for Clipped OFDM Signalling

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***Abstract*— All communication systems in which Orthogonal Frequency Division Multiplexing (OFDM) is applied suffer from a well-known problem: the high Peak-to-Average Power Ratio (PAPR) of the time domain OFDM signal. From many PAPR reduction techniques clipping is one of the simplest: although the PAPR can be easily limited, it also introduces strong nonlinearities, reducing the bit error performance of the system unless it is not compensated at the receiver. In this paper we will investigate one of the receiver oriented iterative (turbo) clipping mitigation methods, the so-called Bussgang Noise Cancellation (BNC). We show that with small modifications to this algorithm, the performance of the system can be further improved.**

***Index Terms*— OFDM, PAPR, Clipping, Turbo, Bussgang, Noise cancellation.**

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation scheme where the digital data are modulated on a large number of orthogonal subcarriers. Modulation and demodulation of the subcarriers can be easily performed using Fast Fourier Transform (FFT) and Inverse FFT. Nowadays, wireless communication systems operate usually using OFDM transmission, which has many advantages but it also suffers from a well known major drawback: the high Peak-to-Average Power Ratio (PAPR). This phenomenon negatively affects the Power Amplifiers (PA) and D/A-A/D converters. Signal preprocessing has to be applied to reduce the high PAPR of the OFDM signal, otherwise the power amplifiers will be very expensive and they will operate very inefficiently. On the other hand, if the PA has a smaller linear range then required, nonlinear effects can also negatively affect the system performance of the OFDM system. Many methods have already been proposed to reduce the PAPR of OFDM signals [3]. In this paper we will apply the deliberate amplitude clipping method [4].

Clipping is used to force the amplitude of the signal into the linear range of the PA. Although the PAPR of the signal can be well controlled by this, it causes power attenuation and error (which may be considered as noise), so clipping needs to be compensated. The receiver oriented turbo principle is a good candidate for compensation of the clipping effects. Two different methods are described in the literature:

- Decision Aided Reconstruction (DAR), where the receiver tries to rebuild the peaks of the time domain signal [5].
- Bussgang Noise Cancellation (BNC), where the objective is to remove the clipping noise in the frequency domain [6].

Both methods were originally presented with a decoding procedure which uses hard decisions. The modified receivers using soft decisions were presented in [7]. With the use of soft information, the receiver takes full advantage of the turbo principle yielding better Bit Error Rate (BER) results than the methods using hard decisions. In this paper we will focus on the BNC algorithm which outperforms the DAR method [7].

In the next section we will introduce the system model which is used for clipped, coded OFDM signals. Then, we explain the soft BNC receiver algorithm [7] in detail and will discuss its convergence behavior based on the Extrinsic Information Transfer (EXIT) chart. At the end of this section we propose some modifications to the described algorithms to further improve the system performance. Finally, the simulation results for the original and the improved BNC are presented and compared.

## II. SYSTEM MODEL

The baseband model used for system simulations is presented in Fig. 1. The binary information data are encoded by a rate-$R$ convolutional encoder,

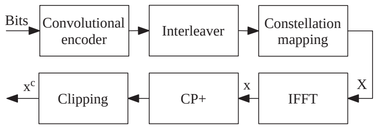


Fig. 1. OFDM transmitter with amplitude clipping

randomly bit-interleaved, mapped to complex constellation symbols $X_n$ – from the set $\mathcal{C}$, where each symbol $c_k \in \mathcal{C}$, maps $M$ bits $b_n$, $n = 1...M$ – and then OFDM modulated. The time-domain OFDM symbol can be expressed as [1]:

$$x_n = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i \exp\left(j2\pi \frac{in}{N}\right), 0 \leq n < N, \quad (1)$$

where $N$ denotes the number of sub-carriers. Clipping to the transmit signal $x_n$ is then applied to reduce the PAPR, where the amplitude values are limited to a threshold $A_{max}$. The clipped signal $x_n^c$ is given by

$$x_n^c = \begin{cases} x_n & |x_n| \leq A_{max} \\ A_{max} e^{j\varphi(x_n)} & |x_n| > A_{max} \end{cases}, \quad (2)$$

where $\varphi(x_n)$ is the phase of the complex signal $x_n$. The limiter is characterized by the clippling ratio (CR),

$$\text{CR} = 20 \log_{10}(\gamma), \quad (3)$$

with $\gamma = A_{max}/\sqrt{P_x}$, where $P_x$ is the average power of the transmit signal $x_n$. According to Bussgang's theorem [2], the signal at the output of the limiter can be expressed as

$$x_n^c = \alpha x_n + d_n, \quad (4)$$

where $\alpha$ is the attenuation factor and $d_n$ is the clipping noise, assumed to be uncorrelated with the signal $x_n$. The attenuation factor $\alpha$ is calculated as [2]

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}}{2} \gamma \ \text{erfc}(\gamma). \quad (5)$$

The output power of the limiter is given by

$$P_{out} = \left(1 - e^{-\gamma^2}\right) P_x, \quad (6)$$

With (4) and (6), the clipping noise power can be calculated as

$$P_d = \left(1 - e^{-\gamma^2} - \alpha^2\right) P_x. \quad (7)$$

In this paper, perfect synchronization and knowledge about the channel coefficients $h_n$ at the receiver are assumed. Further, the the cyclic prefix (CP), with a length of P samples, is assumed to be larger than the channel's maximum excess delay.

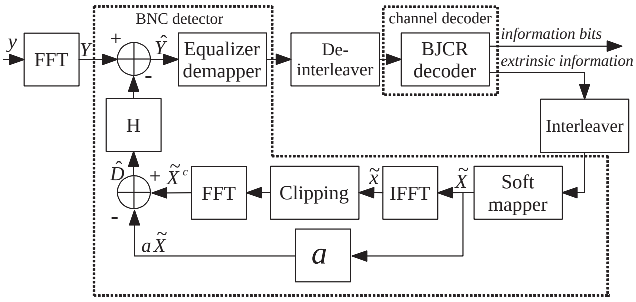


Fig. 2. Block diagram of the Bussgang noise cancellation

At the receiver, the received signal after OFDM demodulation can be expressed as

$$\begin{aligned} Y_n &= X_n^c H_n + W_n \\ &= \alpha X_n H_n + D_n H_n + W_n, 0 \leq n < N, \end{aligned} \quad (8)$$

where $X_n$, $X_n^c$, $D_n$, $H_n$ and $W_n$ are the discrete Fourier transforms of the sampled signals $x_n$, $x_n^c$, $d_n$, $h_n$ and $w_n$, respectively. $w_n$ is the Additive White Gaussian Noise (AWGN) with variance $\sigma_0^2 = N_0/2$. The noise power of the AWGN channel is calculated according to the following definition:

$$E_b/N_0 = P_{out}(N+P)/(N_0 MRN). \quad (9)$$

## III. Bussgang Noise Cancellation (BNC)

### A. BNC turbo detection

The Bussgang noise cancellation (BNC) receiver performs iterative equalization and detection [12]. The basic block diagram of this iterative method is shown in Fig. 2. These blocks can be grouped into two main subblocks (Fig. 2.): the BNC detector and the channel decoder. The BNC detector consists of a forward and feedback signal processing path.

*1) Forward-path:* The extrinsic Log-Likelihood Ratio (LLR) value for each channel observation $\hat{Y}_n$ are calculated according to [8],

$$L(b_{k,m}|\hat{Y}_n) = \ln \frac{\sum\limits_{c_i \in \mathcal{C}_{k,m}^1} p\left(\hat{Y}_n | c_k = c_i\right)}{\sum\limits_{c_i \in \mathcal{C}_{k,m}^0} p\left(\hat{Y}_n | c_k = c_i\right)}, \quad (10)$$

where $\mathcal{C}_{k,m}^1$ and $\mathcal{C}_{k,m}^0$, $1 < k \leq M$ are the subsets of $\mathcal{C}_k$, where the m$^{\text{th}}$ bit in $c_k$ takes the value 1 and 0, respectively. The conditional probability density function $p(\hat{Y} = c_i)$ is given by [5]

$$p(\hat{Y}_n|c) = \exp\left(\frac{(\hat{Y}_n - \alpha H_n c)^2}{N_0 + P_{ch} P_D}\right), \quad (11)$$

where $P_D$ is the power of the remaining clipping noise. Due to a large number of samples and the central limit theorem, the clipping noise term $d_n$ can be modeled as a Gaussian distributed random variable, which is independent from the channel noise $w_n$. Based on this assumption, passing through the linear channel filter, the power of the Bussgang's noise $P_D$ is multiplied by $P_{ch} = E\left\{|H_n|^2\right\}$, $0 \leq n < N$. For the 0$^{\text{th}}$ iteration $P_D$ is calculated according to (7). On the other hand, for each higher iteration, for a large number of samples, $P_D$ can be approximated as

$$P_D = E\{|D_n - \hat{D}_n|^2\}. \quad (12)$$

Of course the receiver does not have knowledge of $D_n$, so for further implementation the power of the remaining clipping noise has to be estimated in another way. This will be discussed in section III-C in detail.

*2) Feedback-path:* After interleaving the exstrinsic LLR values, the soft symbols are computed as [5]

$$\tilde{X}_n = \sum_{k=0}^{2^M-1} c_k \prod_{l=0}^{M-1} P(b_{k,l}), \quad c_k \in C, \quad (13)$$

i.e. each constellation symbol is weighted by the probability of the mapped bits, then they are summed up. Using these soft symbols the time domain estimate of the OFDM signal is formed using IFFT. Then, with the knowledge of the clipping level $A_{max}$, clipping is applied, and the signal is converted back to the frequency domain. Subtracting from these symbols the attenuated symbols, we can express the estimated clipping noise as

$$\hat{D}_n = \tilde{X}_n^c - \alpha\tilde{X}_n, \quad 0 \leq n < N. \quad (14)$$

The estimate noise term $\hat{D}$, multiplied by the channel coefficient, is then subtracted from the received symbols (8) to suppress the clipping noise

$$\hat{Y}_n = \alpha H_n X_n + H_n(D_n - \hat{D}_n) + W_n, \quad 0 \leq n < N. \quad (15)$$

The 0$^{\text{th}}$ iteration is considered as the case when no feedback loop is used, i.e $\hat{Y}_n = Y_n$.

The BJCR channel decoder has the task to compute the extrinsic information of the deinterleaved LLRs, which are provided by the BNC detector. These extrinsic LLRs will be used to suppress the clipping noise in the feedback path of the BNC detector.

## *B. Convergence Analysis*

The EXtrinsic Information Transfer (EXIT) chart was developed by Stephan ten Brink [11]. It is used to investigate the iteration behavior of the turbo loop based on mutual information exchange. With this powerful tool the mutual information exchange between the BNC detector and the channel decoder can be traced over the iterations.

The LLRs defined by (10) are modeled with an equivalent Gaussian channel [11]. The mutual information between these LLRs and the sent symbols $U$ which are the realizations of $u \in \{-1, +1\}$ can be written with the [11] conditional probability density function as

$$I_A(U;\text{LLR}) = \frac{1}{2} \sum_{u=-1,1} \int_{-\infty}^{\infty} p_A(\xi|U=u) \cdot \log_2 \frac{2p_A(\xi|U=u)}{p_A(\xi|U=-1) + p_A(\xi|U=1)} \mathrm{d}\xi, \quad (16)$$

where $0 \leq I_A \leq 1$ and the binary variable $u_k$ can be easily matched to digital bits $b_k \in \{0,1\}$; the binary variable $u_k = -1$ and $u_k = 1$ represents the digital bits $b_k = 0$ and $b_k = 1$, respectively. To measure the mutual information content of the output extrinsic LLR values, the following expression is applied

$$I_E(U;\text{LLR}) = 1 - E\left\{\log_2\left(1 + e^{-\text{LLR}}\right)\right\} \approx \approx 1 - \frac{1}{N}\sum_{n=1}^{N} \log_2\left(1 + e^{-u_n \text{LLR}_n}\right). \quad (17)$$

The EXIT function of the BNC detector is not just a function of the a priori mutual information $I_A$ provided by the channel decoder, what is also dependent on $E_b/N_0$: $I_{E1} = f(I_{A1}, E_b/N_0)$. On the other hand, for the channel decoder the EXIT function is only dependent on the a priori LLRs provided by the BNC detector: $I_{E2} = f(I_{A2})$. With the help of the two EXIT functions, the iteration steps of the turbo loop can be visualized. The output of the channel decoder becomes the input of the BNC detector, and the output of the detector will be the input of the decoder in the next iteration:

$$I_{E1} = f(I_{A1} = I_{E2}, E_b/N_0) \quad (18)$$

$$I_{E2} = f(I_{A2} = I_{E1}). \quad (19)$$

To observe the mutual information transfer of the turbo loop, the EXIT chart is constructed from the two EXIT functions. The EXIT function of the channel decoder is plotted with swapped x-y axes on top of the BNC detectors to visualize the iteration trajectory. An iteration trajectory can be seen for

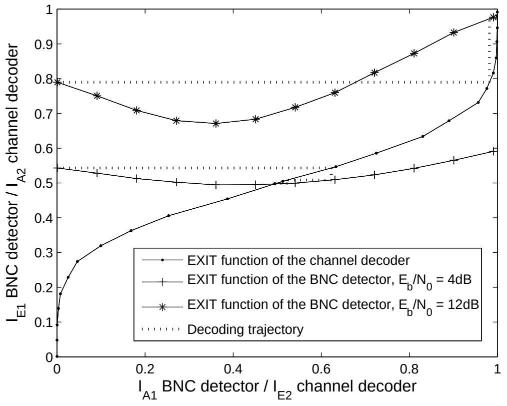


Fig. 3. EXIT chart, with iteration trajectories of the BNC turbo receiver with an $R = \frac{1}{2}$ rate channel decoder for $E_b/N_0 = 4$ dB and $E_b/N_0 = 12$ dB values with CR=1 dB

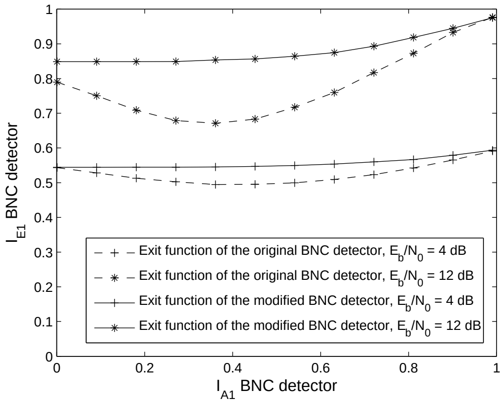


Fig. 4. EXIT function of the original and the modified BNC detector for $E_b/N_0 = 4$ dB and $E_b/N_0 = 12$ dB with CR=1 dB

$E_b/N_0 = 4$ dB and $E_b/N_0 = 12$ dB with a channel decoder rate of $\frac{1}{2}$ in Fig. 3. The divergence for the SNR value of 4 dB is clearly visible, the correction loop can not perform any improvement due to the "minimum" in the EXIT function of the BNC detector. After the first iteration, the mutual information will converge to a lower value than the starting value of the $0^{\text{th}}$ iteration. Despite the "minimum" for 12 dB, the convergence can be clearly observed, the starting mutual information is already high enough to overcome the "minimum" in the EXIT function of the BNC detector and perform convergence.

In iterative receivers, to achieve convergence, the EXIT function of both decoders have to be monotonically growing. It can be observed in Fig. 3 that the monotony of the BNC detector is not satisfactory. As the input mutual information $I_{A1}$ is getting larger, we would expect a growing output mutual information $I_{E1}$, but this can only be observed if the input mutual information value exceed 0.4.

## C. Modified BNC

To answer the question why the "minimum" is in the EXIT function of the BNC detector, the BNC feedback path has to be investigated more carefully. If the mutual information content of the input LLR values of the soft mapper is low, the output power $P_{\tilde{X}}$ will be small. If all constellation symbols have the same probability, the output power of the soft mapper will be zero. With small output power the clipping does not change the time domain signal significantly, since almost all peaks are under the clipping level $A_{max}$. So the estimation of the clipping noise from (14) will be approximated as

$$\hat{D}_n \approx \tilde{X}_n - \alpha\tilde{X}_n = (1-\alpha)\tilde{X}_n, \qquad (20)$$

which can be interpreted as an additional noise, for which the remaining clipping noise will be larger than for the $0^{\text{th}}$ iteration (14): $P_D > P_d$. This effect causes the "minimum" in the EXIT function of the BNC detector. Therefore, a performance gain can be expected by setting dynamically the attenuation factor $\alpha$ according to the output of the soft mapper. The clipping ratio for the $k^{\text{th}}$ iteration is calculated as

$$\gamma_{new} = \frac{A_{max}}{\sqrt{P_{\tilde{x}}}}. \qquad (21)$$

The new attenuation factor can then be expressed according to (5) as

$$\alpha_{new} = 1 - e^{-\gamma_{new}^2} + \frac{\sqrt{\pi}}{2}\gamma_{new}\ \text{erfc}(\gamma_{new}). \qquad (22)$$

During the iterations, the new attenuation factor will decrease from the value 1 to the value $\alpha$ as the estimation becomes more and more accurate.

The simplest way to estimate the clipping noise is to create a lookup table for the remaining clipping noise power according to $\alpha_{new}$ as $P_D^{\alpha} = f(\alpha_{new})$. So, based on these assumptions (11) is modified as

$$p(\hat{Y}_n|c) = \exp\left(\frac{(\hat{Y}_n - \alpha H_n c)^2}{N_0 + P_{ch}P_D^{\alpha}}\right), \qquad (23)$$

The effect of these changes on the EXIT function are visualized in Fig. 4 for $E_b/N_0 = 4$ dB and $E_b/N_0 = 12$ dB. The "minimum" is fully eliminated and the EXIT function is monotonically growing with the input mutual information, so the BNC receiver will converge.

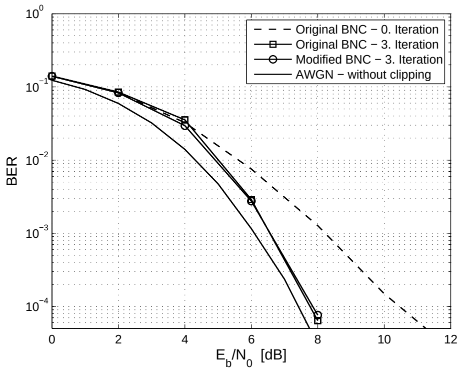


Fig. 5. BER performance of the original and the modified BNC with a code rate R=1/2

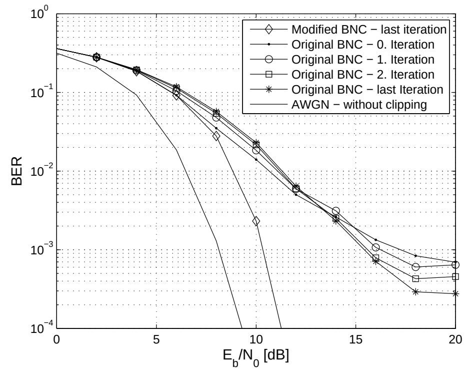


Fig. 6. BER performance of the original and the modified BNC with a code rate R=3/4

## IV. Simulation results

In [7] simulation are made with the following parameters: the binary data are encoded with a code rate 1/2, using a 4-state recursive systematic convolutional encoder with polynomials $(1, 5/7)_8$ in octal notation. The interleaved bits are mapped according to a 16-QAM constellation with Gray mapping, then OFDM modulated on 64 subcarriers and clipped with CR=1 dB. No cyclic prefix is used for the simulation with AWGN channel. Due to arithmetic overflow problems, Log-map decoder [10] is used instead of the BJCR decoder. It can be seen in Fig. 5 that both the original and modified receivers can suppress the clipping noise with these code parameters, and the difference is not significant. In comparison, if we use a punctured 3/4 rate code with the polynomials $(5, 7)_8$, the performance difference is noticeable. It is illustrated in Fig. 6 that the original BNC receiver does not converge any more under 14 dB, and over 14 dB only a small gain is visible over the subsequent iteration steps. On the other hand, the proposed modified BNC algorithm can suppress the the clipping noise, and the gain is clearly visible over 6 dB.

## V. Conclusions

The BNC iterative clipping mitigation method for OFDM signals was studied in detail. The convergence behavior of the BNC receiver was examined using the EXIT chart. The EXIT function of the BNC receiver was investigated and the reason of the divergence was explained. Based on the results of the EXIT functions, a small modification to the structure was proposed, with which the clipping noise for higher coding rates can also be suppressed, and over a certain $E_b/N_0$ value it can be fully eliminated.

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